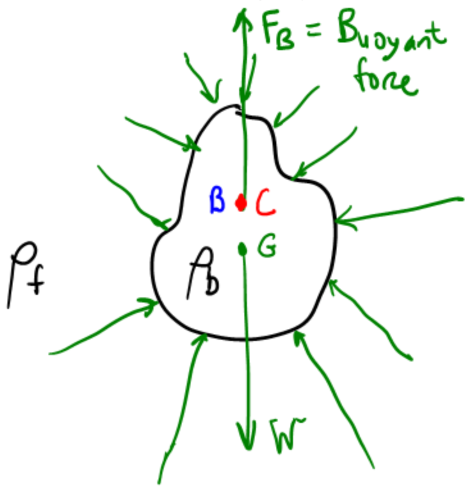
BUOYANCY AND STABILITY

In this lesson, we will:

- Define the **buoyant force** on a submerged body and how to calculate it
- Discuss **Archimedes' Principle**
- Discuss how to predict the stability of a boat or ship
- Do some example problems

Buoyancy and Archimedes' Principle

We are still discussing hydrostatics, so our workhorse equation is $\boxed{P_{\text{below}} = P_{\text{above}} + \rho g \left|\Delta z\right|}$.



G = center of gravity (center of mass)

C = centroid of the volume

B = center of buoyancy

$$\boxed{F_B = \rho_f g \forall_{sub}}$$

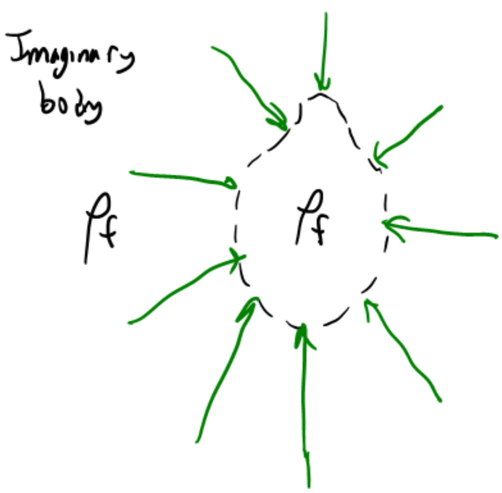
Submerged portion of the volume ★

★ $$\boxed{W = \rho_b g \forall}$$

(total volume)

★

Archimedes' Principle: The buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body, and it acts upward through the centroid of the displaced volume.



The pressure distribution is identical to that on the real body

$$\boxed{W = \rho_f g \forall_{sub}}$$

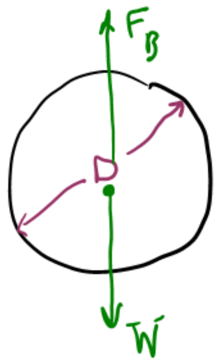
F_B must = W here

Example: Buoyancy

Given: A sphere of diameter D = 0.0550 m and density ρ_{body} = 1700 kg/m^3 falls into a tank of water (ρ_f = 1000 kg/m^3).

To do: Calculate the net downward body force on the sphere due to gravity in units of N.

Solution:



Net downward body force = F

$$F = W - F_B$$

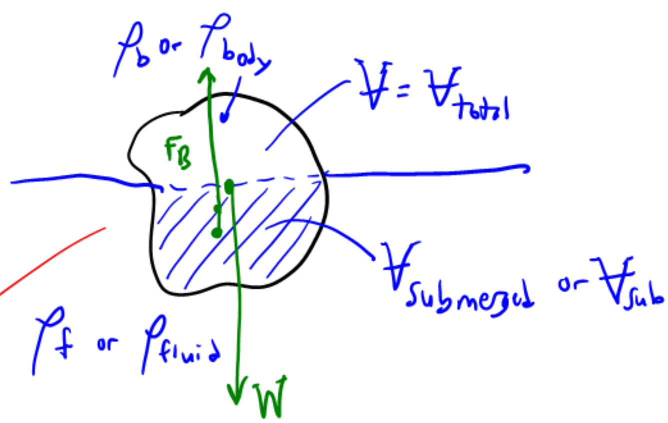
$$W = \rho_{body}\, g\, \frac{\pi D^3}{6} \qquad F_B = \rho_f\, g\, \frac{\pi D^3}{6}$$

$$\boxed{F = (\rho_{body} - \rho_f)\, g\, \frac{\pi D^3}{6}}$$

#s $$F = (1700 - 1000)\frac{\text{kg}}{\text{m}^3}\left(9.807\,\frac{\text{m}}{\text{s}^2}\right)(\pi)\frac{(0.0550\ \text{m})^3}{6}\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right) = \boxed{0.598\ \text{N down}}$$

Floating (Partially Submerged) Bodies

Person floating in the dense salty water of the Dead Sea. Photo by the author's son, Andy Cimbala.



- Weight acts on the total volume
- Buoyancy acts only on the submerged vol.

Balance: $F_B = W$

$$\rho_{fluid}\, \cancel{g}\, \forall_{submerged} = \rho_{body}\, \cancel{g}\, \forall_{total} \quad\rightarrow\quad \boxed{\frac{\forall_{submerged}}{\forall_{total}} = \frac{\rho_{body}}{\rho_{fluid}}}$$

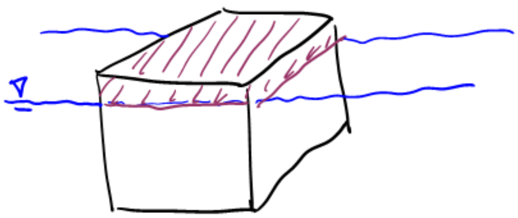
☆

Example: Partially Submerged Body

Given: An ice cube floats in a glass of cold water.

To do: Calculate the percentage of the ice cube volume that is above the water.

Solution:



$$\rho_{ice} \approx 916 \frac{kg}{m^3} = \rho_b$$

$$\rho_{cold\ water} \approx 1000 \frac{kg}{m^3} = \rho_f$$

$$\frac{\forall_{sub}}{\forall_{total}} = \frac{\rho_b}{\rho_f} = \frac{916}{1000} = 0.916$$ or 91.6% of the volume is submerged

$\therefore$ $\boxed{\text{8.4\% of the ice cube is above water}}$

Hydrometer – A simple instrument to measure the S.G. of a liquid

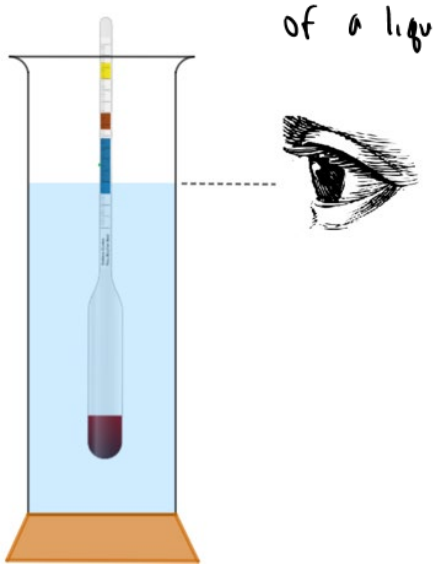
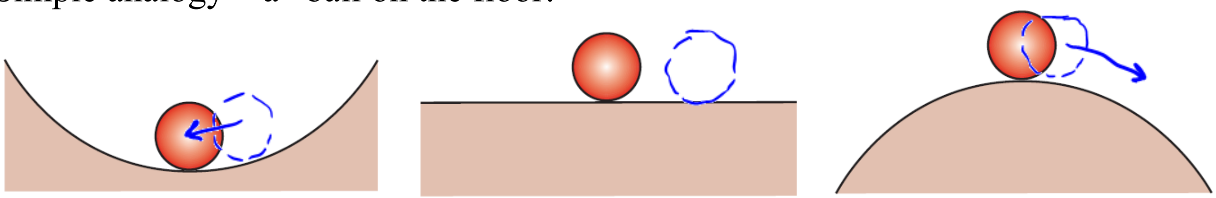


Photo from https://study.com/

Photo from https://brewtogether.com/

Stability of Ships and Boats

Simple analogy – a "ball on the floor:"

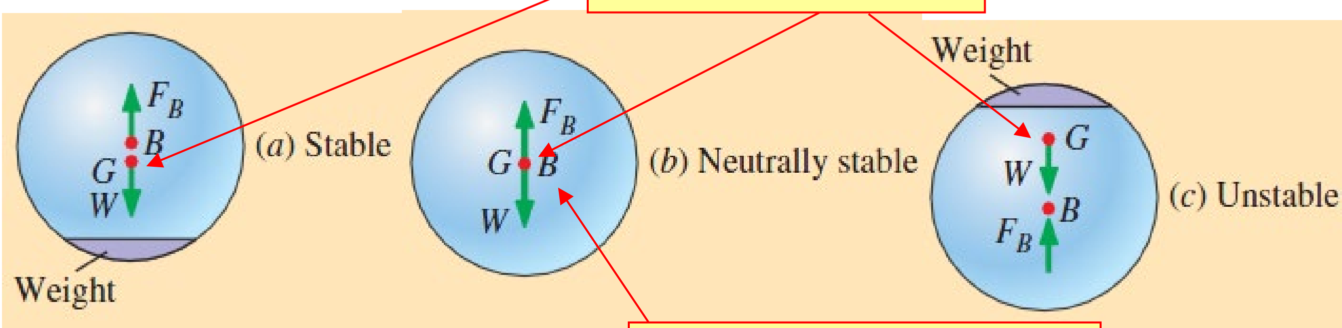


(*a*) Stable (*b*) Neutrally stable (*c*) Unstable

From Çengel and Cimbala, Ed. 4.

G is the center of gravity (depends on how the weight is distributed inside the body)

Stability of Fully Submerged Bodies



From Çengel and Cimbala, Ed. 4.

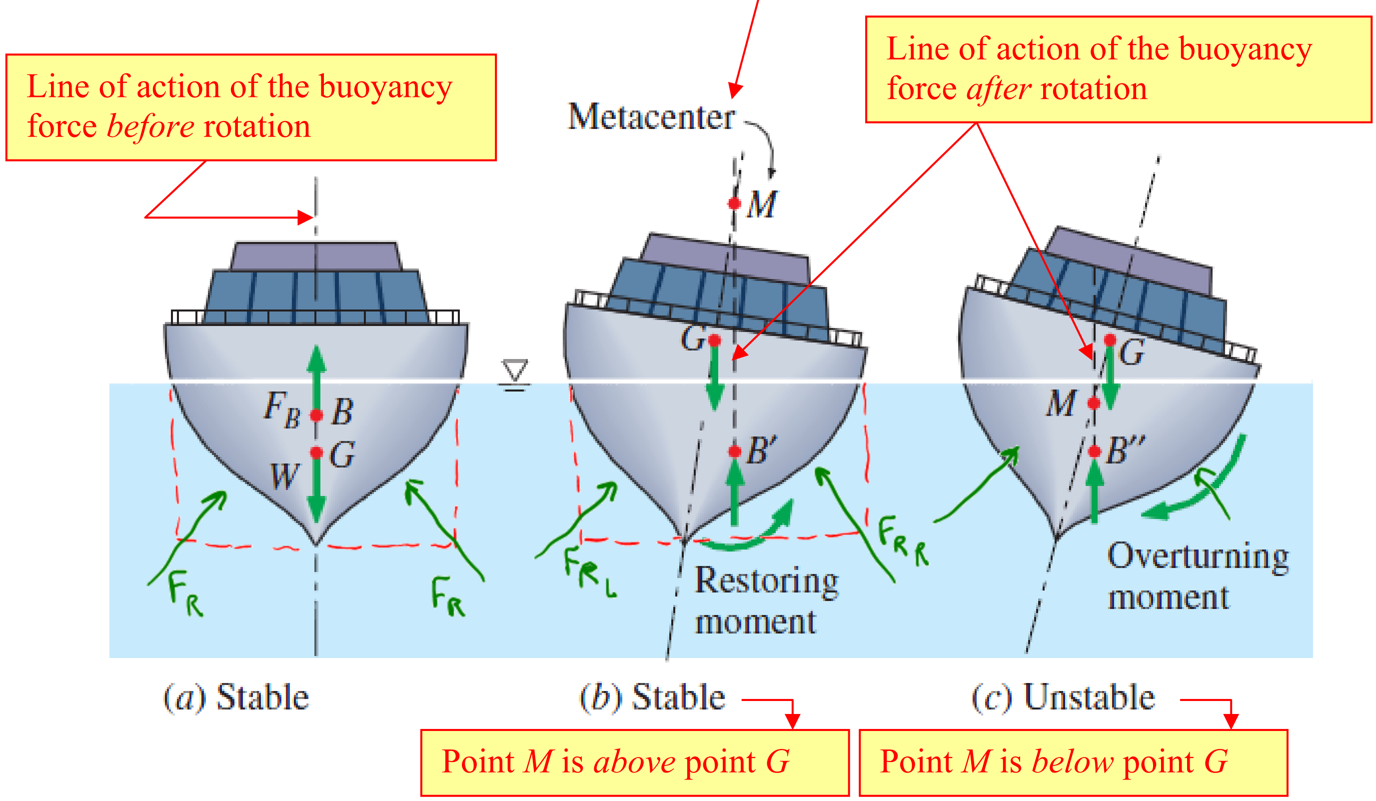
B is the centroid of the submerged portion of the body = center of buoyancy, and also = center of rotation.

Bottom line: **The body is *unstable* if center of buoyancy *B* is *below* center of gravity *G*.**

Stability of Partially Submerged Bodies (Ships and Boats)

Note: As the boat tips ("lists") to one side, the submerged part of the hull is no longer symmetric, and it is difficult to calculate the new line of action of the hydrostatic pressure force. We thus do only a qualitative analysis here.

Define M = the **metacenter** = the point where the line of action of the buoyancy force *before* rotation and the line of action of the buoyancy force *after* rotation intersect.



Bottom line: **The boat is *unstable* if metacenter *M* is *below* center of gravity *G*.**