# AERODYNAMIC DRAG ON VARIOUS OBJECTS

**In this lesson, we will**:

- Look at tables of drag coefficient for various objects (geometric shapes, parachutes, trees, people, buildings, etc.)
- Do a "student-friendly" example problem – drag on a bicycle

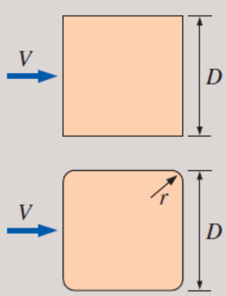
## Drag Coefficients on Objects of Various Shapes

See tables in the textbook, copied here. Table 11-1: 2-D bodies; Table 11-2: 3-D bodies.

**TABLE 11–1** 2-D BODIES

Drag coefficients $C_D$ of various two-dimensional bodies for Re > $10^4$ based on the frontal area $A = bD$, where $b$ is the length in direction normal to the page (for use in the drag force relation $F_D = C_D A \rho V^2/2$ where $V$ is the upstream velocity)

**Square rod**

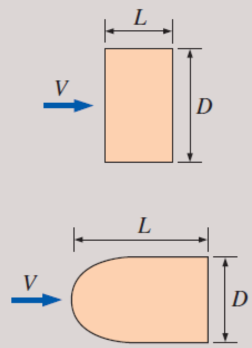



*Sharp corners*: $C_D = 2.2$

*Round corners* ($r/D = 0.2$): $C_D = 1.2$

b >> D for 2D body

**Rectangular rod**




*Sharp corners*:

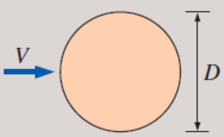
| $L/D$ | $C_D$ |
|---|---|
| 0.0* | 1.9 |
| 0.1 | 1.9 |
| 0.5 | 2.5 |
| 1.0 | 2.2 |
| 2.0 | 1.7 |
| 3.0 | 1.3 |

* Corresponds to thin plate

*Round front edge*:

| $L/D$ | $C_D$ |
|---|---|
| 0.5 | 1.2 |
| 1.0 | 0.9 |
| 2.0 | 0.7 |
| 4.0 | 0.7 |

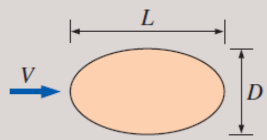
**Circular rod (cylinder)**




*Laminar*: $C_D = 1.2$

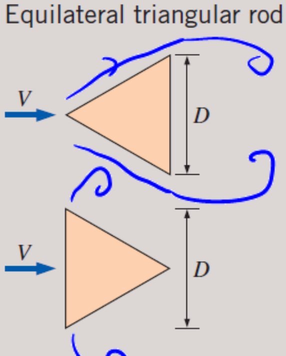
*Turbulent*: $C_D = 0.3$

**Elliptical rod**




| | $C_D$ | |
|---|---|---|
| $L/D$ | Laminar | Turbulent |
| 2 | 0.60 | 0.20 |
| 4 | 0.35 | 0.15 |
| 8 | 0.25 | 0.10 |

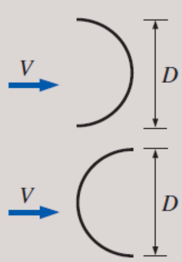
**Equilateral triangular rod**




$C_D = 1.5$

$C_D = 2.0$

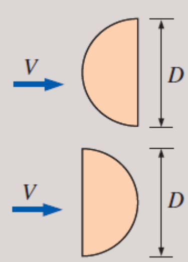
**Semicircular shell**




$C_D = 2.3$

$C_D = 1.2$

**Semicircular rod**




$C_D = 1.2$

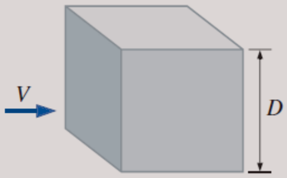
$C_D = 1.7$

Tables from Çengel and Cimbala, Ed. 4.

## TABLE 11–2 3-D BODIES

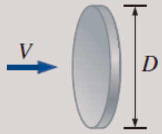
Representative drag coefficients $C_D$ for various three-dimensional bodies based on the frontal area for Re > $10^4$ unless stated otherwise (for use in the drag force relation $F_D = C_D A \rho V^2/2$ where $V$ is the upstream velocity)

**Cube, $A = D^2$**



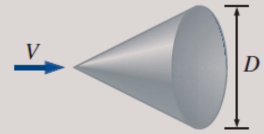

$C_D = 1.05$

**Thin circular disk, $A = \pi D^2/4$**



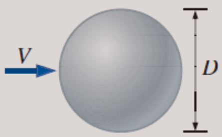

$C_D = 1.1$

**Cone (for $\theta = 30°$), $A = \pi D^2/4$**




$C_D = 0.5$

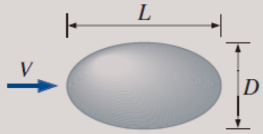
**Sphere, $A = \pi D^2/4$**




*Laminar*:
Re $\lesssim 2 \times 10^5$
$C_D = 0.5$
*Turbulent*:
Re $\gtrsim 2 \times 10^6$
$C_D = 0.2$

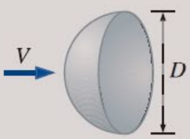
See Fig. 11–36 for $C_D$ vs. Re for smooth and rough spheres.

**Ellipsoid, $A = \pi D^2/4$**

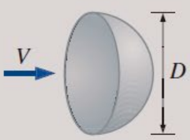



| $L/D$ | $C_D$ Laminar Re $\lesssim 2 \times 10^5$ | $C_D$ Turbulent Re $\gtrsim 2 \times 10^6$ |
|---|---|---|
| 0.75 | 0.5 | 0.2 |
| 1 | 0.5 | 0.2 |
| 2 | 0.3 | 0.1 |
| 4 | 0.3 | 0.1 |
| 8 | 0.2 | 0.1 |

**Hemisphere, $A = \pi D^2/4$**

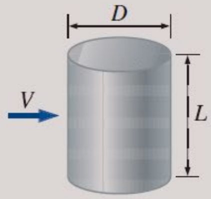



$C_D = 0.4$




$C_D = 1.2$

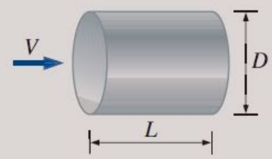
**Finite cylinder, vertical, $A = LD$**




| $L/D$ | $C_D$ |
|---|---|
| 1 | 0.6 |
| 2 | 0.7 |
| 5 | 0.8 |
| 10 | 0.9 |
| 40 | 1.0 |
| ∞ | 1.2 |

Values are for laminar flow (Re $\lesssim 2 \times 10^5$)

**Finite cylinder, horizontal, $A = \pi D^2/4$**

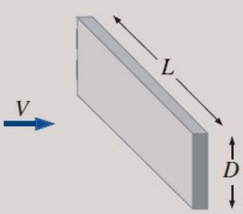



| $L/D$ | $C_D$ |
|---|---|
| 0.5 | 1.1 |
| 1 | 0.9 |
| 2 | 0.9 |
| 4 | 0.9 |
| 8 | 1.0 |

**Streamlined body, $A = \pi D^2/4$**

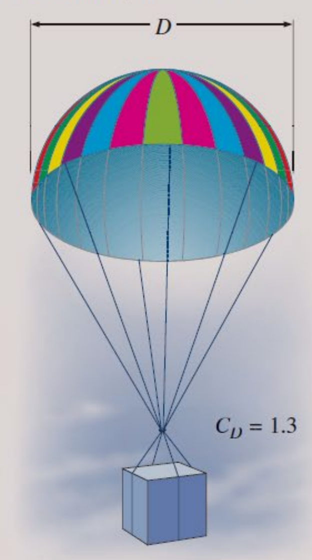

$C_D = 0.04$

**Rectangular plate, $A = LD$**



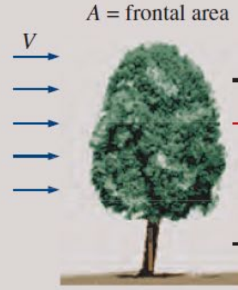

$C_D = 1.10 + 0.02\,(L/D + D/L)$
for $1/30 < (L/D) < 30$

**Parachute, $A = \pi D^2/4$**

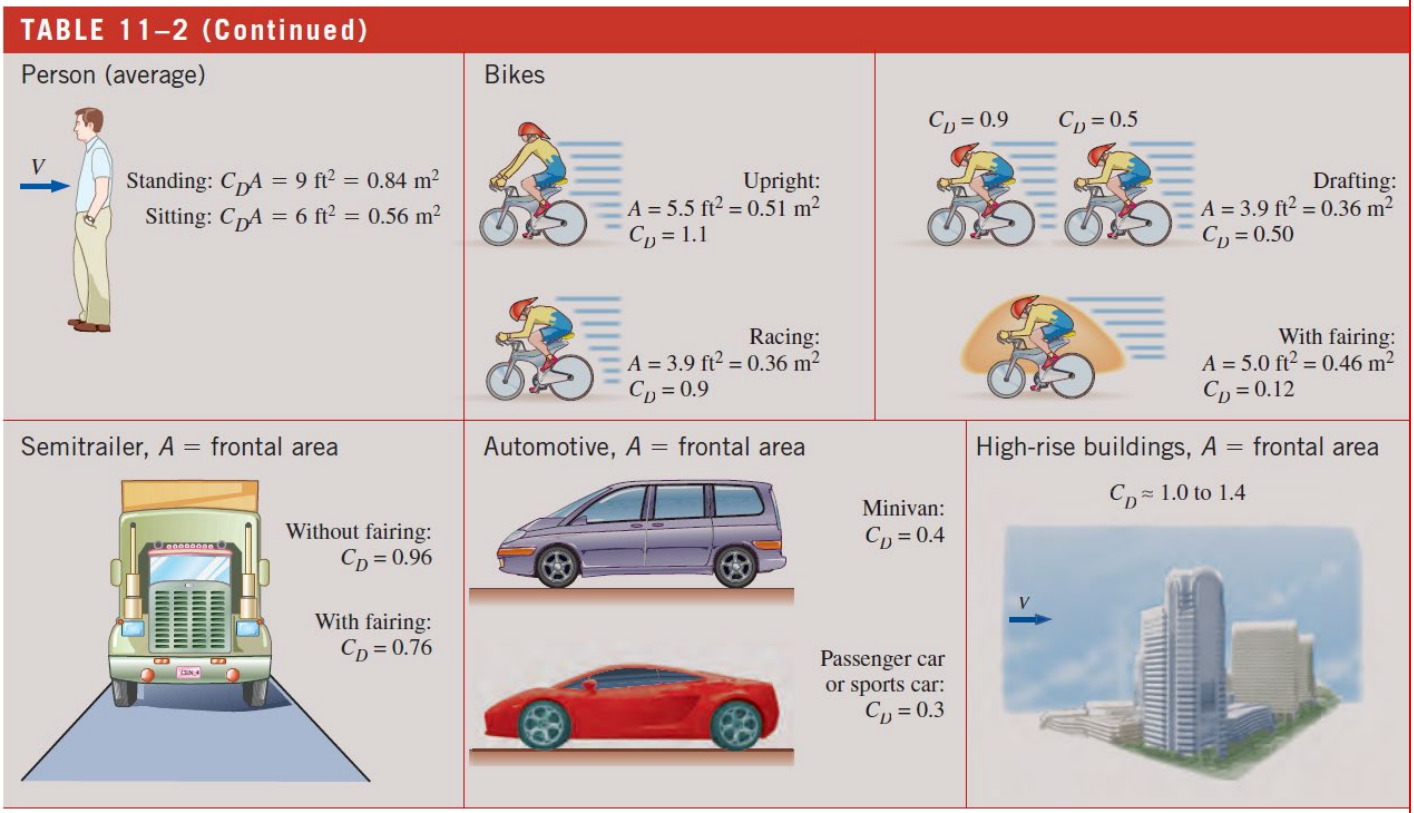



**Tree, $A$ = frontal area**

$A$ = frontal area



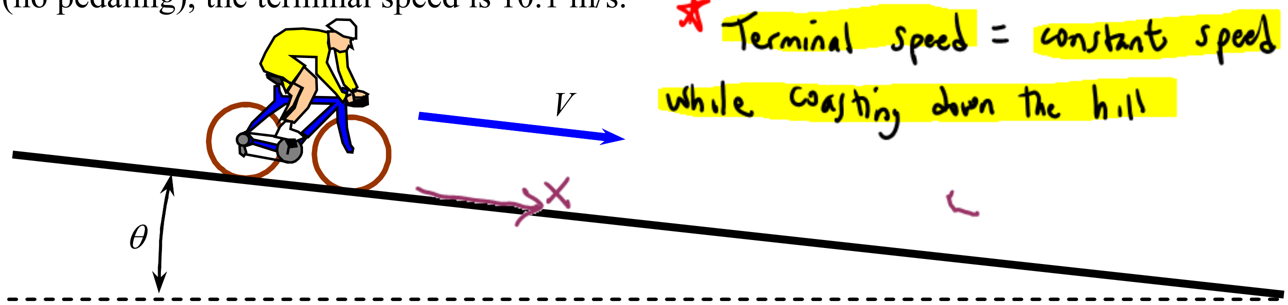

| $V$, m/s | $C_D$ |
|---|---|
| 10 | 0.4–1.2 |
| 20 | 0.3–1.0 |
| 30 | 0.2–0.7 |




## Example – Drag on a Bicycle Rolling Down a Hill

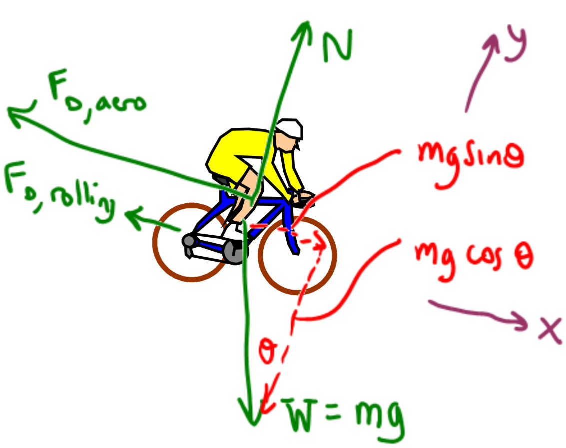
**Given**: A person coasts a bicycle down a long hill with a slope of 5° in order to measure the drag area of the bike and rider. The mass of the bike is 7.0 kg, the mass of the rider is 70.0 kg, and the rolling resistance of the bike is measured separately – it is 19.0 N. When the rider coasts down the hill (no pedaling), the terminal speed is 10.1 m/s.

★ Terminal speed = constant speed while coasting down the hill




(a) **To do**: Calculate the drag area $C_DA$ of the rider/bicycle combination in m².

**Solution**: First draw a free-body diagram of the bicycle and rider, showing all forces acting.




Since no acceleration

$$\sum \vec{F} = 0$$

$$\sum F_x = 0 \quad ; \quad \sum F_y = 0$$

$$mg\sin\theta - F_{D,\text{rolling}} - F_{D,\text{aero}} = 0$$

$$mg\sin\theta = F_{D,\text{rolling}} + \tfrac{1}{2}\rho V^2 C_D A$$

This is what we want to calculate

$$C_D A = \text{Drag area} = \frac{mg\sin\theta - F_{D,\text{rolling}}}{\tfrac{1}{2}\rho V^2}$$

answer in variable form

Plug in the values:

$$C_D A = \frac{(70+7)\,\text{kg}\left(9.807\,\frac{\text{m}}{\text{s}^2}\right)\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)\sin 5^\circ - 19.0\ \text{N}}{\tfrac{1}{2}\left(1.204\,\frac{\text{kg}}{\text{m}^3}\right)\left(10.1\,\frac{\text{m}}{\text{s}}\right)^2\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)} = 0.762\ \text{m}^2$$

$$C_D A = 0.762\ \text{m}^2$$

(**b**) **To do**: Calculate how much power in Watts (to the wheel) it would take for the person to ride this bike on a level road at the same speed (10.1 m/s).

**Solution**: Use the same equation we had in a previous lesson for automobiles:

$$\dot{W} = \mu_{\text{rolling}} W V + \tfrac{1}{2}\rho V^3 C_D A$$

$F_{D,\text{rolling}}$ — Does not change with speed

numbers:

$$\dot{W} = \left[(19.0\ \text{N})\left(10.1\,\frac{\text{m}}{\text{s}}\right) + \tfrac{1}{2}\left(1.204\,\frac{\text{kg}}{\text{m}^3}\right)\left(10.1\,\frac{\text{m}}{\text{s}}\right)^3\left(0.76233\ \text{m}^2\right)\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)\right]\left(\frac{\text{W}\cdot\text{s}}{\text{N}\cdot\text{m}}\right)$$

$$\dot{W} = 665.\ \text{W}$$

$\approx 0.89$ hp!