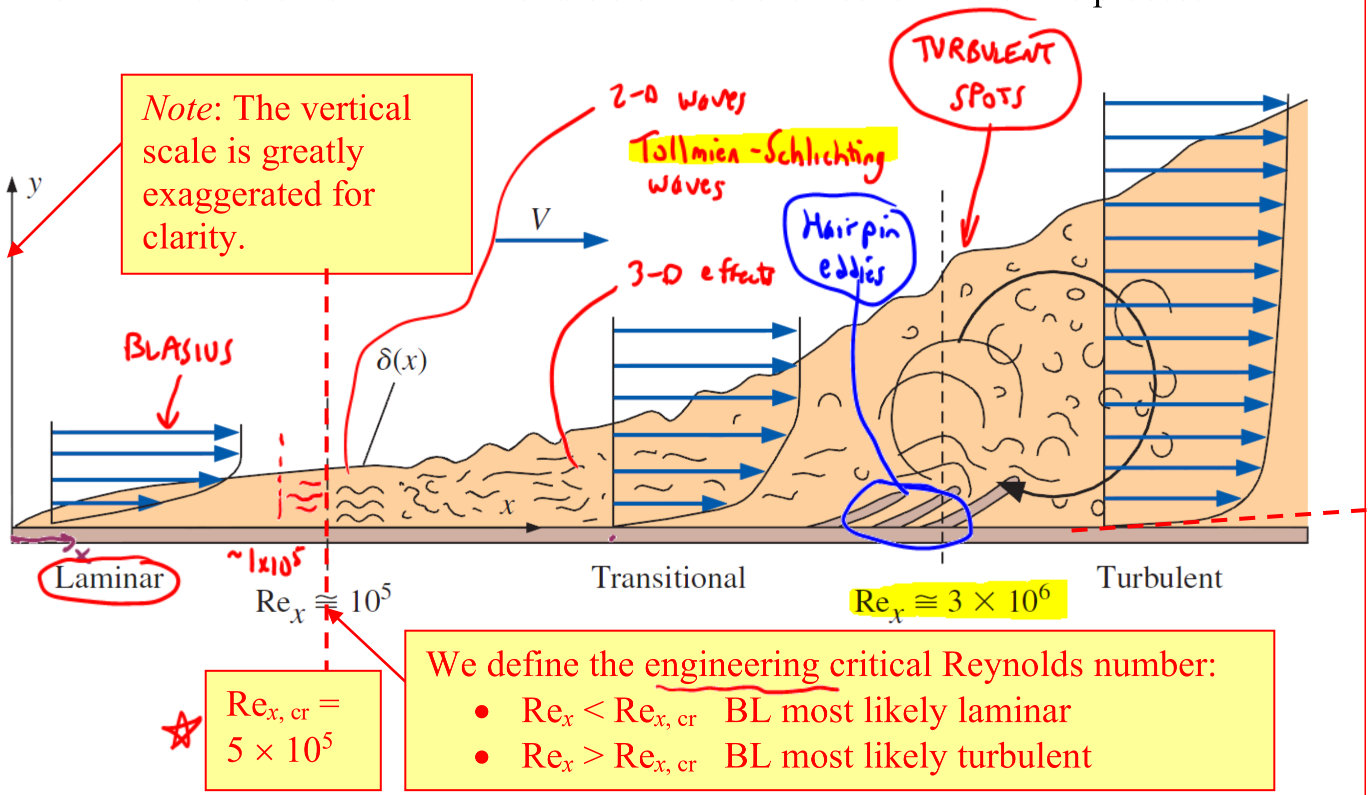
TURBULENT FLAT PLATE BOUNDARY LAYER

In this lesson, we will:

- Describe **Transition from Laminar to Turbulent Boundary Layer** on a flat plate
- Discuss the differences between laminar and turbulent **Boundary Layer Profiles**
- Show equations for the **Flat Plate Turbulent Boundary Layer** (BL thickness, displacement thickness, skin friction coefficient, etc.) compared to those of laminar flow
- Discuss the effect of **Surface Roughness** on a turbulent boundary layer
- Do an example problem

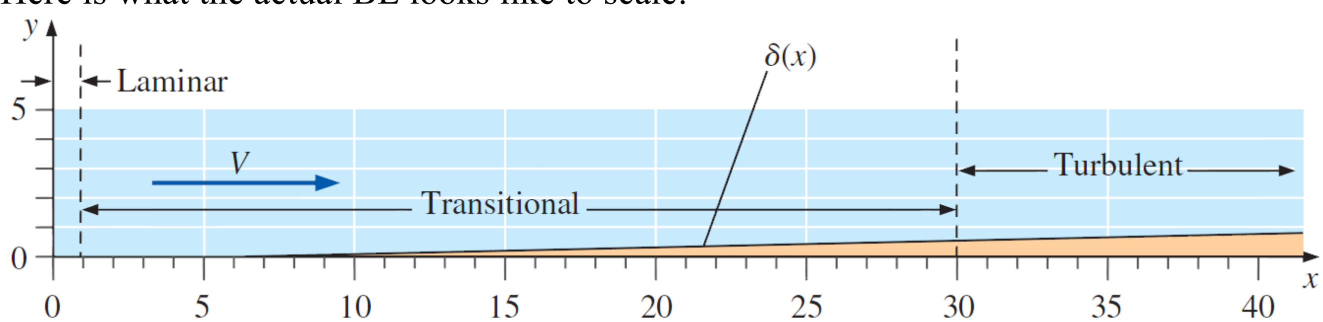
Transition to Turbulence on a Flat Plate

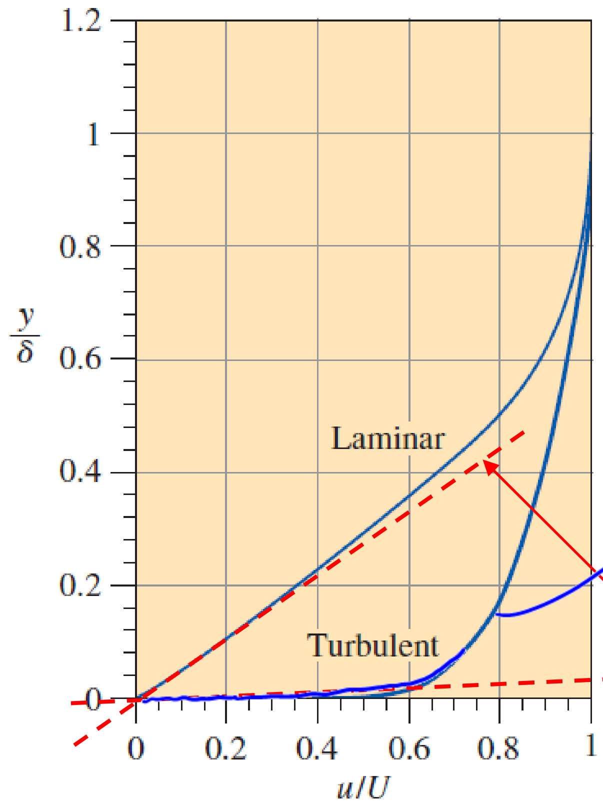
Since $\mathrm{Re}_x = Ux/\nu$ increases with x (distance down the plate), eventually Re_x gets big enough that the BL transitions from laminar to turbulent. Here is a schematic of the process:



Figures and tables from Çengel and Cimbala, Ed. 4.

Here is what the actual BL looks like to scale:

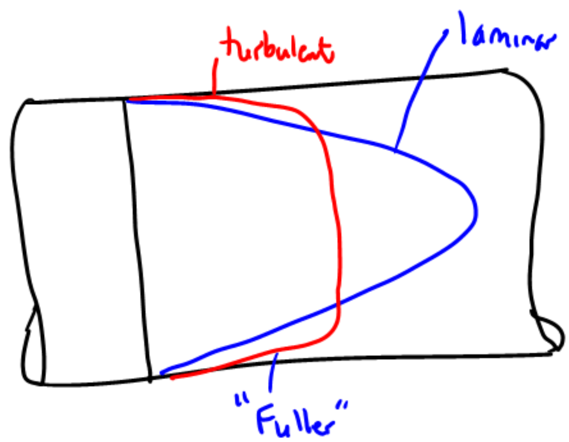




The time-averaged turbulent flat plate (zero pressure gradient) boundary layer velocity profile is much *fuller* than the laminar flat plate boundary layer profile, and therefore has a larger slope $\partial u/\partial y$ at the wall, leading to greater skin friction drag along the wall.

Note: This is a nondimensional plot in terms of u/U vs. y/δ. The actual turbulent BL is of course much *thicker* than the laminar one in physical dimensions, as sketched above.

Similar to <u>pipe flow</u>



Quantities of interest for the turbulent flat plate boundary layer:

There are ***empirical equations*** for the turbulent flat plate boundary layer velocity profile. We can use these curve fits to estimate quantities of interest such as 99% boundary layer thickness δ, displacement thickness δ^*, local skin friction coefficient $C_{f,x}$, etc., as we did for the laminar (Blasius) flat plate boundary layer. These are summarized in the table below.

Column (b) expressions are generally preferred for engineering analysis.

Property	Laminar	(a) Turbulent(†) (1/7th law)	(b) Turbulent(‡) (1/5th law)
Boundary layer thickness	$\frac{\delta}{x} = \frac{4.91}{\sqrt{\mathrm{Re}_x}}$	$\frac{\delta}{x} \cong \frac{0.16}{(\mathrm{Re}_x)^{1/7}}$	$\frac{\delta}{x} \cong \frac{0.38}{(\mathrm{Re}_x)^{1/5}}$
Displacement thickness	$\frac{\delta^*}{x} = \frac{1.72}{\sqrt{\mathrm{Re}_x}}$	$\frac{\delta^*}{x} \cong \frac{0.020}{(\mathrm{Re}_x)^{1/7}}$	$\frac{\delta^*}{x} \cong \frac{0.048}{(\mathrm{Re}_x)^{1/5}}$
Momentum thickness	$\frac{\theta}{x} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$	$\frac{\theta}{x} \cong \frac{0.016}{(\mathrm{Re}_x)^{1/7}}$	$\frac{\theta}{x} \cong \frac{0.037}{(\mathrm{Re}_x)^{1/5}}$
Local skin friction coefficient	$C_{f,x} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$	$C_{f,x} \cong \frac{0.027}{(\mathrm{Re}_x)^{1/7}}$	$C_{f,x} \cong \frac{0.059}{(\mathrm{Re}_x)^{1/5}}$

Note that $C_{f,x}$ is the *local* skin friction coefficient, applied at only *one* value of x.

To these we add the integrated ***average skin friction coefficients*** for *one side* of a flat plate of length L, noting that C_f applies to the entire plate from $x = 0$ to $x = L$:

Laminar: $$C_f = \frac{1.33}{\mathrm{Re}_L^{1/2}} \qquad \mathrm{Re}_L \lesssim 5 \times 10^5$$

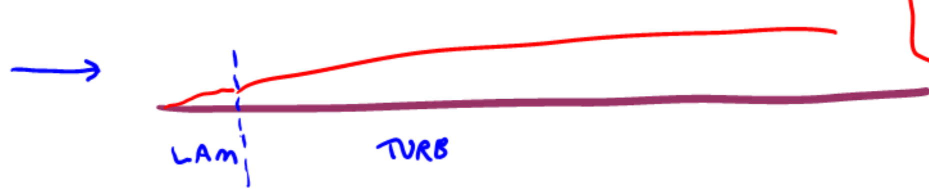
Turbulent: $$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} \qquad 5 \times 10^5 \lesssim \mathrm{Re}_L \lesssim 10^7$$

These equations are for ***smooth*** flat plates and for ***one*** side of the plate only.

For cases in which the laminar portion of the plate is taken into consideration, we use:

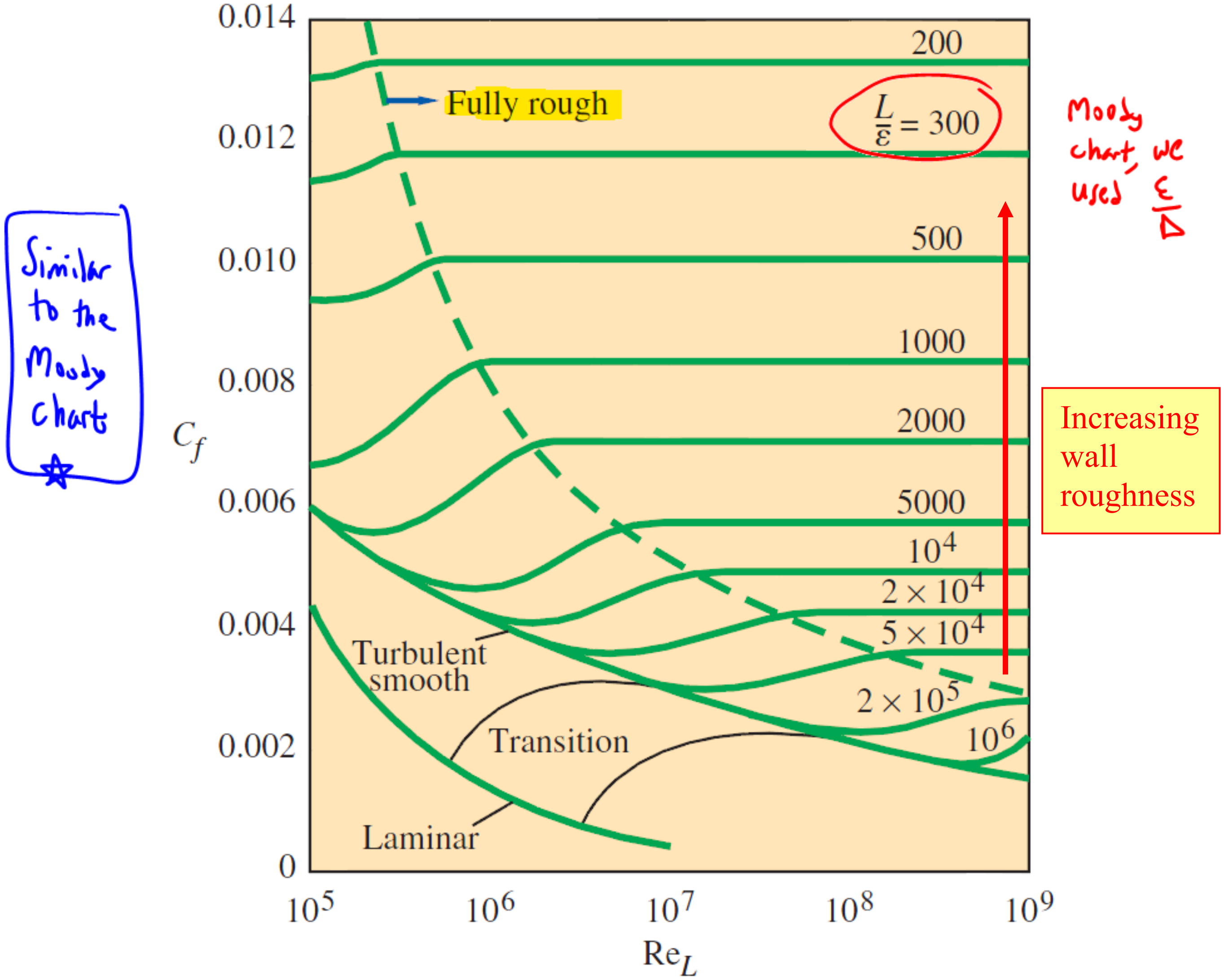
$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} \qquad 5 \times 10^5 \lesssim \mathrm{Re}_L \lesssim 10^7$$

ALL THESE EQ's ARE FOR SMOOTH PLATES



Turbulent flat plate boundary layers with wall roughness:

Finally, all of the above are for *smooth* flat plates. However, if the plate is *rough*, the average skin friction coefficient C_f increases with roughness ε. This is similar to the situation in pipe flows, in which Darcy friction factor f increases with pipe wall roughness.



Friction coefficient for parallel flow over smooth and rough flat plates.

Just as with pipe flows, at high enough Reynolds numbers, the boundary layer becomes "fully rough". For a ***fully rough flat plate turbulent boundary layer*** with average wall roughness height ε,

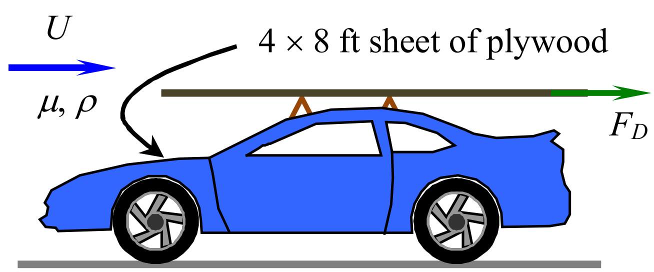
Fully rough turbulent regime:

$$C_f = \left(1.89 - 1.62 \log \frac{\varepsilon}{L}\right)^{-2.5}$$

This equation represents the flat portions of the above figure that are labeled "Fully rough".

Example: Drag on a Sheet of Plywood (Turbulent BL Case)

Given: Craig buys a standard 4 × 8 ft (1.219 × 2.438 m) sheet of plywood at Lowe's and mounts it on the roof rack of his car. He drives (carefully) at 15.65 m/s. The air density and kinematic viscosity are $\rho = 1.204$ kg/m^3 and $\nu = 1.516 \times 10^{-5}$ m^2/s, respectively.



To do: Estimate the boundary layer thickness at the end of the plate (at $x = L$) in millimeters and the drag force on the plate in newtons.

Solution:

THIS BL IS TURBULENT

We solved this problem previously assuming that the BL remained ***laminar***. We first solved for the Reynolds number at the end of the plate, $\text{Re}_x = \text{Re}_L$ (at $x = L$) $= UL/\nu = 2.52 \times 10^6$. Results: $\delta = 7.55$ **mm**, $F_D = 0.735$ **N**. Now let's repeat the calculations for a ***turbulent*** BL.

LAMINAR FLOW RESULTS

- Assume $\varepsilon = 0$ (No roughness)
- Use column (b) ($\frac{1}{5}$th law) equations
- δ (BL thickness)

$$\frac{\delta}{x} = \frac{0.38}{(\text{Re}_x)^{1/5}} \quad @\, x = L, \quad \delta = L \frac{0.38}{(\text{Re}_L)^{1/5}}$$

$$\delta = (2.438 \text{ m}) \frac{0.38}{(2.5168 \times 10^6)^{1/5}} \left(\frac{1000 \text{ mm}}{\text{m}}\right) \rightarrow \delta_{turb} = 48.6 \text{ mm}$$

≈ 6.4 times larger than laminar case

- δ^* (Displacement thickness)

$\delta^*_{laminar} = 2.64$ mm

$\delta^*_{turbulent} = 6.14$ mm (higher) @ same Re_L

- F_D (Skin friction drag)

$$F_D = \rho U^2 C_f L \cdot W \quad \text{(both sides)}, \quad C_f = \frac{0.074}{\text{Re}^{1/5}} \text{ (smooth)}$$

correct → $F_{D_{turb}} = 3.40$ N ≈ 4.6 x larger than $F_{D_{lam}}$ (0.735 N)

★ A TURBULENT BL HAS MUCH MORE FRICTION DRAG THAN A LAMINAR BL

Example: Displacement Thickness in a Wind Tunnel Test Section – *Turbulent BL*

Given: Professor Wakeflow is again studying the far wakes of objects using a small low-speed wind tunnel. This time she runs the tunnel at much higher speed. For her experiments, she needs the freestream flow to remain constant with downstream distance in the test section of the wind tunnel. Because of displacement thickness, however, the air speed increases downstream. To counteract this effect, she decides to expand the cross-sectional area of the test section as discussed in a previous lesson. Here are some values:

- Air density $\rho = 1.204$ kg/m^3 and kinematic viscosity $\nu = 1.516\times10^{-5}$ m^2/s
- The wind tunnel test section speed is 65.5 m/s [was 6.50 m/s in laminar case]
- The length of the test section is 0.510 m
- The cross section of the wind tunnel is rectangular, 0.633 m wide × 0.412 m high

To do: Professor Wakeflow decides to tilt only the *bottom* wall of the wind tunnel to counteract displacement effects. Calculate how much she should increase the test section height at the end of the test section.

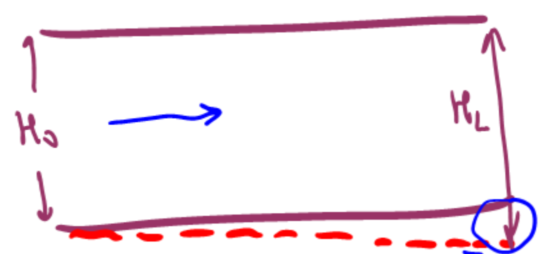
Solution: $U = 65.5$ m/s instead of 6.50 m/s, but all else is the same as in the previous lesson (laminar BL)

$$Re_L = \frac{UL}{\nu} = 2.2035\times10^6 \quad \text{TURBULENT}$$

$$\delta^* = L\cdot\frac{0.048}{(Re_L)^{1/5}} = 1.3188 \text{ mm}$$

$$H_L = \frac{WH_0 + 2\delta^*(W+H)}{W} = 0.41635 \text{ m}$$

$$\therefore H_L - H_0 = 4.35 \text{ mm} \star$$



COMPARED TO LAMINAR CASE: WE HAD $H_L - H_0 = 6.19$ mm (larger)

HERE, OUR Re_L IS $\approx$ 10 times larger

δ, δ^*, C_f, etc $\downarrow$ as $Re_L \uparrow$