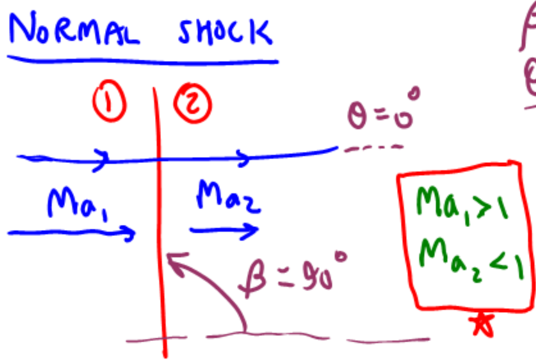
OBLIQUE SHOCKS

In this lesson, we will:

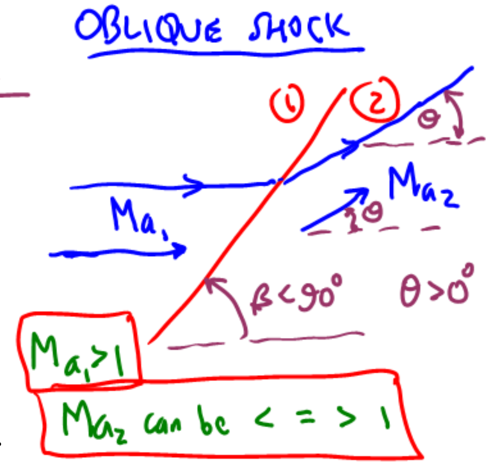
- Define **Oblique Shocks** and compare to normal shocks
- Discuss **The θ-β-Ma Relationship** and provide some qualitative comments about it
- Do two example problems

Definition and Geometry of an Oblique Shock

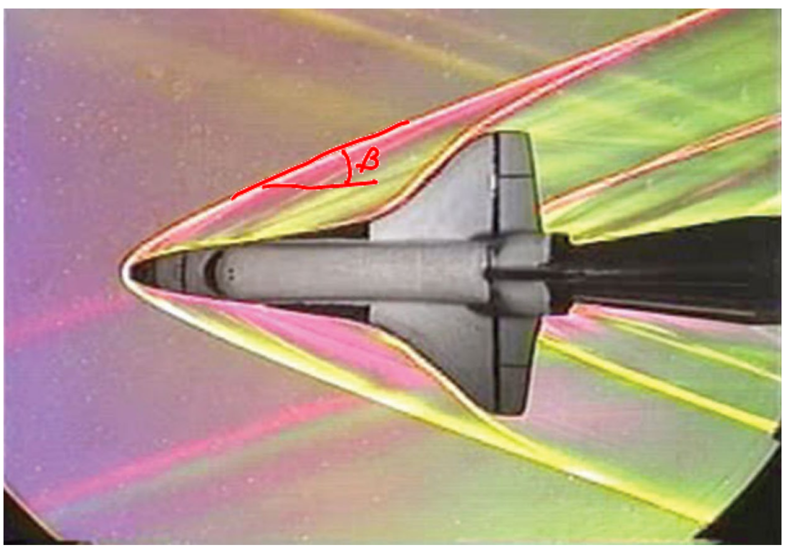
An oblique shock is a shock aligned at some angle other than 90° to the direction of flow.

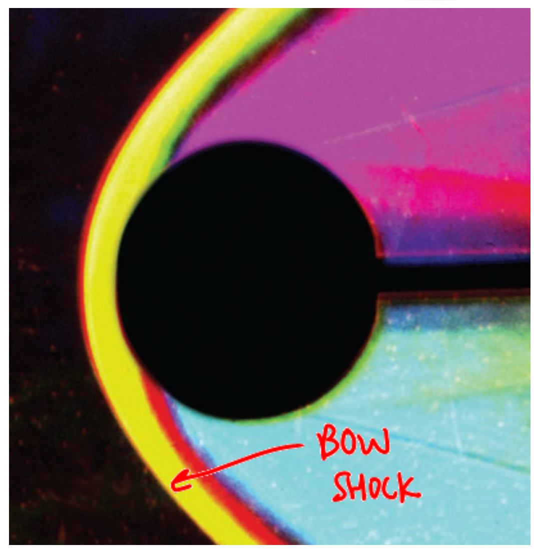


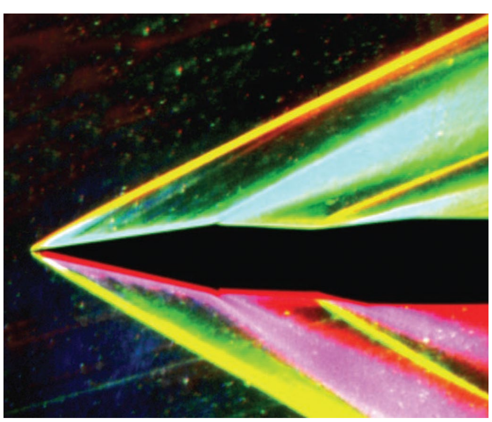
β = shock angle
θ = turning angle

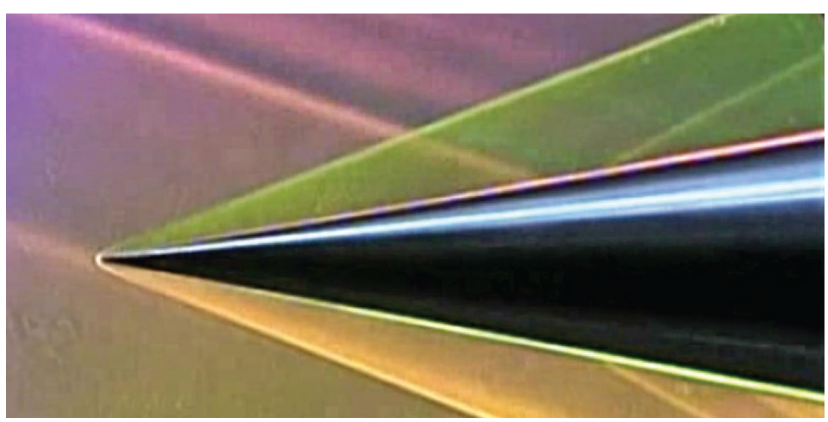


Below are some color schlieren images of oblique shocks.









2-D wedge   Figures from Çengel and Cimbala, Ed. 4.   cone (axisymmetric)

The sketch below shows the geometry of a typical oblique shock.

- Split $\vec{V}_1$ & $\vec{V}_2$ into <u>normal</u> & <u>tangential</u> components

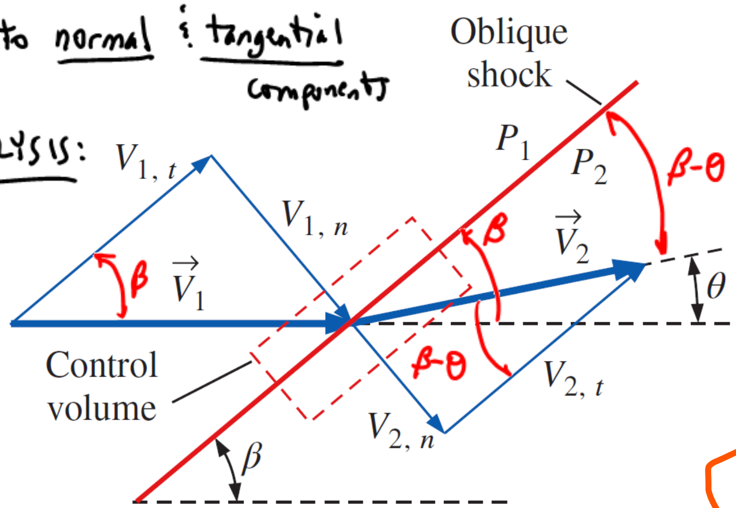


Figure from Çengel and Cimbala, Ed. 4.

KEYS TO OUR ANALYSIS:

1) $\boxed{V_{1,t} = V_{2,t}}$

Tangential speed is not affected by the shock

2) An oblique shock behaves exactly like a normal shock in terms of the normal velocity components.

$$\boxed{\begin{aligned} V_{1,n} &= V_1 \sin\beta & \quad V_{1,t} &= V_1 \cos\theta \\ V_{2,n} &= V_2 \sin(\beta-\theta) & \quad V_{2,t} &= V_2 \cos(\beta-\theta) \end{aligned}}$$

This should be beta, not theta

- To analyze oblique shocks,

ALL EQUATIONS FOR A NORMAL SHOCK STILL APPLY, BUT USE $M_{1,n}$ INSTEAD OF M_1 IN THE EQUATIONS

ROTATED REFERENCE FRAME

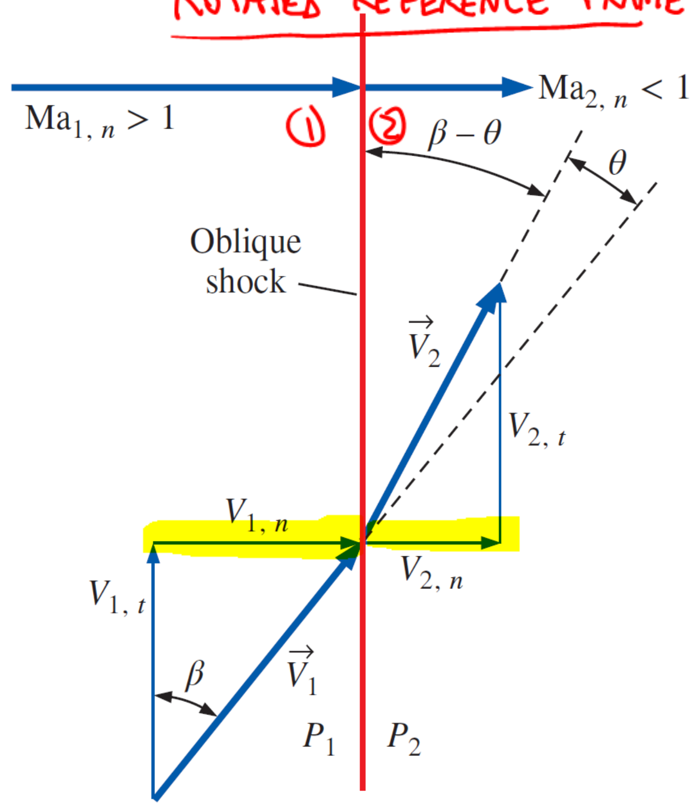
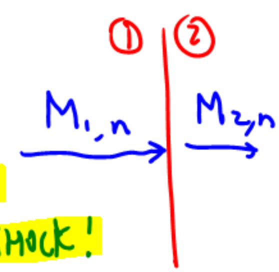


Figure from Çengel and Cimbala, Ed. 4.

IF MOVE UP AT SPEED $V_{1,t}$

WE SEE A NORMAL SHOCK!



The Theta-Beta-Mach Number Equation and Plot

- Do some algebra & trig

$$\boxed{V_{1,t} = \frac{V_{1,n}}{\tan\beta}} \quad \& \quad \boxed{V_{2,t} = \frac{V_{2,n}}{\tan(\beta-\theta)}}$$

BUT $V_{1,t} = V_{2,t}$!!

Equate these

& use
- cons. of mass
- normal shock equations
- some trig identities

(see textbook)

The θ–β–Ma equation for oblique shocks:

$$\tan\theta = \frac{2\cot\beta\left(\mathrm{Ma}_1^{\,2}\sin^2\beta - 1\right)}{\mathrm{Ma}_1^{\,2}\left[k + \cos(2\beta)\right] + 2}$$

OUR WORKHORSE EQUATION FOR OBLIQUE SHOCKS

NOTE: IT IS EXPLICIT IF CALCULATING θ FOR KNOWN k & Ma_1 & β
IT IS IMPLICIT IF CALCULATING β FOR KNOWN k & Ma_1 & θ

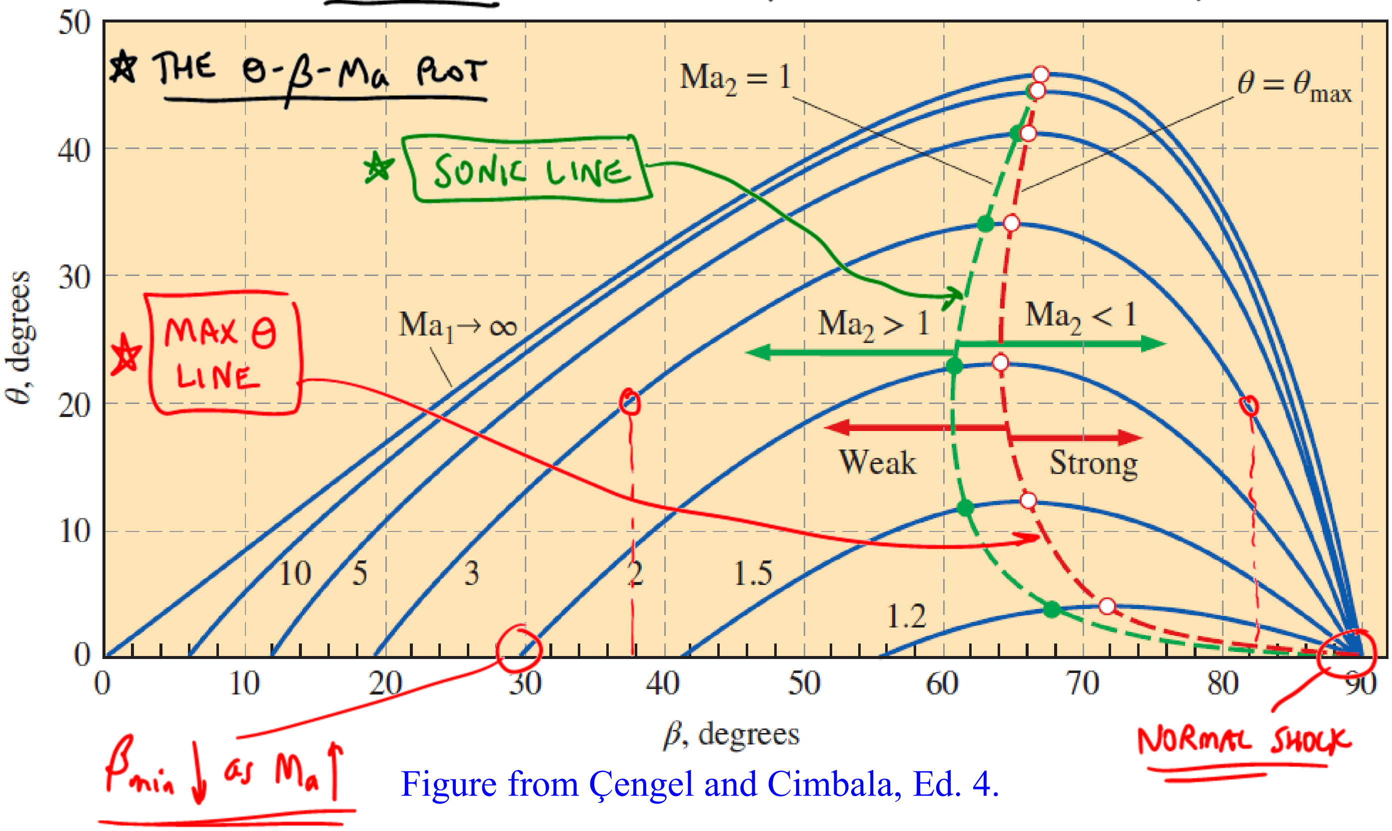
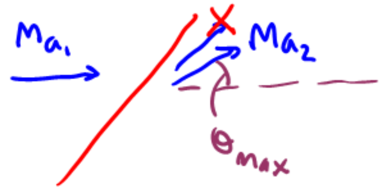


Figure from Çengel and Cimbala, Ed. 4.

COMMENTS:

- Curves grow in width & height as Ma ↑
- For each Ma there is a maximum value of θ (θ_{max} ↑ as Ma ↑)



- Each Mach # curve has a maximum β of 90°

A NORMAL SHOCK IS THE STRONGEST POSSIBLE OBLIQUE SHOCK

- Each Mach # curve also has a <u>minimum</u> β

AT $\beta = \beta_{min}$, THIS IS THE WEAKEST POSSIBLE SHOCK

$\boxed{\beta_{min} = \mu}$ = Mach angle = angle of a Mach wave = weakest possible shock (nearly isentropic)

An oblique shock with $\beta < \mu$ cannot exist

- — — — = SONIC LINE
- — — — = MAX θ LINE

IF TRY TO FORCE $\theta > \theta_{max}$, SHOCK JUMPS FORWARD & BECOMES A <u>BOW SHOCK</u>

CFD Example – 2-D wedge of various half angles (CFD by J. M. Cimbala):

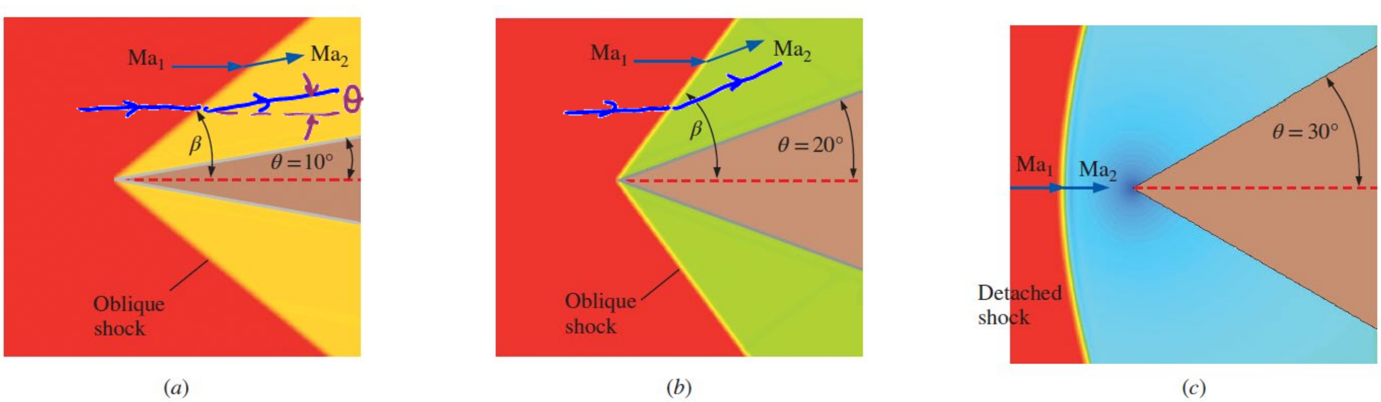
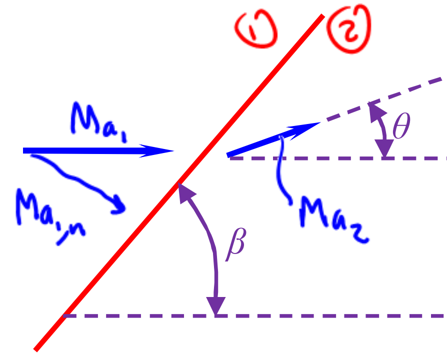


Figure from Çengel and Cimbala, Ed. 4.

Oblique shock formed by supersonic flow impinging on three two-dimensional wedges with different wedge angles: θ = (a) 10°, (b) 20°, and (c) 30°. Upstream Mach number = 2.00. Contours of Mach number are shown, ranging from 0.2 (dark blue) to 2.0 (dark red).

Example: Steady 2-D Oblique Shock



Given: Air flowing at supersonic speed encounters an oblique shock. Here are some values:

- Upstream: P_1 = 101.325 kPa, T_1 = 288.0 K, Ma_1 = 3.00
- $\beta = 50^{\circ}$ (oblique shock angle)

To do: Calculate turning angle θ, $\mathrm{Ma}_{1,n}$, air properties P_2 and T_2, and Mach number Ma_2.

Assumptions and Approximations: Steady, ideal gas.

Solution:

We apply the θ–β–Ma equation for oblique shocks, where we have solved for θ:

$$\theta = \arctan\left[\frac{2\cot\beta\left(\mathrm{Ma}_1^{\,2}\sin^2\beta - 1\right)}{\mathrm{Ma}_1^{\,2}\left[k + \cos(2\beta)\right] + 2}\right] = \arctan\left[\frac{2\cot(50^{\circ})\left(3.00^2 \sin^2(50^{\circ}) - 1\right)}{3.00^2\left[1.40 + \cos(2\times 50^{\circ})\right] + 2}\right]$$

$$\theta = 28.8601^{\circ} \qquad \boxed{\theta = 28.9^{\circ}}$$

- Calculate $\boxed{Ma_{1,n} = Ma_1 \sin\beta} = 3.00 \sin(50^{\circ}) = 2.29813$

$$\boxed{Ma_{1,n} = 2.30}$$

- Use normal shock eqs to calculate P_2, Ma_2 & T_2

★ BUT USE $Ma_{1,n}$ INSTEAD OF Ma_1 IN THE EQUATIONS ★

$$P_2 = \left(\frac{P_2}{P_1}\right)P_1 = \left(\frac{2k\,Ma_{1,n}^{\,2} - k + 1}{k+1}\right)P_1 = \left(\frac{2(1.40)(2.29813^2) - 1.40 + 1}{1.40 + 1}\right)(101.325\text{ kPa})$$

$$P_2 = 607.442\text{ kPa} \Rightarrow \boxed{P_2 = 607.\text{ kPa}}$$

$$Ma_{2,n} = \sqrt{\frac{(k-1)\,Ma_{1,n}^{\,2} + 2}{2k\,Ma_{1,n}^{\,2} - k + 1}} = 0.534634 = Ma_{2,n}$$

But $\boxed{Ma_{2,n} = Ma_2 \sin(\beta - \theta)} \rightarrow \boxed{Ma_2 = \frac{Ma_{2,n}}{\sin(\beta - \theta)}} = 1.4824$

$$\boxed{Ma_2 = 1.48}$$

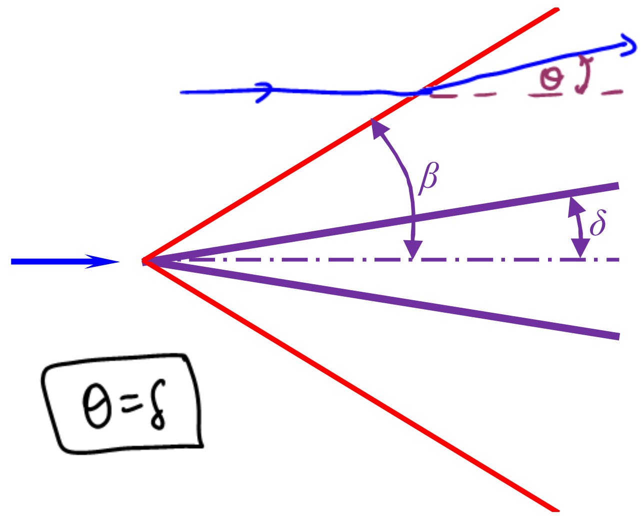
SUPERSONIC !!

$$T_2 = \left(\frac{T_2}{T_1}\right)T_1 = \left(\frac{2 + Ma_{1,n}^{\,2}(k-1)}{2 + Ma_{2,n}^{\,2}(k-1)}\right)T_1 \rightarrow \boxed{T_2 = 560\text{ K}}$$

Example: Steady 2-D Supersonic Flow Over a Wedge

Given: Air flowing at supersonic speed strikes a two-dimensional wedge. Here are some values:

- Upstream: P_1 = 101.325 kPa, T_1 = 288.0 K, Ma_1 = 3.00
- δ = 20° (wedge half angle)



To do: Calculate shock angle β, air properties P_2 and T_2, and Mach number Ma_2.

Assumptions and Approximations: Steady, ideal gas, adiabatic, ignore boundary layer effects along the walls.

Solution:

We apply the θ–β–Ma equation for oblique shocks:

$$\boxed{\tan\theta = \frac{2\cot\beta\left(Ma_1^{\,2}\sin^2\beta - 1\right)}{Ma_1^{\,2}\left[k + \cos(2\beta)\right] + 2}}$$

MUST SOLVE IMPLICITLY FOR β !!
(ITERATE, USE FOR EXAMPLE THE F.P.M)

• FOR $k = 1.40$, $Ma_1 = 3.00$, $\theta = 20°$, we get 2 roots

$\beta = 37.7636°$ $\boxed{\beta \simeq 37.8°}$; $\beta = 82.1470°$ $\boxed{\beta \simeq 82.1°}$

☆ WEAK OBLIQUE SHOCK

☆ STRONG OBLIQUE SHOCK

FROM HERE ON THE PROCEDURE IS IDENTICAL TO THAT OF THE PREVIOUS PROBLEM

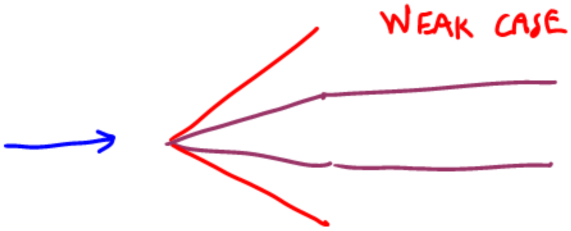
(TRY IT ON YOUR OWN)

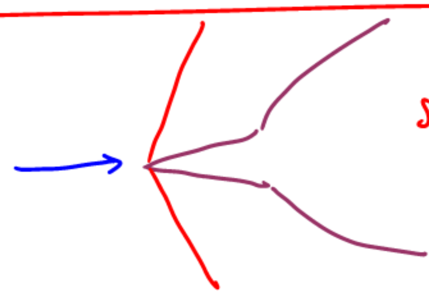
ANSWERS:

$\boxed{\text{WEAK CASE: } \beta = 37.8° \quad P_2 = 382.\ \text{kPa} \quad Ma_2 = 1.99 \quad T_2 = 449.\ \text{K}}$

$\boxed{\text{STRONG CASE: } \beta = 82.1° \quad P_2 = 1020\ \text{kPa} \quad Ma_2 = 0.539 \quad T_2 = 762.\ \text{K}}$

WEAK CASE





STRONG CASE