PIPE FLOW MINOR LOSSES

In this lesson, we will:

- Discuss how to account for minor losses: **Equivalent Length** or **Minor Loss Coefficient**
- Show how to incorporate minor losses into the head form of the energy equation
- Show values of minor loss coefficients for elbows, valves, expansions, inlets, outlets, etc.
- Do some example problems

Review of Major and Minor Losses and the Energy Equation

Our "**workhorse**" equation: **Head form of the energy equation from inlet 1 to outlet 2**:

$$\boxed{\frac{P_1}{\rho_1 g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, }u}=\frac{P_2}{\rho_2 g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, }e}+h_{L,\text{ total}}}$$

where $\boxed{h_{L,\text{ total}}=\sum h_{L,\text{ major}}+\sum h_{L,\text{ minor}}}$

We previously discussed how to deal with **Major Losses** (fully developed sections of pipe):

$$\boxed{h_{L,\text{ major}}=f\frac{L}{D}\frac{V^2}{2g}} \text{ where } \boxed{f=\frac{8\tau_w}{\rho V^2}=\text{fnc}\left(\text{Re},\frac{\varepsilon}{D}\right)} \text{ and } \boxed{\text{Re}=\frac{\rho VD}{\mu}=\frac{VD}{\nu}}$$

We calculate Darcy friction factor f from the **Churchill Equation**,

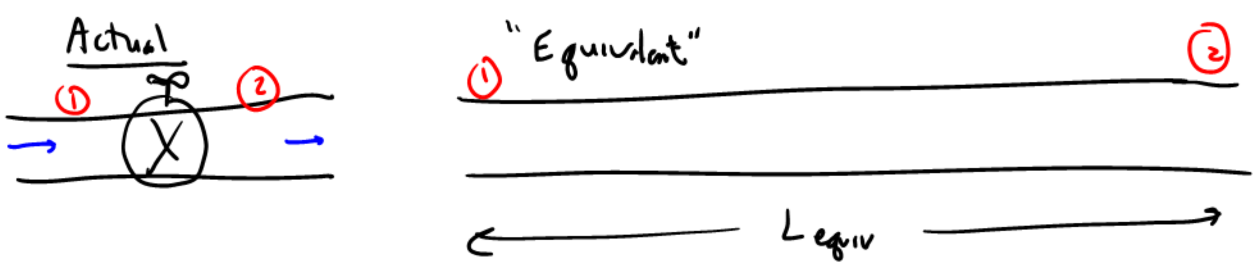
$$\boxed{f=8\left[\left(\frac{8}{\text{Re}}\right)^{12}+\left(A+B\right)^{-1.5}\right]^{\frac{1}{12}}} \quad \boxed{A=\left\{-2.457\cdot\ln\left[\left(\frac{7}{\text{Re}}\right)^{0.9}+0.27\frac{\varepsilon}{D}\right]\right\}^{16}} \quad \boxed{B=\left(\frac{37530}{\text{Re}}\right)^{16}}$$

Note: We drop the negative sign for parameter A since exponent 16 is even.
[Pointed out to me by **Dr. John Torczynski**, Private Communication, September 2022.]

Now let's consider **Minor Losses** (components other than fully developed sections of pipe).

Equivalent Length

$$\boxed{h_{L\,\text{minor}}=f\,\frac{L_{\text{equiv}}}{D}\,\frac{V^2}{2g}}$$



Minor Loss Coefficient

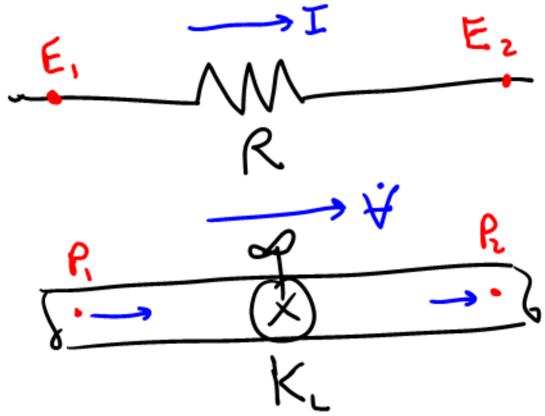
$$\boxed{h_{L,\,minor} = K_L \frac{V^2}{2g}}$$

where K_L = minor loss coefficient or resistance coefficient

$\{K_L\} = \{1\} \qquad [K_L] = \{1\}$

* NOTATION: HVAC → use C_0 instead of K_L

ANALOGY:



$E_1 > E_2$

I = const

$P_1 > P_2$

$\dot{V}$ = const

Head form of energy equation

$$\boxed{h_{L,\,total} = h_{L,\,major} + h_{L,\,minor}}$$

$$\boxed{h_{L,\,total} = \sum_i f_i \frac{L_i}{D_i} \frac{V_i^2}{2g} + \sum_j K_{L,j} \frac{V_j^2}{2g}} \; *$$

[i = each pipe section] [j = each minor loss]

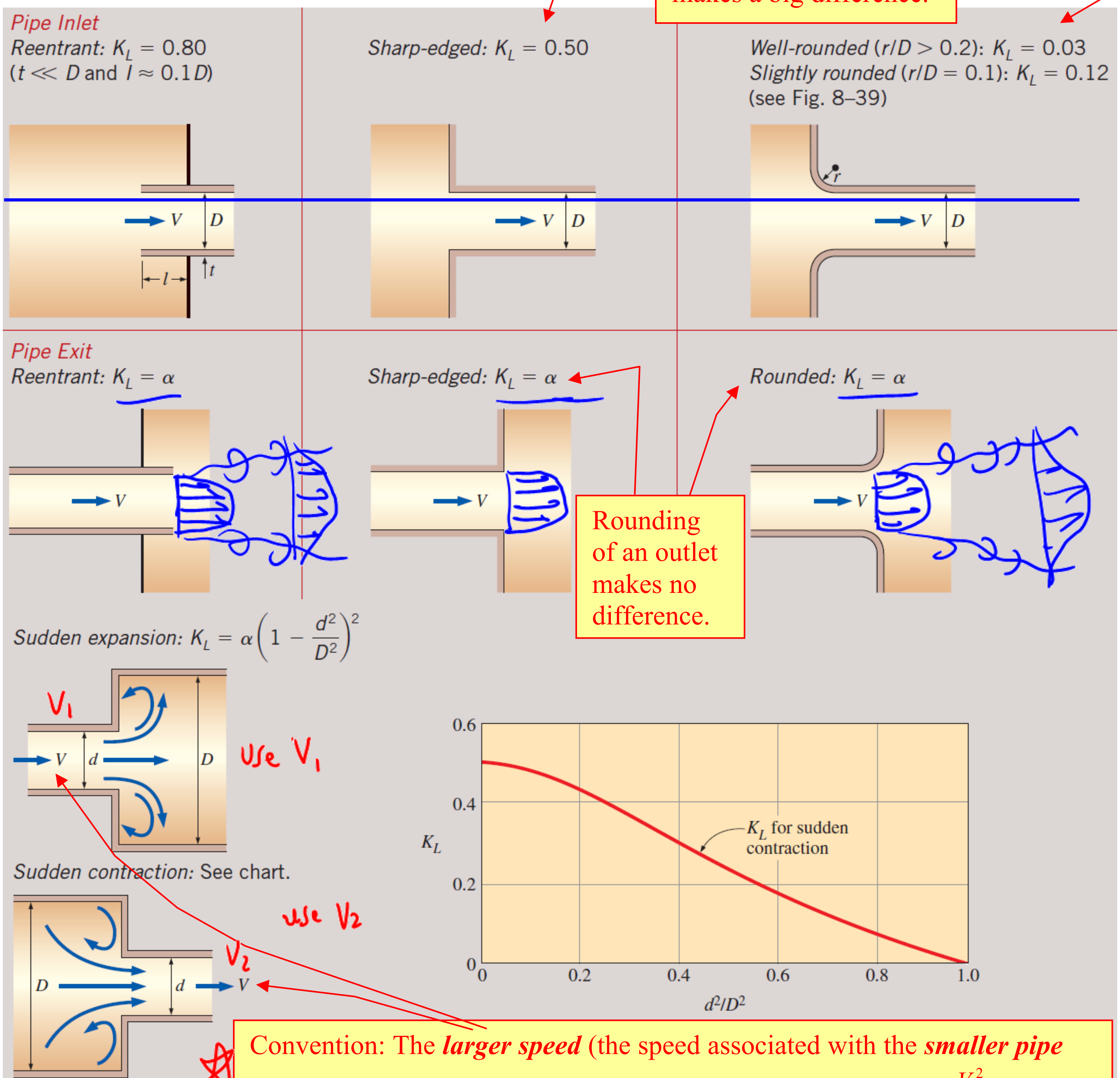
In most cases, D = constant throughout → V = const throughout

$$\boxed{h_{L,\,total} = \frac{V^2}{2g}\left[f\frac{L}{D} + \sum K_L\right]} \; *$$

Sample Values of Minor Loss Coefficients

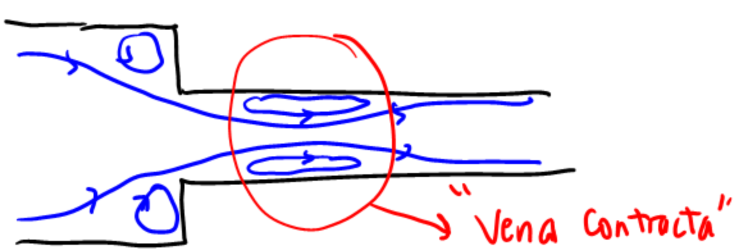
Table 8-4 of Çengel and Cimbala, Ed. 4.

Rounding of an inlet makes a big difference.



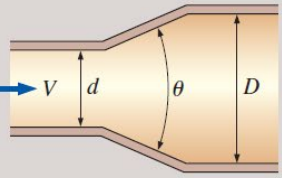
Rounding of an outlet makes no difference.

Convention: The ***larger speed*** (the speed associated with the ***smaller pipe section***) is used in the equation for minor head loss, $h_{L,\text{ minor}} = K_L \dfrac{V^2}{2g}$.

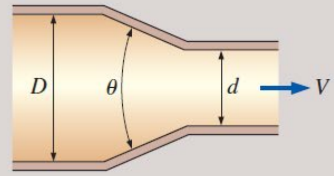


Gradual Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

Expansion (for $\theta = 20°$):
$K_L = 0.30$ for $d/D = 0.2$
$K_L = 0.25$ for $d/D = 0.4$
$K_L = 0.15$ for $d/D = 0.6$
$K_L = 0.10$ for $d/D = 0.8$

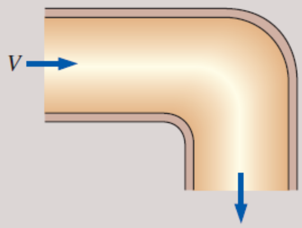


Contraction:
$K_L = 0.02$ for $\theta = 30°$
$K_L = 0.04$ for $\theta = 45°$
$K_L = 0.07$ for $\theta = 60°$

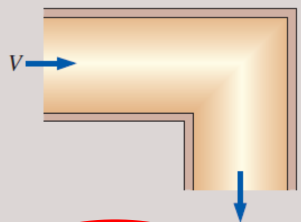


Bends and Branches

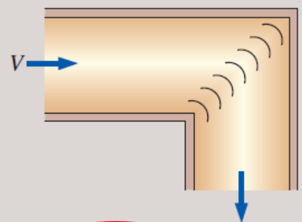
90° smooth bend:
Flanged: $K_L = 0.3$
Threaded: $K_L = 0.9$



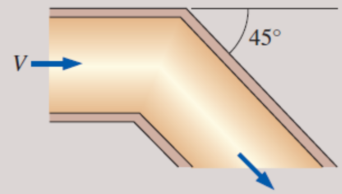
90° miter bend
(without vanes): $K_L = 1.1$



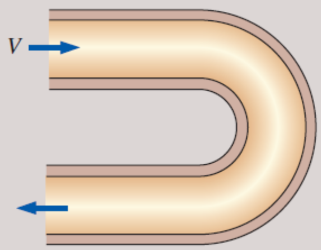
90° miter bend
(with vanes): $K_L = 0.2$



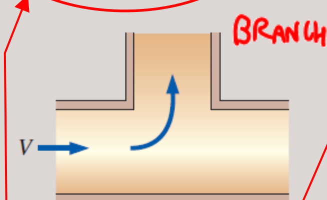
45° threaded elbow:
$K_L = 0.4$



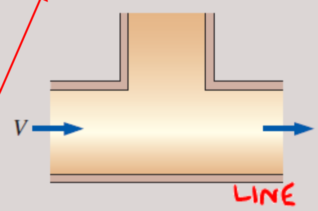
180° return bend:
Flanged: $K_L = 0.2$
Threaded: $K_L = 1.5$



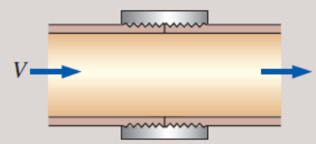
Tee (branch flow):
Flanged: $K_L = 1.0$
Threaded: $K_L = 2.0$



Tee (line flow):
Flanged: $K_L = 0.2$
Threaded: $K_L = 0.9$



Threaded union:
$K_L = 0.08$



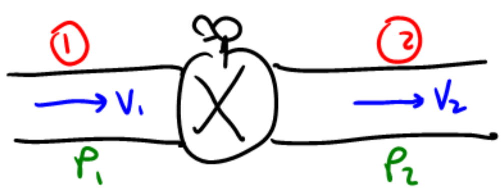
For tees, there are <u>*two*</u> values of K_L, one for *branch flow* and one for *line flow*.

Valves

Globe valve, fully open: $K_L = 10$
Angle valve, fully open: $K_L = 5$
Ball valve, fully open: $K_L = 0.05$
Swing check valve: $K_L = 2$

Gate valve, fully open: $K_L = 0.2$
$\frac{1}{4}$ closed: $K_L = 0.3$
$\frac{1}{2}$ closed: $K_L = 2.1$
$\frac{3}{4}$ closed: $K_L = 17$

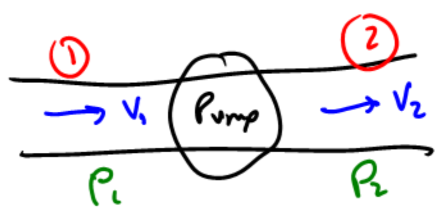
VALVES:



$V_1 = V_2$ $P_2 < P_1$

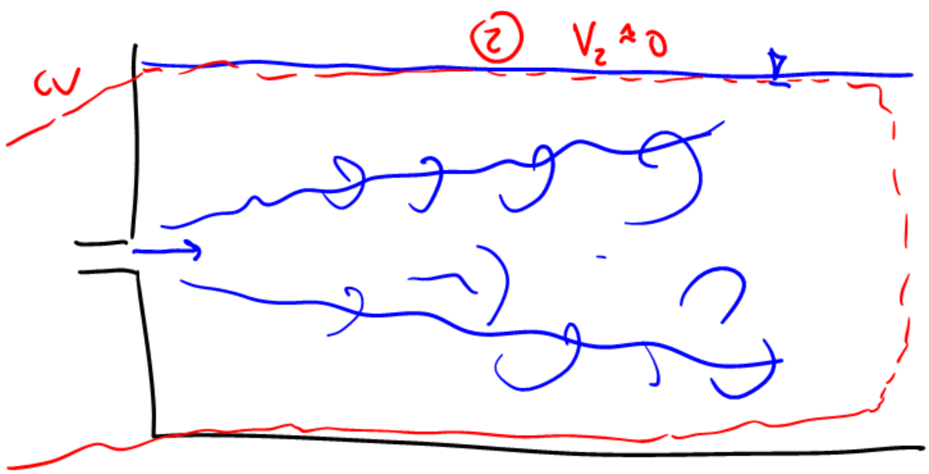
VALVES DO NOT SLOW DOWN THE FLOW FROM 1 to 2.
THEY CREATE A HEAD LOSS (Pressure drop) → SLOWS DOWN THE WHOLE SYSTEM

PUMPS:



$V_1 = V_2$ $P_2 > P_1$

Pumps do not speed up the flow from 1 to 2. They provide a head <u>gain</u> that speeds up the flow in the whole system



ALL the kinetic energy of the jet eventually dissipates into heat

It is all wasted

In energy eq.

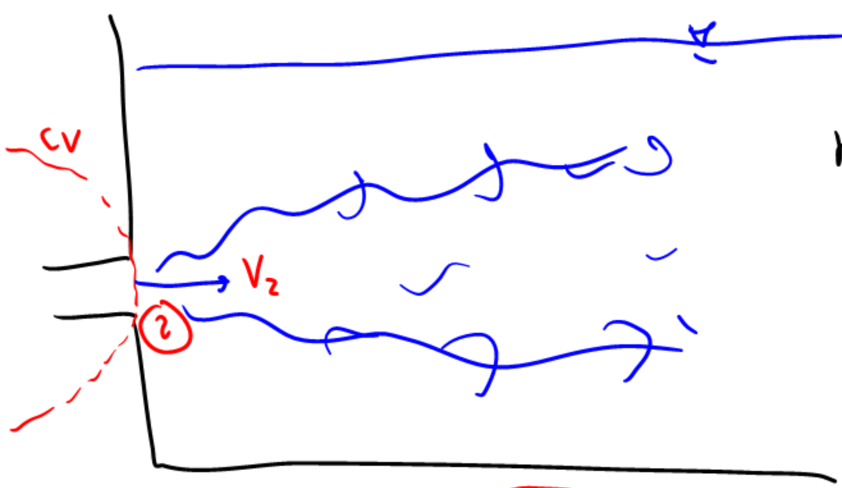
$$\ldots + \alpha \frac{V^2}{2g}$$

ALL OF THIS IS WASTED

$\therefore K_L = \alpha$ at an outlet

THIS HOLDS FOR SUBMERGED OUTLETS WHERE THE JET IS INCLUDED IN THE C.V.

$$\boxed{h_{L\,outlet} = \underset{\alpha}{\underset{\shortparallel}{K_{L\,outlet}}} \frac{V^2}{2g}} \rightarrow \alpha \frac{V^2}{2g} = \text{minor loss}$$

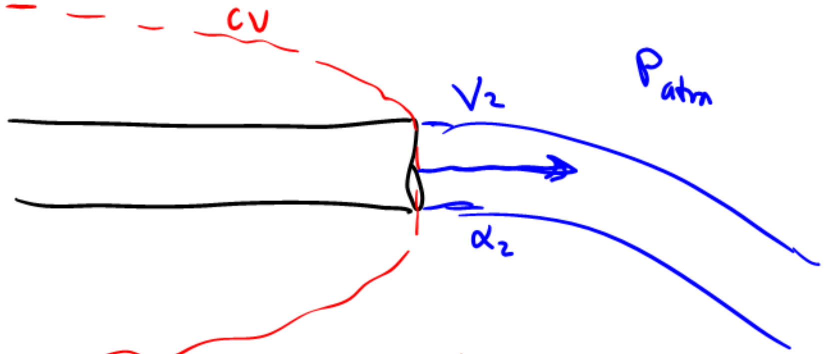


Here the outlet is NOT a minor loss since the jet is not included in the CV

$\boxed{\text{DO NOT INCLUDE } K_{L\,outlet}}$

IN energy eq. the $\alpha_2 \frac{V_2^2}{2g}$ term is not zero

THE FIRST CV IS A WISER CHOICE

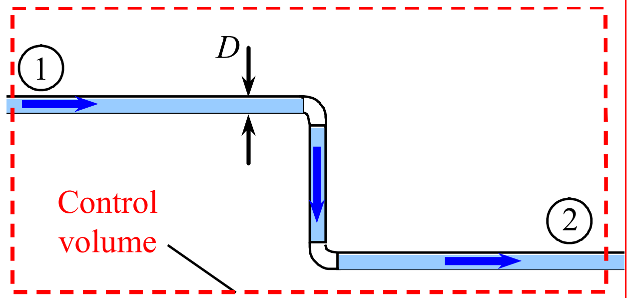


NO MINOR LOSS AT THIS OUTLET ☆

(our CV does not include any jet dissipation)

Example: Major and Minor Losses

Given: Water at 20°C ($\rho = 998.0$ kg/m^3, $\mu = 1.002 \times 10^{-3}$ kg/m·s) flows at a steady average velocity of 6.45 m/s through a smooth pipe of diameter 2.54 cm. The flow is fully developed through the entire section of pipe. The total pipe length is 10.56 m, and there are two elbows, each with $K_L = 0.90$.



To do:
(a) Calculate the total irreversible head loss in meters and the pressure drop in kPa through this section of piping due to both major and minor losses. (b)

Solution:

(a) • Draw a CV

• $Re = \dfrac{\rho V D}{\mu} = \dfrac{\left(998.0 \frac{kg}{m^3}\right)\left(6.45 \frac{m}{s}\right)\left(0.0254\ m\right)}{1.002 \times 10^{-3} \frac{kg}{m \cdot s}} = 163176 = Re$

• Churchill eq. @ this Re & $\frac{\varepsilon}{D} = 0 \rightarrow \boxed{f = 0.016176}$

• $h_{L,total} = \dfrac{V^2}{2g}\left(f \dfrac{L}{D} + \Sigma K_L\right)$

$\Sigma K_L \rightarrow 2(0.90) = 1.80$ (2 elbows)

$$h_{L,total} = \frac{\left(6.45 \frac{m}{s}\right)^2}{2\left(9.807 \frac{m}{s^2}\right)}\left(0.016176 \frac{10.56\ m}{0.0254\ m} + 1.80\right) = 18.083\ m$$

$$\boxed{h_{L,total} = 18.1\ m}$$

(b) Pressure drop due to irreversibilities (not counting elevation changes)

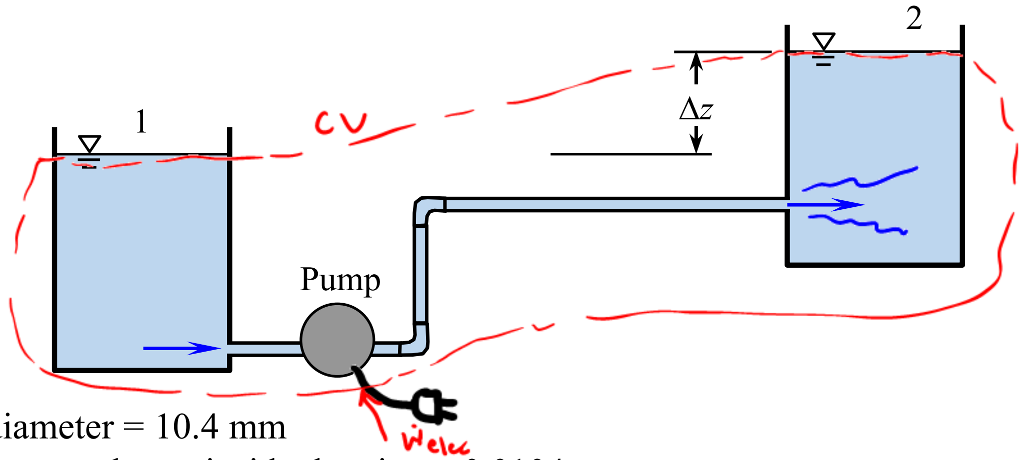
Accounted for in the energy eq.

$$\Delta P_{irrev.\ losses} = \rho g h_{L,total}$$

$$\Delta P_{irrev.\ losses} = \left(998.0 \frac{kg}{m^3}\right)\left(9.807 \frac{m}{s^2}\right)\left(18.083\ m\right)\left(\frac{N \cdot s^2}{kg \cdot m}\right)\left(\frac{kPa \cdot m^2}{1000\ N}\right) = \boxed{177.\ kPa}$$

Example: Fully Developed Turbulent Pipe Flow With Calculated Minor Losses

Given: Water at 20ºC (ρ = 998.0 kg/m^3, μ = 1.002 × 10^{-3} kg/m·s) is pumped by a small aquarium pump from a lower tank to an upper tank as sketched (not to scale).



- Pipe diameter = 10.4 mm
- Average roughness inside the pipe = 0.0104 mm
- Pipe length = 15.8 m
- The surface elevation difference is 4.13 m
- Volume flow rate = 2.06 L/min
- The elbows are smooth and threaded; the inlet and outlet are sharp
- The efficiency of the pump/motor assembly is 76.7%

To do: Calculate the electrical power (in W) that must be delivered to the pump motor in order to pump the water at the given flow conditions.

Solution: Everything is the same as the example of the previous lesson except that we need to ***calculate*** the minor losses, which were ***given*** in the previous lesson.

OUR NEW WORKHORSE EQUATION

$$\cancel{\frac{P_1}{\rho_1 g}} + \cancel{\alpha_1 \frac{V_1^2}{2g}} + z_1 + h_{\text{pump, }u} = \cancel{\frac{P_2}{\rho_2 g}} + \cancel{\alpha_2 \frac{V_2^2}{2g}} + z_2 + \cancel{h_{\text{turbine, }e}} + \sum h_{L,\text{ major}} + \sum h_{L,\text{ minor}}$$

$$\boxed{h_{pump,u} = z_2 - z_1 + \sum h_{L,major} + \sum h_{L,minor}}$$

$$\sum K_L = \underset{\text{sharp inlet}}{0.50} + \underset{\text{2 elbows}}{2(0.90)} + \underset{\alpha\text{ @ outlet}}{1.05} = \underline{\underline{3.35}}$$

$Re = 4186.54$, $\varepsilon/D = 0.00100 \rightarrow f = 0.04118$

SAME AS PREVIOUS LESSON

$$\dot{W}_{elec} = \frac{\rho \dot{\forall} g}{\eta_{pump-motor}} h_{pump,u}$$

$$\boxed{\dot{W}_{elec} = \frac{\rho \dot{\forall} g}{\eta_{pump-motor}} \left[(z_2 - z_1) + \frac{V^2}{2g}\left(f\frac{L}{D} + \sum K_L \right) \right]}$$ ANS

Error found. This exponent should be -5 not -3

$$\dot{W}_{elec} = \frac{\left(998.0\ \frac{kg}{m^3}\right)\left(3.4333\times 10^{-3}\ \frac{m^3}{s}\right)\left(9.807\ \frac{m}{s^2}\right)}{0.767} \cdot \left[4.13\ m + \frac{\left(0.404166\ \frac{m}{s}\right)^2}{2\left(9.807\ \frac{m}{s^2}\right)} \left(0.04118\ \frac{15.8\ m}{0.0104\ m} + 3.35 \right) \right] \left(\frac{N\cdot s^2}{kg\cdot m}\right)\left(\frac{W\cdot s}{N\cdot m}\right)$$

$$\boxed{\dot{W}_{elec} = 2.05\ W}$$