An Analytical Model for the Basic Design Calculations of Journal Bearings

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This paper presents an analytical model for the basic design calculations of plain journal bearings. The model yields reasonable accuracy as compared with published numerical solutions under the same conditions. The principles and procedures of the formulations are presented along with accuracy analyses.

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1 Formulation of the Model

Figure 1 shows a schematic of a basic journal bearing in a steady-state configuration. The lubricant is supplied from the top region of the bearing, referred to as the inlet. The hydrodynamic action generates pressure in the lubricant, primarily in the convergent part of the journal-bearing gap, to counteract the load thereby separating the journal surface from the bearing surface with a thin lubricant film. The hydrodynamic pressure eventually terminates in the divergent part of the gap, referred to as the outlet, and the lubricant film ruptures into streamers. When a steady-state condition is reached, the journal is displaced from the bearing with a center distance e, which is referred to as the eccentricity. The ratio of the eccentricity and the radial clearance $\varepsilon = e/c$ is an important measure of the load capacity of the bearing. It also provides a measure of the thickness of the lubricant film separating the journal and the bearing.

The analytical model to be developed is divided into two parts: the load-capacity module and the lubricant-temperature module. The load-capacity module determines a value of the bearing eccentricity ratio for a given set of bearing geometry, operating condition, and lubricant viscosity. The temperature module calculates an effective temperature of the lubricant and lubricant viscosity at this temperature for given sizes, load, speed, and eccentricity ratio of the bearing. The two modules are then integrated to form the complete model that may be used to perform computer-aided bearing design calculations consistent with the use of the design tables of Ref. [1] or the design charts of Ref. [2].

1.1 The Load-Capacity Module. First-order analytical approximations have been developed for the bearing load capacity when the length-to-diameter ratio L/D of the bearing is either sufficiently small or sufficiently large [1]. The former is referred to as the short bearings and the latter, the long bearings. The approximations lead to the following equations [1]:

$$W = \frac{\omega\eta D L^3}{8c^2}\frac{\pi\varepsilon(0.62\varepsilon^2+1)^{1/2}}{(1-\varepsilon^2)^2} \quad \text{for short bearings} \tag{1}$$

and

$$W = \frac{3\omega\eta D^3 L}{4c^2}\frac{\varepsilon(4\varepsilon^2+\pi^2-\pi^2\varepsilon^2)^{1/2}}{(2+\varepsilon^2)(1-\varepsilon^2)} \quad \text{for long bearings} \tag{2}$$

Equations (1) and (2) may be used as primary anchor expressions to develop a load-capacity expression for finite bearings across short bearings to long bearings. A dimensionless load is first defined

$$\bar{W} = \frac{c^2}{\omega\eta D^4}W \tag{3}$$

Then, Eqs. (1) and (2) can be rewritten as

$$\bar{W} = \frac{\pi\varepsilon(0.62\varepsilon^2+1)^{1/2}}{8(1-\varepsilon^2)^2}\left(\frac{L}{D}\right)^3 \quad \text{for short bearings} \tag{4}$$

and

$$\bar{W} = \frac{3\varepsilon(4\varepsilon^2+\pi^2-\pi^2\varepsilon^2)^{1/2}}{4(2+\varepsilon^2)(1-\varepsilon^2)}\left(\frac{L}{D}\right) \quad \text{for long bearings} \tag{5}$$

Taking the logarithmic on both sides of Eqs. (4) and (5) yields

$$\log(\bar{W}) = \log\left(\frac{\pi\varepsilon(0.62\varepsilon^2+1)^{1/2}}{8(1-\varepsilon^2)^2}\right) + 3\log\left(\frac{L}{D}\right) \tag{6}$$

and

$$\log(\bar{W}) = \log\left(\frac{3\varepsilon(4\varepsilon^2+\pi^2-\pi^2\varepsilon^2)^{1/2}}{4(2+\varepsilon^2)(1-\varepsilon^2)}\right) + \log\left(\frac{L}{D}\right) \tag{7}$$

Defining

$$Y = \log(\bar{W}) \tag{8}$$

$$X = \log\left(\frac{L}{D}\right) \tag{9}$$

$$f_S(\varepsilon) = \log\left(\frac{\pi\varepsilon(0.62\varepsilon^2+1)^{1/2}}{8(1-\varepsilon^2)^2}\right) \tag{10}$$

$$f_L(\varepsilon) = \log\left(\frac{3\varepsilon(4\varepsilon^2+\pi^2-\pi^2\varepsilon^2)^{1/2}}{4(2+\varepsilon^2)(1-\varepsilon^2)}\right) \tag{11}$$

Equations (6) and (7) are rewritten as

$$Y = f_S(\varepsilon) + 3X \quad \text{for short bearings} \tag{12}$$

and

$$Y = f_L(\varepsilon) + X \quad \text{for long bearings} \tag{13}$$

Equations (12) and (13) characterize the load capacity of the short and long bearings by two families of straight lines in the X-Y plane. The slope of the lines is three for the short bearings, and it is one for the long bearings. For finite bearings of L/D ratios in between of sufficiently small and large, it is plausible to expect the load capacities of the bearings to be described by a family of smooth curves of the form $Y = f(\varepsilon, X)$. The curves make smooth transitions from the lines of Eq. (12) to the lines of Eq. (13) as the L/D ratio of the bearing is increased.

A third-order polynomial may be chosen to describe the load-capacity function for the finite bearings

$$Y = f(\varepsilon, X) = C_3X^3 + C_2X^2 + C_1X + C_o \tag{14}$$

Let X_S be the X location at which the load-capacity expression makes a transition from Eq. (12) to Eq. (14). A smooth transition is obtained by equating Y and $\partial Y/\partial X$ from Eq. (14) to those from Eq. (12) at X_S. Or

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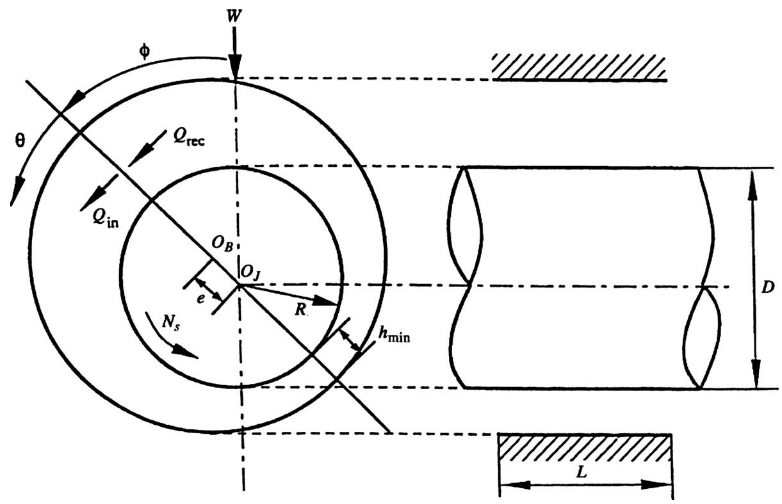

Fig. 1 Schematic of a basic journal bearing (taken from Fig. 8.5 of Ref. [1])

$$C_3X_S^3 + C_2X_S^2 + C_1X_S + C_o = f_S(\varepsilon) + 3X_S \tag{15}$$

and

$$3C_3X_S^2 + 2C_2X_S + C_1 = 3 \tag{16}$$

Similarly, a smooth transition from the finite bearing to the long-bearing results in

$$C_3X_L^3 + C_2X_L^2 + C_1X_L + C_o = f_L(\varepsilon) + X_L \tag{17}$$

and

$$3C_3X_L^2 + 2C_2X_L + C_1 = 1 \tag{18}$$

where X_L is the X location at which the load-capacity expression makes a transition from Eq. (14) to Eq. (13). The four coefficients of the polynomial of Eq. (14) can be obtained in terms of ε, X_S, and X_L by simultaneously solving Eqs. (15)–(18) to give

$$C_3 = \frac{2[f_S(\varepsilon) - f_L(\varepsilon) + X_L + X_S]}{(X_L - X_S)^3} \tag{19}$$

$$C_2 = \frac{-2 - 3C_3(X_L^2 - X_S^2)}{2(X_L - X_S)} \tag{20}$$

$$C_1 = -3C_3X_S^2 - 2C_2X_S + 3 \tag{21}$$

$$C_o = -C_3X_L^3 - C_2X_L^2 + (1 - C_1)X_L + f_L(\varepsilon) \tag{22}$$

The load-capacity model of Eq. (14) may be further defined and evaluated by comparing its calculations with results from the design tables of Ref. [1]. A given pair of L/D and ε corresponds to a Sommerfeld number S in the table entries. The Sommerfeld number is defined as [1]

$$S = \frac{\eta NDL}{W}\left(\frac{R}{c}\right)^2 \tag{23}$$

where N is the speed of the journal in rev/s. Thus, the Sommerfeld number is related to the dimensionless load of Eq. (3) by

$$\bar{W} = \frac{1}{8\pi}\frac{L}{D}\frac{1}{S} \tag{24}$$

Considering the load determined by the design tables to be the reference value, a percentage error may be defined for the load calculation using the analytical model of Eq. (14)

$$\bar{W}_{\mathrm{err}} = \left(\frac{\bar{W}_t - \bar{W}_m}{\bar{W}_t}\right) \times 100 \tag{25}$$

where $\bar{W}_t$ is the load determined from the table and $\bar{W}_m$, the load calculated by Eq. (14). Through a trial-and-error process, a "best" L/D ratio for the transition from the short-bearing model to the finite-bearing model is determined to be $L/D=1/8$ and that from the finite bearing to the long bearing to be $L/D=4.75$, giving $X_S=\log(1/8)$ and $X_L=\log(4.75)$ in Eqs. (19)–(22). Table 1 tabulates the percentage errors for practically the entire ranges of both L/D and ε of the design tables of Ref. [1]. For the bearings of small L/D ratios, the analytical model overpredicts the load capacity (i.e., negative errors). The error grows progressively to above 10% as ε or the bearing load is increased. That the error is relatively large at a high eccentricity ratio of the bearing is evidently due to the fact that the pressure variation in the circumferential direction of the bearing is no longer very small compared with its variation in the axial direction, which is a key assumption leading to the

Table 1 Percentage errors of the preliminary load-capacity module

$\varepsilon \backslash L/D$	1/8	1/6	1/4	1/3	1/2	3/4	1	1.5	2
0.9	−10.88	−13.68	−11.45	−7.48	−3.08	−0.16	0.75	2.11	3.36
0.8	−5.07	−5.94	−2.91	0.76	5.46	7.62	7.36	5.24	3.65
0.7	−3.07	−3.63	−1.55	1.18	5.12	7.32	7.12	4.79	2.28
0.6	−2.07	−2.65	−1.72	−0.04	2.84	4.88	5.08	3.12	0.46
0.5	−1.48	−2.16	−2.32	−1.68	0.05	1.8	2.3	1	−1.25
0.4	−1.1	−1.9	−3.02	−3.29	−2.67	−1.33	−0.63	−1.24	−2.95
0.3	−0.86	−1.76	−3.64	−4.69	−5.05	−4.19	−3.45	−3.48	−4.58
0.2	−0.71	−1.7	−4.15	−5.78	−6.96	−6.6	−5.92	−5.59	−6.08
0.1	−0.63	−1.69	−4.51	−6.54	−8.33	−8.48	−7.98	−7.43	−7.43

Table 2 Percentage errors of the load-capacity module

$\varepsilon \backslash L/D$	1/8	1/6	1/4	1/3	1/2	3/4	1	1.5	2
0.9	−2.54	−5.47	−4.62	−2.09	−0.29	0.02	−0.91	−1.97	−2.17
0.8	−2.99	−4.03	−1.64	1.46	4.89	5.83	4.67	1.27	−1.15
0.7	−2.64	−3.32	−1.57	0.94	4.09	5.6	4.87	1.78	−1.25
0.6	−2	−2.66	−1.88	−0.23	2.14	3.8	3.73	1.37	−1.57
0.5	−1.47	−2.2	−2.37	−1.59	−0.11	1.57	2.01	0.63	−1.68
0.4	−1.1	−1.91	−2.88	−2.84	−2.2	−0.63	0.23	−0.16	−1.72
0.3	−0.86	−1.74	−3.31	−3.86	−3.91	−2.49	−1.36	−0.87	−1.61
0.2	−0.71	−1.65	−3.62	−4.57	−5.12	−3.85	−2.54	−1.38	−1.33
0.1	−0.63	−1.6	−3.78	−4.94	−5.77	−4.64	−3.26	−1.56	−0.86

short-bearing load-capacity equation, Eq. (1). For the bearings of large L/D ratios, on the other hand, the model underpredicts the load at high ε and overpredicts at low ε with the errors reaching about 8%. The relatively large percentage error at low ε (i.e., light load) may be due to the fact that pressure variation in the axial direction of the bearing may not be negligibly small compared with pressure variation in the circumferential direction, which is a key assumption leading to the long-bearing load-capacity equation, Eq. (2). The other source of errors is the assumption that the pressure is terminated at the end of the convergent half of the journal-bearing gap.

Based on the error trends with respect to ε at small and large L/D ratios, some offset refinements in $f_S(\varepsilon)$ and $f_L(\varepsilon)$ may be made to the short-bearing and long-bearing load-capacity equations, Eqs. (12) and (13), on which the finite-bearing load-capacity model is anchored. Through a trial-and-error process, $f_S(\varepsilon)$ and $f_L(\varepsilon)$ of Eqs. (10) and (11) are best refined to

$$f_S(\varepsilon) = \log\left([1.0 - 0.7(\varepsilon - 0.1)^{10}]\frac{\pi\varepsilon(0.62\varepsilon^2 + 1)^{1/2}}{8(1-\varepsilon^2)^2}\right) \quad (26)$$

and

$$f_L(\varepsilon) = \log\left((0.91 + 0.19\varepsilon)\frac{3\varepsilon(4\varepsilon^2 + \pi^2 - \pi^2\varepsilon^2)^{1/2}}{4(2+\varepsilon^2)(1-\varepsilon^2)}\right) \quad (27)$$

Table 2 tabulates the percentage errors again after the refinements. Compared with Table 1, the errors are significantly reduced to largely within 5% throughout the $L/D \sim \varepsilon$ domain of the design tables of Ref. [1]. In addition, the trends of the errors suggest that the errors would not be appreciably increased if the domain is modestly extended, particularly to a larger L/D ratio of the bearing. This refined model is taken to be the load-capacity module. It is given by Eqs. (3), (8), (9), (14), (19)–(22), (26), and (27), in which

$$X_S = \log(1/8) \quad (28)$$

and

$$X_L = \log(4.75) \quad (29)$$

1.2 The Lubricant-Temperature Module. The viscosity of the lubricant used in the load-capacity module is the viscosity determined at an effective temperature of the lubricant in the bearing. Raimondi and Boyd [2] took this effective temperature to be the average temperature of the lubricant at the bearing inlet and outlet

$$T_{\text{eff}} = \frac{T_i + T_o}{2} = T_i + \frac{\Delta T}{2} \quad (30)$$

where $\Delta T = T_o - T_i$ is the temperature rise of the lubricant from the inlet to the outlet. While the inlet temperature of the lubricant may be meaningfully estimated for a particular bearing design arrangement, it is beyond the scope of this paper and T_i is taken to be an input parameter. In obtaining the design solutions, Raimondi and Boyd [2] also assumed that the heat generated in the bearing is primarily carried out by the flow of the lubricant to give

$$\rho c_h Q_{\text{avg}} \Delta T = fWU \quad (31)$$

where ρ and c_h are the density and specific heat of the lubricant and f is the coefficient of friction of the bearing (i.e., the ratio of the drag force around the journal and the applied load). The variable Q_{avg} in Eq. (31) represents the effective rate of lubricant flow leaving the bearing at the average temperature. It is approximated by [2,3]

$$Q_{\text{avg}} = Q_i - \frac{Q_L}{2} \quad (32)$$

where Q_i is the rate of lubricant flow into the bearing and Q_L is the rate of the hydrodynamic-pressure-driven leakage flow through the axial ends of the bearing. Equation (31) can then be rearranged to

$$\Delta T = \frac{fW2\pi RN}{\rho c_h(Q_i - 0.5Q_L)} \quad (33)$$

Define, as in Refs. [1,3], the dimensionless inlet-flow rate and leakage-flow rate:

$$Q_i = \frac{\pi NDLc}{2}\bar{Q}_i \quad (34)$$

and

$$Q_L = \frac{N\pi DLc}{2}\bar{Q}_L \quad (35)$$

Then, Eq. (33) can be expressed as

$$\Delta T = \frac{4W}{\rho c_h LD}\frac{(R/c)f}{(\bar{Q}_i - 0.5\bar{Q}_L)} \quad (36)$$

The values of the three dimensionless variables, $(R/c)f$, $\bar{Q}_i$, and $\bar{Q}_L$, are tabulated in the design tables published in Refs. [1,3] at a pair of given values of L/D and ε of the bearing. A value of ΔT can then be obtained using Eq. (36) and subsequently the effective temperature of the lubricant and the corresponding viscosity. A solution to the design problem is obtained using the design tables through manual iterations and interpolations until agreement is reached among all the variables.

The temperature module to be developed in this paper aims to produce analytical expressions for $(R/c)f$, $\bar{Q}_i$, and $\bar{Q}_L$ as functions of L/D and ε, making use of basic theories developed for the friction and lubricant flows in the journal bearings. The design tables are used to refine these expressions and to ensure their accuracy. First, the expression for the friction variable $(R/c)f$ is developed. A simple approximation of the friction force in a journal bearing is the Petroff equation [4]

Table 3 Percentage errors of the preliminary friction equation

$\varepsilon \backslash L/D$	1/8	1/6	1/4	1/3	1/2	3/4	1	1.5	2
0.9	57.06	57.51	58.47	59.58	61.31	63.38	64.69	66.4	67.36
0.8	40.4	40.69	41.42	42.31	44.08	46.52	48.38	50.86	52.42
0.7	28.85	29.04	29.55	30.18	31.65	33.93	35.83	38.71	40.54
0.6	20.17	20.3	20.66	21.12	22.23	24.07	25.81	28.57	30.33
0.5	13.51	13.6	13.83	14.13	14.91	16.3	17.68	20.01	21.65
0.4	8.42	8.47	8.62	8.81	9.31	10.22	11.22	12.97	14.28
0.3	4.64	4.67	4.75	4.86	5.14	5.68	6.26	7.39	8.25
0.2	2.04	2.05	2.09	2.13	2.26	2.51	2.78	3.31	3.77
0.1	0.51	0.51	0.52	0.53	0.56	0.63	0.7	0.85	0.97

$$F = 2\pi\eta URL/c \tag{37}$$

Substituting $f=F/W$ and with the use of Eq. (23) yields, after rearrangements, the following expression:

$$(R/c)f = 2\pi^2 S \tag{38}$$

The accuracy of this expression for $(R/c)f$ may be evaluated by comparing its calculations with the numerical solutions in the design tables. Define a percentage-error function for $(R/c)f$

$$f_{\text{err}} = \left(\frac{((R/c)f)_t - ((R/c)f)_m}{((R/c)f)_t} \right) \times 100 \tag{39}$$

where $((R/c)f)_t$ is the value of $(R/c)f$ from the design tables at a given pair of L/D and ε, while $((R/c)f)_m$ is the value by Eq. (38) using the value of S from the design tables under the same pair of L/D and ε. Table 3 shows the errors for practically the entire ranges of L/D and ε of the design tables of Ref. [1]. The errors are fairly small at relatively low ε or light loading conditions throughout the tabulated L/D ratios. However, the errors become very large at high values of ε, where Eq. (38) underpredicts the friction by over 60%. That the errors are large at high ε is largely attributed to the fact that the Petroff equation is derived assuming $\varepsilon=0$. The trends of the errors appear to be relatively simple. The errors grow progressively along ε and modestly along L/D, suggesting that they may be largely reduced with a simple correction factor to Eq. (38) along L/D and ε. Through a trial-and-error process, a best correction is obtained to refine Eq. (38) to

$$(R/c)f = \left[1.0 + \left(0.56\frac{L}{D} + 1.93 \right) \varepsilon^4 \right] 2\pi^2 S \tag{40}$$

The errors of this refined equation are shown in Table 4. They are much reduced to within 5% at most $L/D \sim \varepsilon$ locations, with a few exceptions where the errors reach 8%. Since the pattern of the errors is no longer simple, it is difficult to devise simple corrections to further reduce the errors. Equation (40) is taken to be the friction equation for the temperature module.

The expression for the inflow variable $\bar{Q}_i$ is to be developed next. The inflow of the lubricant may be assumed to be primarily Couette. Then a first-order approximation for the inflow is given by [5]

$$Q_i = \frac{\pi NDLc}{2}(1+\varepsilon) \tag{41}$$

Comparing Eq. (41) with Eq. (34) leads to the following expression for the dimensionless inflow

$$\bar{Q}_i = 1 + \varepsilon \tag{42}$$

This inflow expression may be evaluated and refined by studying the percentage errors in the $L/D \sim \varepsilon$ domain of the bearing, as defined below

Table 4 Percentage errors of the friction equation

$\varepsilon \backslash L/D$	1/8	1/6	1/4	1/3	1/2	3/4	1	1.5	2
0.9	0.72	1.09	2.06	3.44	5.22	6.91	7.02	5.33	2.04
0.8	−8.43	−8.46	−8.25	−7.7	−6.54	−4.96	−4.26	−4.9	−7.01
0.7	−5.32	−5.43	−5.46	−5.29	−4.62	−3.35	−2.53	−2.05	−3.01
0.6	−0.52	−0.59	−0.62	−0.52	−0.04	0.94	1.87	2.93	2.79
0.5	2.7	2.67	2.69	2.78	3.15	4.01	4.86	6.16	6.71
0.4	3.73	3.73	3.77	3.87	4.17	4.82	5.57	6.8	7.58
0.3	3.1	3.11	3.16	3.23	3.44	3.89	4.37	5.31	5.99
0.2	1.72	1.73	1.76	1.8	1.91	2.14	2.39	2.88	3.3
0.1	0.49	0.49	0.5	0.51	0.54	0.61	0.68	0.82	0.94

Table 5 Percentage errors of the preliminary inflow equation

$\varepsilon \backslash L/D$	1/8	1/6	1/4	1/3	1/2	3/4	1	1.5	2
0.9	−0.25	−0.45	−1.04	−1.94	−4.91	−13.2	−25.83	−60.03	−98.97
0.8	−0.23	−0.42	−0.97	−1.8	−4.48	−11.67	−22.47	−50.5	−81.52
0.7	−0.22	−0.39	−0.9	−1.64	−4.03	−10.19	−19.18	−42.05	−66
0.6	−0.19	−0.35	−0.81	−1.48	−3.57	−8.74	−16.07	−34.05	−52.38
0.5	−0.14	−0.31	−0.71	−1.3	−3.08	−7.3	−13.11	−26.81	−40.29
0.4	−0.15	−0.26	−0.61	−1.1	−2.55	−5.88	−10.27	−20.29	−29.8
0.3	−0.12	−0.22	−0.49	−0.87	−1.98	−4.44	−7.55	−14.4	−20.68
0.2	−0.08	−0.15	−0.35	−0.62	−1.38	−3	−4.95	−9.08	−12.75
0.1	−0.05	−0.08	−0.19	−0.33	−0.72	−1.51	−2.43	−4.3	−5.89

Table 6 Percentage errors of the inflow equation

$\varepsilon\backslash L/D$	1/8	1/6	1/4	1/3	1/2	3/4	1	1.5	2
0.9	0.04	0.5	1.49	2.39	3.62	3.27	1.23	−1.56	5.91
0.8	0.03	0.43	1.29	2.07	3.1	2.85	1.05	−1.37	3.96
0.7	0.01	0.36	1.09	1.75	2.62	2.43	0.98	−1.21	2.85
0.6	0	0.3	0.9	1.45	2.16	2.02	0.91	−0.73	2.26
0.5	0.03	0.23	0.74	1.17	1.73	1.65	0.85	−0.22	2.14
0.4	−0.01	0.18	0.57	0.91	1.34	1.32	0.81	0.25	2.17
0.3	−0.02	0.13	0.42	0.68	1	1.04	0.79	0.68	2.26
0.2	−0.01	0.09	0.29	0.47	0.72	0.81	0.78	1.05	2.35
0.1	0	0.06	0.18	0.3	0.48	0.67	0.82	1.33	2.34

$$\bar{Q}_i^{\mathrm{err}} = \left(\frac{\bar{Q}_i^{\,t} - \bar{Q}_i^{\,m}}{\bar{Q}_i^{\,t}}\right) \times 100 \tag{43}$$

where $\bar{Q}_i^{\,t}$ is the value of $\bar{Q}_i$ from the design tables of Ref. [1] at a given pair of L/D and ε and $\bar{Q}_i^{\,m}$ is calculated by Eq. (42). Table 5 shows the errors in the $L/D \sim \varepsilon$ domain of the tables. The errors are very small for low L/D ratios and still fairly small for large L/D under light loading conditions. These small errors are evidently due to the relatively small pressure gradient along the circumferential direction of the bearing, making the Poiseuille flow quite negligible at the inlet. On the other hand, the errors are big at relatively high L/D ratios and under heavy loading conditions, reaching 100% at the extreme. This is the situation at which the Poiseuille flow at the inlet is no longer small compared with the Couette flow. As a result, the actual inflow is much smaller than that predicted by Eq. (42). Nevertheless the variations in the errors in the $L/D \sim \varepsilon$ domain are of a relatively simple pattern, making it possible to refine Eq. (42) with a relatively simple correction factor along L/D and ε. A trial-and-error procedure leads to the following refined expression:

$$\bar{Q}_i = \left[1.0 - (0.26\varepsilon + 0.01)\left(\frac{L}{D} - 0.1\right)^{1.2}\right](1+\varepsilon) \quad L/D \geq 1/8 \tag{44}$$

No correction is made for a L/D ratio less than 1/8 and the original expression of Eq. (42) is used. The errors of this refined equation are shown in Table 6. They are sufficiently small and Eq. (44) is taken to be the inflow equation for the temperature module.

Finally, the expression for the leakage variable $\bar{Q}_L$ is developed. The hydrodynamic-pressure-driven leakage flow through the axial ends of the journal bearing may be taken to be the difference between the inflow at the inlet and the outflow at the outlet of the bearing loaded region $Q_L = Q_i - Q_o$. A first-order approximation of the outflow is the Couette flow at the location of the minimum film thickness [5] to give

$$Q_o = \frac{\pi NDLc}{2}(1-\varepsilon) \tag{45}$$

Therefore, a simple expression for $\bar{Q}_L$, making use of Eqs. (35), (41), and (45), is given by

Table 7 Percentage errors of the preliminary leakage equation

$\varepsilon\backslash L/D$	1/8	1/6	1/4	1/3	1/2	3/4	1	1.5	2
0.9	−0.4	−0.72	−1.5	−2.7	−6.16	−15.44	−29.83	−69.67	−118.2
0.8	−0.4	−0.72	−1.57	−2.81	−6.38	−15.48	−29.27	−67.52	−114.3
0.7	−0.41	−0.73	−1.6	−2.83	−6.5	−15.45	−28.71	−65.27	−110.3
0.6	−0.41	−0.73	−1.62	−2.86	−6.57	−15.35	−28.1	−63.04	−106.2
0.5	−0.41	−0.74	−1.63	−2.89	−6.55	−15.19	−27.44	−60.75	−102
0.4	−0.41	−0.74	−1.64	−2.89	−6.57	−14.98	−26.74	−58.38	−97.58
0.3	−0.42	−0.74	−1.63	−2.9	−6.53	−14.72	−25.97	−55.93	−93.05
0.2	−0.43	−0.76	−1.63	−2.88	−6.47	−14.42	−25.16	−53.43	−88.41
0.1	−0.4	−0.76	−1.63	−2.88	−6.38	−14.09	−24.38	−50.94	−83.82

Table 8 Percentage errors of the leakage equation

$\varepsilon\backslash L/D$	1/8	1/6	1/4	1/3	1/2	3/4	1	1.5	2
0.9	0.08	0.69	1.96	2.98	4.49	4.32	1.96	−2.11	3.36
0.8	0.07	0.66	1.83	2.78	4.11	3.93	1.81	−2.03	2.93
0.7	0.05	0.63	1.74	2.66	3.8	3.59	1.66	−1.86	2.6
0.6	0.04	0.6	1.66	2.52	3.54	3.32	1.56	−1.66	2.42
0.5	0.03	0.57	1.59	2.39	3.36	3.09	1.5	−1.4	2.37
0.4	0.02	0.54	1.51	2.29	3.16	2.92	1.48	−1.05	2.49
0.3	0.01	0.52	1.46	2.18	2.99	2.78	1.51	−0.61	2.77
0.2	−0.01	0.47	1.4	2.09	2.86	2.68	1.59	−0.12	3.2
0.1	0.01	0.45	1.34	1.99	2.74	2.6	1.65	0.42	3.69

$$\bar{Q}_L = 2\varepsilon \tag{46}$$

Defining the percentage error the same as that for the inflow defined in Eq. (43), Eq. (46) is evaluated and refined. The errors are shown in Table 7. The errors are uniformly small at small L/D ratios but uniformly progress to very large values at large L/D ratios. A refinement of the expression through the use of a relatively simple correction factor is in order, leading to the following refined expression:

$$\bar{Q}_L = \left[1.0 - (0.05\varepsilon + 0.23)\left(\frac{L}{D} - 0.1\right)^{1.1}\right](2\varepsilon) \quad L/D \geq 1/8 \tag{47}$$

As in the case of inflow, no correction is made for a L/D less than 1/8 and Eq. (46) is used. Table 8 shows the errors of Eq. (47); the maximum is below 5%. Equation (47) is taken to be the leakage-flow equation for the temperature module.

1.3 Integration of the Model. Two analytical modules have been developed for the model of the bearing. The load-capacity module is used to perform design calculations such as the film thickness in the bearing along with other load-capacity related variables. The lubricant-temperature module is used to determine the average temperature of the lubricant and its effective viscosity. This module also yields values for other important bearing variables such as friction, power loss and lubricant flows. The two modules are interrelated. The load-capacity module needs the viscosity solution from the temperature module, and the temperature module needs the eccentricity-ratio solution from the load-capacity module. Therefore, a calculation procedure needs to be developed to integrate the two modules forming a complete analytical model. The equations for the two modules are summarized below followed by the descriptions of the model integration.

The load-capacity module is given by

$$\bar{W} = \frac{c^2}{\omega \eta D^4} W \tag{3'}$$

$$Y = \log(\bar{W}) \tag{8'}$$

$$X = \log\left(\frac{L}{D}\right) \tag{9'}$$

$$Y = C_3X^3 + C_2X^2 + C_1X + C_o \tag{14'}$$

$$C_3 = \frac{2[f_S(\varepsilon) - f_L(\varepsilon) + X_L + X_S]}{(X_L - X_S)^3} \tag{19'}$$

$$C_2 = \frac{-2 - 3C_3({X_L}^2 - {X_S}^2)}{2(X_L - X_S)} \tag{20'}$$

$$C_1 = -3C_3{X_S}^2 - 2C_2X_S + 3 \tag{21'}$$

$$C_o = -C_3{X_L}^3 - C_2{X_L}^2 + (1 - C_1)X_L + f_L(\varepsilon) \tag{22'}$$

$$f_S(\varepsilon) = \log\left([1.0 - 0.7(\varepsilon - 0.1)^{10}]\frac{\pi\varepsilon(0.62\varepsilon^2 + 1)^{1/2}}{8(1 - \varepsilon^2)^2}\right) \tag{26'}$$

$$f_L(\varepsilon) = \log\left((0.91 + 0.19\varepsilon)\frac{3\varepsilon(4\varepsilon^2 + \pi^2 - \pi^2\varepsilon^2)^{1/2}}{4(2 + \varepsilon^2)(1 - \varepsilon^2)}\right) \tag{27'}$$

$$X_S = \log(1/8) \tag{28'}$$

$$X_L = \log(4.75) \tag{29'}$$

The nominal domain of application of the module is $1/8 \leq L/D \leq 2.0$ and $0 < \varepsilon \leq 0.9$. It may be modestly extended beyond, especially for L/D ratios larger than 2.0. If calculations are needed for a L/D ratio less than 1/8, the short-bearing model of Eq. (12) should be used instead of Eq. (14).

The lubricant-temperature module is given by

$$S = \frac{\eta NDL}{W}\left(\frac{R}{c}\right)^2 \tag{23'}$$

$$T_{\text{eff}} = T_i + \frac{\Delta T}{2} \tag{30'}$$

$$\Delta T = \frac{4W}{\rho c_h LD}\frac{(R/c)f}{(\bar{Q}_i - 0.5\bar{Q}_L)} \tag{36'}$$

$$(R/c)f = \left[1.0 + \left(0.56\frac{L}{D} + 1.93\right)\varepsilon^4\right]2\pi^2 S \tag{40'}$$

$$\bar{Q}_i = \left[1.0 - (0.26\varepsilon + 0.01)\left(\frac{L}{D} - 0.1\right)^{1.2}\right](1 + \varepsilon) \quad L/D \geq 1/8 \tag{44'}$$

$$\bar{Q}_L = \left[1.0 - (0.05\varepsilon + 0.23)\left(\frac{L}{D} - 0.1\right)^{1.1}\right](2\varepsilon) \quad L/D \geq 1/8 \tag{47'}$$

and also

$$\eta = f(T_{\text{eff}}) \tag{48}$$

where $f(T_{\text{eff}})$ is a plausible viscosity-temperature relation of the lubricant such as the Barus law [1]

$$\eta = \eta_i e^{-\beta(T_{\text{eff}} - T_i)} \tag{49}$$

where η_i is the lubricant viscosity at the inlet temperature and β is the viscosity-temperature coefficient. The nominal domain of application of the temperature module is the same as the load-capacity module with modest extensions. If calculations are needed for a L/D ratio less than 1/8, Eq. (44) is replaced by Eq. (42) and Eq. (47) replaced by Eq. (46).

Figure 2 shows a flowchart highlighting the integration of the model. The input parameters of the model include the length, diameter, radial clearance, load, and speed of the bearing. They also include the density and specific heat of the lubricant, the inlet temperature and viscosity of the lubricant, and an initial estimate of the effective temperature of the lubricant in the bearing. The initial estimate of the effective temperature may be taken to be the inlet temperature. A value of the viscosity is calculated at this effective temperature of the lubricant. Then, the load-capacity module is called upon to iteratively determine a value of the eccentricity ratio. The bisection method of calculation [6] is very effective to determine ε. A residual function $r(\varepsilon)$ is defined by moving all terms of Eq. (14) to one side of the equation. This function is equal to zero when a solution for ε satisfying Eq. (14) is found. Select two initial ε values, $\varepsilon_l = \Delta\varepsilon$ and $\varepsilon_h = 1 - \Delta\varepsilon$, where $\Delta\varepsilon$ is the prescribed error tolerance for ε in search of a solution. Calculate $r(\varepsilon)$ with both ε_l and ε_h. If the product of $r(\varepsilon_l)$ and $r(\varepsilon_h)$ is negative (i.e., $r(\varepsilon_l)^* r(\varepsilon_h) < 0$), the solution for ε lies in the range of ε_l and ε_h. This range of ε is bisected, and the value of either ε_l or ε_h is updated depending on which half of the range contains the solution. The updating process continues to narrow the range of ε_l to ε_h until $\varepsilon_h - \varepsilon_l \leq \Delta\varepsilon$, and the solution is assigned to ε_h with an error less than $\Delta\varepsilon$. The ε solution may also theoretically fall below $\Delta\varepsilon$ or above $1 - \Delta\varepsilon$, corresponding, respectively, to a large or a small Sommerfeld number. The occurrence of either situation is detected when $r(\varepsilon_l)^* r(\varepsilon_h) > 0$ to begin with. The Sommerfeld number is then calculated. If $S > 0.5$, based on the design tables [1], the ε solution is assigned to $\Delta\varepsilon$; otherwise, it is assigned to $1 - \Delta\varepsilon$ with an error less than $\Delta\varepsilon$.

With a solution of ε determined by the load-capacity module,

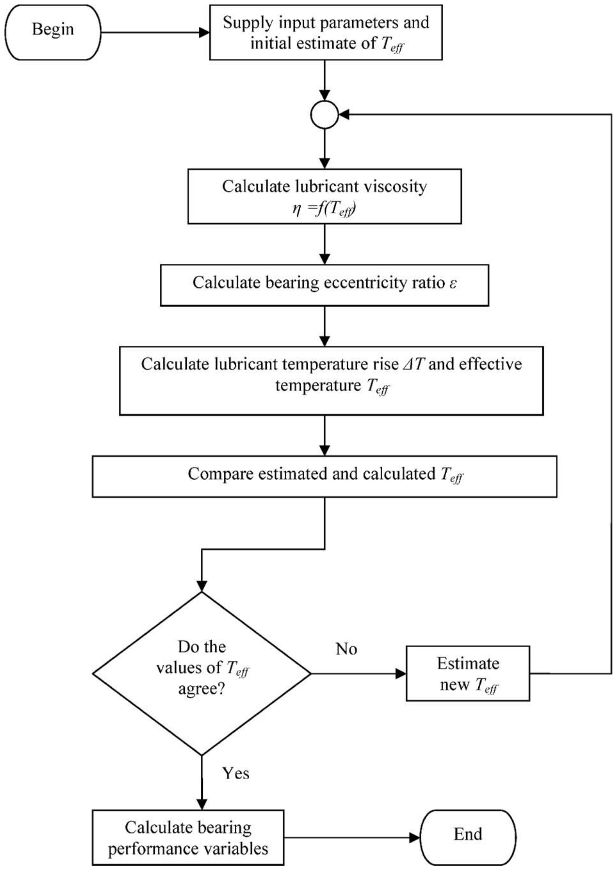

Fig. 2 Flowchart of model integration

the lubricant-temperature module is called upon to calculate an effective temperature of the lubricant. A Sommerfeld number is first calculated followed by the calculations of $(R/c)f$, $\bar{Q}_i$, and $\bar{Q}_L$. A temperature rise is determined and then an effective temperature. This effective temperature is compared with that estimated previously. If the difference between the two values of the effective temperature is greater in magnitude than the prescribed error tolerance, the average of the two is taken to be the new estimate of the effective temperature. Then the viscosity of the lubricant is updated and the iterative calculations are repeated until convergence of the effective temperature. Upon convergence of the solution, variables of design interest are calculated. They include the minimum film thickness $(h_{\min}=c(1-\varepsilon))$, coefficient of friction, power loss $(P_{\text{loss}}=fW\pi ND)$, and rate of leakage flow, to name just a few. Other variables such as the peak pressure and bearing attitude angles, for which no analytical expressions are developed, may be obtained from the design tables of Ref. [1] under the values of L/D and ε.

2 Conclusion

An analytical model with reasonable accuracy is developed in this paper for the basic calculations of journal bearings in the practical design space. The model is composed of two modules, the load-capacity module and the lubricant-temperature module. The load-capacity module calculates the bearing eccentricity ratio and film thickness along with other load-capacity variables. The lubricant-temperature module determines the average temperature of the lubricant and its effective viscosity along with other bearing variables such as lubricant flow, bearing friction, and power loss. Analytical equations in each of the two modules are derived making use of simple theories of journal-bearing lubrication and heat transfer in conjunction with published design tables of numerical solutions. The modules are integrated into one cohesive analytical model. The analytical equations of the model can be easily programmed into a computer to yield continuous solutions in the design space of length-to-diameter ratio of the bearing and the bearing eccentricity ratio. The model effectively replaces the discrete design charts and tables and eliminates the need for manual iterations and interpolations in the solution process. It helps reduce human errors in design calculations and facilitate optimal-design and design-sensitivity analyses. Furthermore, the analytical model may be integrated into a computer-aided-design environment for more advanced design and analysis of journal-bearing systems, and its functional cause-and-effect relations may help gain more insights into the bearing problems.

Nomenclature

c = bearing radial clearance
c_h = specific heat of lubricant
D = bearing diameter
f = coefficient of friction
L = bearing length
N = journal rotational speed in rev/s
Q_{ave} = average flow rate of lubricant through bearing
Q_i = rate of flow of lubricant at bearing inlet
Q_L = rate of leakage flow of lubricant through bearing axial ends
$\bar{Q}_i$ = dimensionless inlet-flow rate (Eq. (34))
$\bar{Q}_L$ = dimensionless leakage-flow rate (Eq. (35))
R = bearing radius
S = Sommerfeld number
T_{eff} = lubricant effective temperature
U = surface velocity of journal
W = bearing radial load
$\bar{W}$ = dimensionless load (Eq. (3))
ΔT = lubricant-temperature rise
ε = bearing eccentricity ratio
η = lubricant viscosity at an effective temperature
ρ = lubricant density
ω = journal angular velocity in rad/s

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