BAROMETERS

In this lesson, we will:

- Learn about a common application of fluid statics – the ***liquid barometer***
- Develop a relationship between the variables associated with a liquid barometer
- Do an example problem for a ***mercury barometer***

Quick Review of Hydrostatics

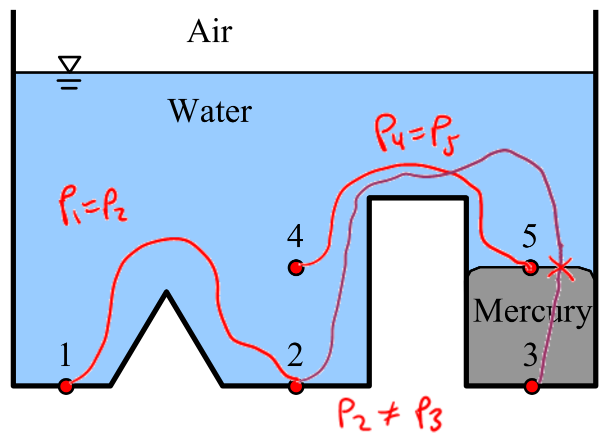
- The general equation for fluid statics is $\boxed{\frac{dP}{dz} = -\rho g}$.
- For an incompressible liquid, the above equation reduces to our workhorse equation for hydrostatics, $\boxed{P_{\text{below}} = P_{\text{above}} + \rho g |\Delta z|}$.

Rules for Hydrostatics

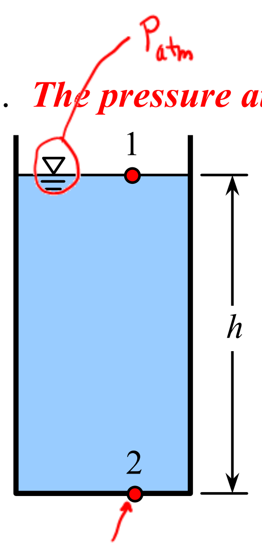
Several "rules" result from the above equation of hydrostatics:

1. ***If you can draw a continuous curve through the* same *fluid from point* 1 *to point* 2, *then $P_1 = P_2$ if $z_1 = z_2$.***

   E.g., consider the oddly shaped container in the sketch. By this rule, $P_1 = P_2$ and $P_4 = P_5$ since these points are at the same elevation in the same fluid. However, P_2 does not equal P_3 even though they are at the same elevation, because one cannot draw a line connecting these points through the *same* fluid. In fact, P_2 is *less than* P_3 since mercury is denser than water.



2. ***The pressure at any free surface open to the atmosphere is atmospheric pressure, P_{atm}.***



This rule holds not only for hydrostatics, by the way, but for *any* free surface exposed to the atmosphere, whether that surface is moving, stationary, flat, or curved. Consider a container of water at rest. The little upside-down triangle indicates a free surface and means that the pressure there is atmospheric pressure, P_{atm}. In other words, in this example, $P_1 = P_{\text{atm}}$. To find the pressure at point 2, our hydrostatics equation is used:

$$P_{\text{below}} = P_{\text{above}} + \rho g |\Delta z|$$

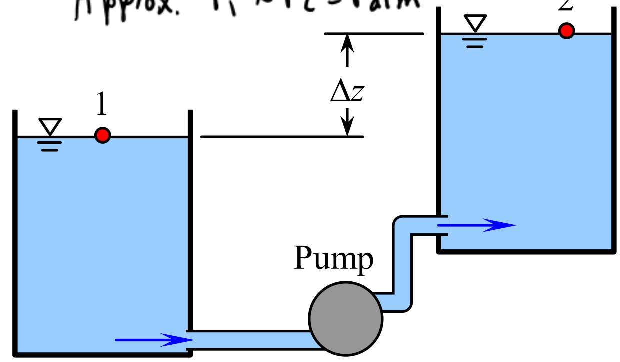
$$P_2 = P_1 + \rho g h$$

$$P_2 = P_{\text{atm}} + \rho g h \quad \text{(absolute pressure)}$$

Or, $\boxed{P_{2,\text{ gage}} = \rho g h \ \ \text{(gage pressure)}}$

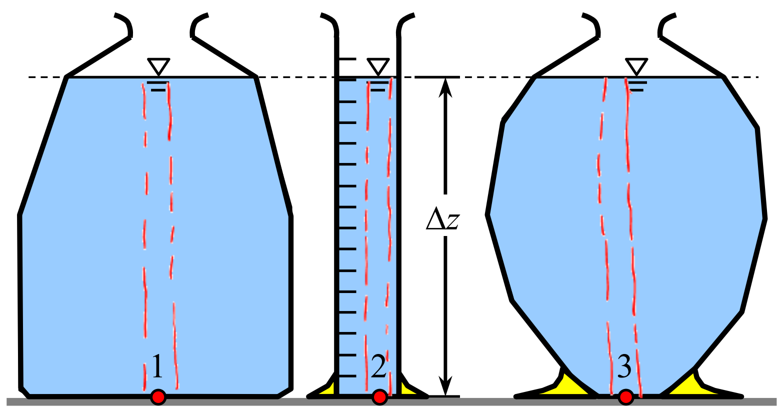
3. ***In most practical problems, atmospheric pressure may be approximated as constant at all elevations (unless the change in elevation is large).***

Approx. $P_1 \approx P_2 = P_{atm}$



Consider the example shown, in which water is pumped from one large reservoir to another. The pressure at both 1 and 2 is atmospheric. But since point 2 is higher in elevation than point 1, the local atmospheric pressure at 2 is a little lower than that at point 1. To be precise, our hydrostatics equation may be used to account for the difference in elevation between points 1 and 2. However, since the density of water is so much greater than that of air, it is common to ignore the difference between P_1 and P_2, and approximate both pressures as the same value of atmospheric pressure, P_{atm}.

4. ***The shape of a container does not matter in hydrostatics except for very small diameter tubes, where surface tension and the capillary effect become important.***



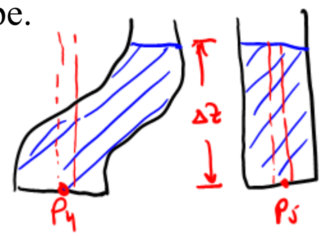
☆ $P_1 = P_2 = P_3$

Consider the three containers in the figure. At first glance, it may seem that the pressure at point 1 is greater than that at point 2 since the jug of water weighs much more than the graduated cylinder of water. Others might argue that the pressure at point 3 is greater than that at point 2, since the weight of the water is more "concentrated" on the small area at the bottom of the vase. However, **all three pressures are identical**. Use of our hydrostatics equation confirms this,

$$P_{\text{below}} = P_{\text{above}} + \rho g |\Delta z|$$

$$P_1 = P_2 = P_3 = P_{\text{atm}} + \rho g \Delta z$$

In all three cases, imagine a thin column of water above the point in question at the bottom. These three imaginary columns are identical, and thus the pressures at the three points are also identical. Pressure is a force per unit area, and over a small area at the bottom, that force is due to the weight of the water above it, which is the same in all three cases, regardless of the container shape.



$P_4 = P_5$

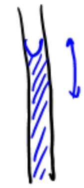
5. ***Pressure is constant across a flat fluid-fluid interface.***

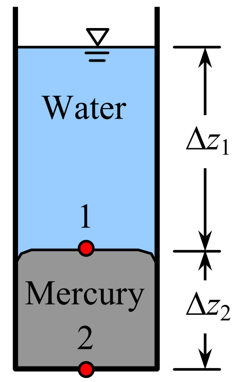
For example, consider the container in the figure, which is partially filled with mercury, and partially with water. In this case, our hydrostatics equation must be used *twice*, once in each of the liquids.

$$P_{\text{below}} = P_{\text{above}} + \rho g |\Delta z|$$

$$P_1 = P_{\text{atm}} + \rho_{\text{water}} g \Delta z_1$$

$$P_2 = P_1 + \rho_{\text{mercury}} g \Delta z_2 = P_{\text{atm}} + \rho_{\text{water}} g \Delta z_1 + \rho_{\text{mercury}} g \Delta z_2$$

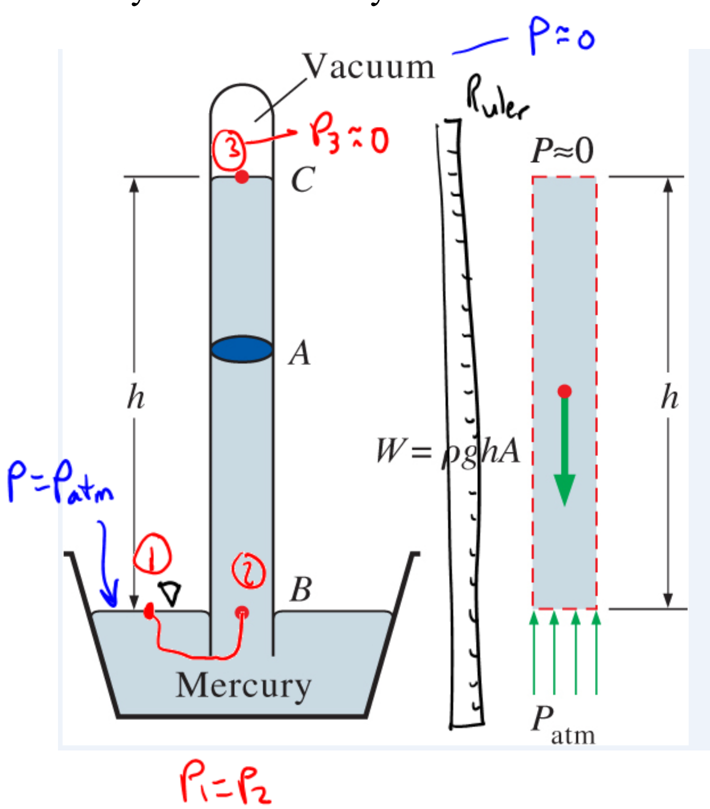




Note that for very small tube diameters, the interface would *not* be flat, but *curved*. In that case, there *would* be a pressure difference across the interface due to surface tension effects.

Liquid Barometer

- A common application of hydrostatics is the ***liquid barometer***.
- The purpose of a liquid barometer is **to measure the local value of atmospheric pressure**.
- Mercury is the most common liquid used in barometers; we call such a barometer a ***mercury barometer***.
- Analysis of a mercury barometer:



- $P_1 = P_2 = P_{atm}$
- From ② to ③

$$\underbrace{P_2}_{P_{atm}} = \underbrace{\cancel{P_3}}_{\approx 0} + \underbrace{\rho}_{\rho_{Hg}} g h$$

$$\boxed{P_{atm} = \rho_{Hg}\, g\, h}$$

Absolute Pressure

Mercury Barometer Equation

Example: Mercury Manometer

Given: Serena looks at the weather app on her phone. The local atmospheric pressure is indicated as 737 mm of mercury.

To do: Calculate the local atmospheric pressure in units of kPa.

Solution:

$$P_{atm} = \rho_{Hg} g h$$

$$= \left(13{,}580 \frac{kg}{m^3}\right)\left(9.807 \frac{m}{s^2}\right)\left(737 \text{ mm}\right)\left(\frac{1 \text{ m}}{1000 \text{ mm}}\right)\left(\frac{N \cdot s^2}{kg \cdot m}\right)\left(\frac{kPa \cdot m^2}{1000 \text{ N}}\right)$$

$$= 98.153 \text{ kPa}$$

$$\boxed{P_{atm} = 98.2 \text{ kPa}}$$