SURFACE TENSION

In this lesson, we will:

- Define and describe **surface tension** and **contact angle**
- Do some example problems that involve surface tension, namely, bubbles and capillary rise in a tube

Surface Tension

- The ***coefficient of surface tension*** (or simply ***surface tension***) σ_s is a measure of the force per unit length required to stretch the surface of a liquid.

$$\{\sigma_s\} = \left\{\frac{F}{L}\right\} \quad [\sigma_s] = \left[\frac{N}{m}\right]$$

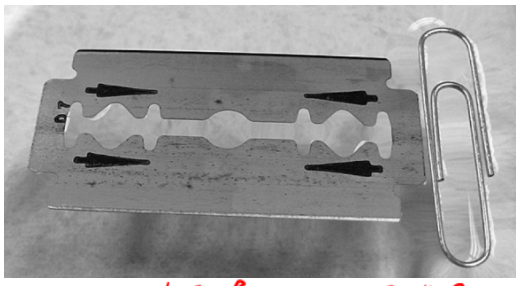
- Alternatively, surface tension can also be thought of as the surface energy (or work) per unit area required to stretch the liquid surface.

$$\{\sigma_s\} = \left\{\frac{F \cdot L}{L^2}\right\} = \left\{\frac{F}{L}\right\}$$

- Surface tension always acts ***parallel*** to the liquid surface.
- Some interesting consequences of surface tension:
  - A liquid when sprayed breaks into small spherical droplets (photo from https://www.pesticidewise.com).

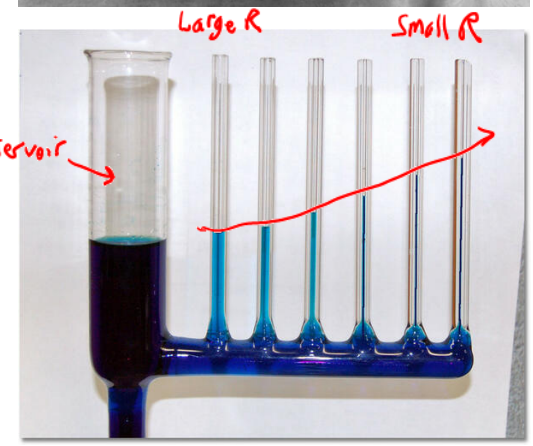
$P_{in} > P_{out}$

  - The surface acts like a stretched film in tension; thus, objects heavier than the liquid can float on the surface (photo by the author).



$\rho_{object} > \rho_{water}$ yet they float!

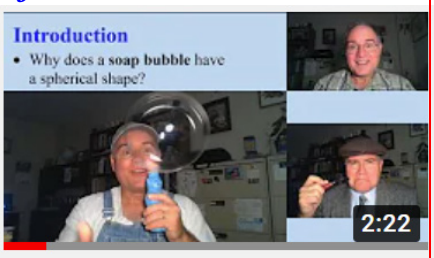
  - Liquids rise (or fall) vertically in small-diameter capillary tubes (photo from https://www.usgs.gov).



Notice that capillary rise height ***increases*** as tube diameter ***decreases***.

The top surface of the liquid is not flat, but rounded into what is known as a ***meniscus***.

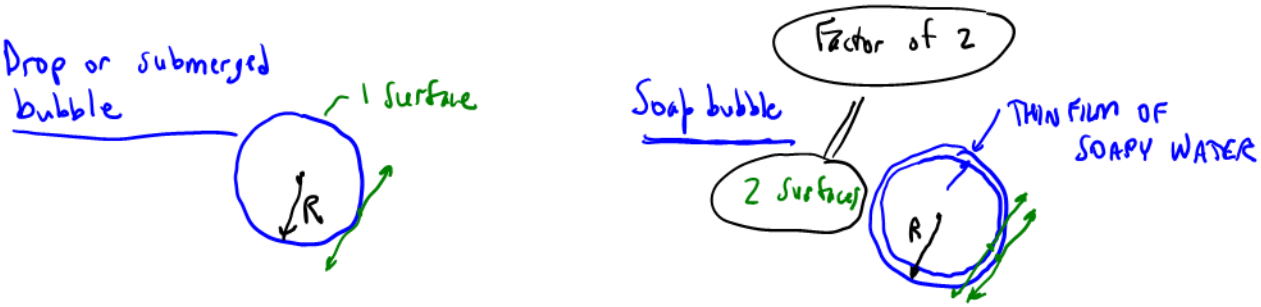
See my short YouTube video called "***Surface Tension: It's a Bit of a Stretch!***" for more about surface tension, including how to measure it and how to analyze soap bubbles. https://youtu.be/50W95-a6DVM



TMFM: Surface Tension: It's a Bit of a Stretch!

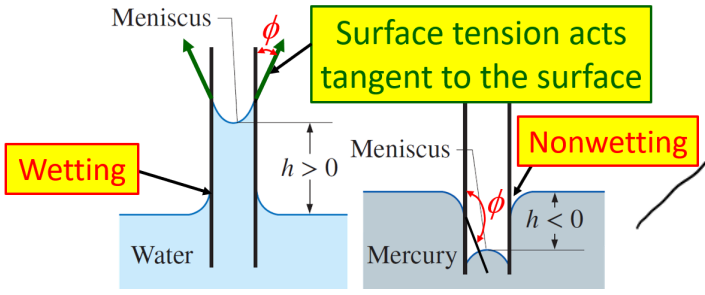
From the video:

- U-frame measurement rig: $F = 2b\sigma_s$
- Spherical **droplet**: $\Delta P_{\text{bubble}} = P_{\text{inside}} - P_{\text{outside}} = 2\dfrac{\sigma_s}{R}$
- Spherical **soap bubble**: $\Delta P_{\text{bubble}} = P_{\text{inside}} - P_{\text{outside}} = 4\dfrac{\sigma_s}{R}$



Contact Angle

- ***Contact angle*** ϕ is the angle between the tangents to the liquid and solid surfaces at the point of contact.
  - For a ***wetting fluid*** $\phi < 90^\circ$ and capillary liquid rises
  - For a ***nonwetting fluid*** $\phi > 90^\circ$ and capillary liquid falls

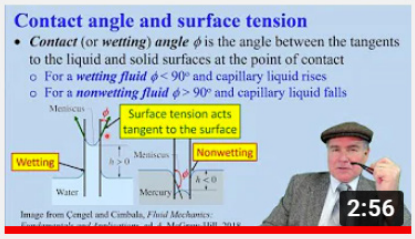


- Capillary rise height = h
- Water in a glass tube
  - Pure water: $\phi \simeq 0^\circ$
  - Tap water: $\phi \approx 10^\circ$-20°
- Mercury in a glass tube
  - $\phi \simeq 135^\circ > 90^\circ$

See my short YouTube video called "***Calculation of Capillary Rise in a Tube***" for capillary rise height calculation. https://youtu.be/x195xKWobhg

From the video:

- Capillary rise height: $h = \dfrac{2\sigma_s}{\rho g R}\cos\phi$



Calculation of Capillary Rise in a Tube

- The table to the right lists surface tension of some common liquid surfaces exposed to air.

$\sigma_{s\ \text{soapy water}} < \sigma_{s\ \text{water}}$

Surface tension of some fluids in air at 1 atm and 20°C (unless otherwise stated)

Fluid	Surface Tension σ_s, N/m*
†Water:	
0°C	0.076
20°C	0.073
100°C	0.059
300°C	0.014
Glycerin	0.063
SAE 30 oil	0.035
Mercury	0.440
Ethyl alcohol	0.023
Blood, 37°C	0.058
Gasoline	0.022
Ammonia	0.021
Soap solution	0.025
Kerosene	0.028

Example: Capillary Rise in a Tube

Given: A tube of radius 1.2 mm is inserted through the surface of pure water at 20°C and one atmosphere.

To do: Calculate the capillary rise height of this water in the tube.

Solution: Table: $\sigma_s = 0.073 \frac{N}{m}$

Should be R → ~~X~~ = 0.0012 m

Contact angle → assume tap water $\phi \approx 10^\circ$

@ 20°C look up $\rho = 998.0 \frac{kg}{m^3}$

Equation for capillary rise height: $h = \frac{2\sigma_s}{\rho g R}\cos\phi$

$$h = \frac{2(0.073\ N/m)}{(998.0\ kg/m^3)(9.807\ m/s^2)(0.0012\ m)}\cos(10^\circ)\left(\frac{kg \cdot m}{s^2 \cdot N}\right) = 0.0122\ m$$

$h \approx 1.2$ cm

See my short YouTube video called "***Why Do Coffee Rings Form?***" for another interesting and ubiquitous consequence of surface tension.
https://youtu.be/gwzHnNqBfAw

TMFM: Why Do Coffee Rings Form?