

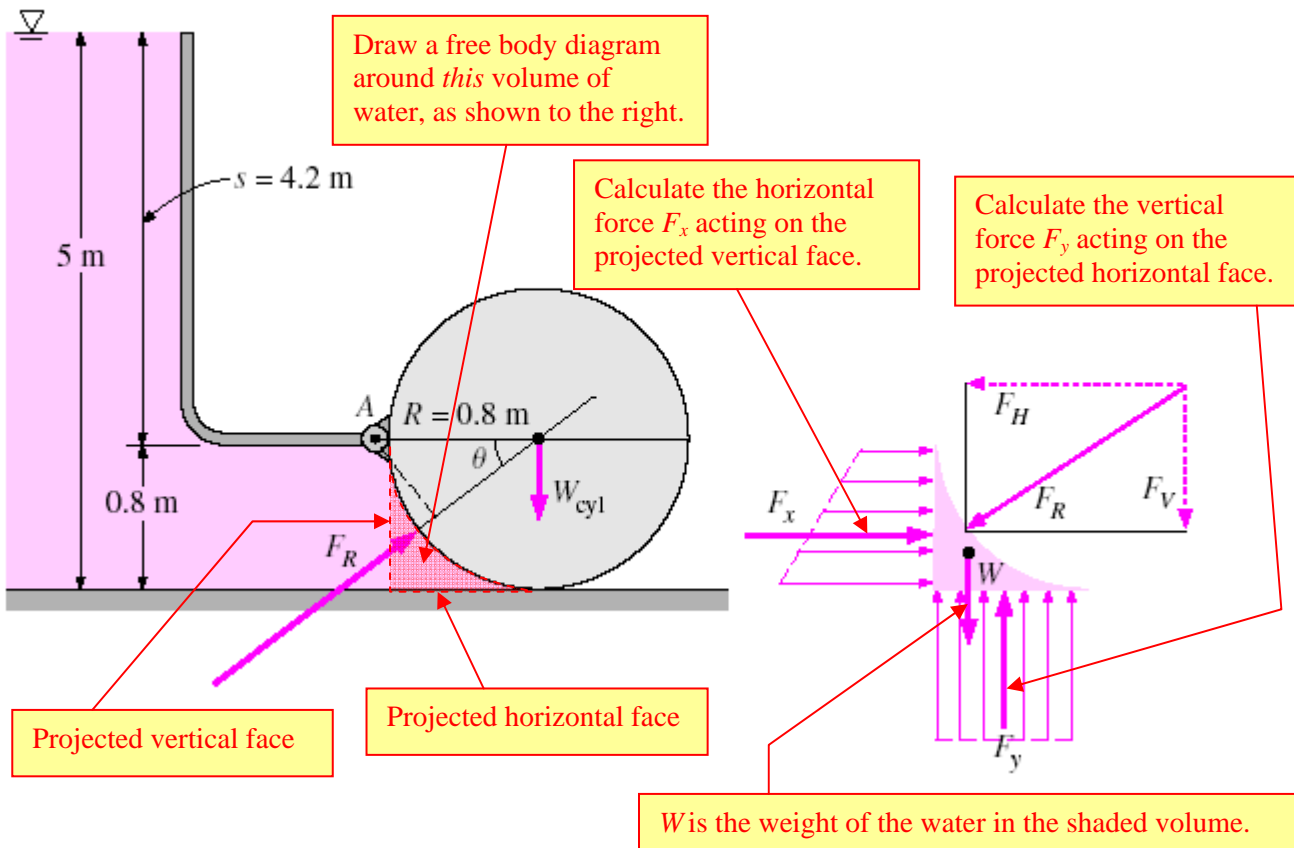
# Example Problem – Hydrostatic Pressure Force on Curved Surfaces

(Example Problem 3-9, Çengel and Cimbala)

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## EXAMPLE 3-9 A Gravity-Controlled Cylindrical Gate

A long solid cylinder of radius 0.8 m hinged at point A is used as an automatic gate, as shown in Fig. 3-38. When the water level reaches 5 m, the gate opens by turning about the hinge at point A. Determine (a) the hydrostatic force acting on the cylinder and its line of action when the gate opens and (b) the weight of the cylinder per m length of the cylinder.



**SOLUTION** The height of a water reservoir is controlled by a cylindrical gate hinged to the reservoir. The hydrostatic force on the cylinder and the weight of the cylinder per m length are to be determined.

**Assumptions** 1 Friction at the hinge is negligible. 2 Atmospheric pressure acts on both sides of the gate, and thus it cancels out.

**Properties** We take the density of water to be  $1000 \text{ kg/m}^3$  throughout.

**Analysis** (a) We consider the free-body diagram of the liquid block enclosed by the circular surface of the cylinder and its vertical and horizontal projections. The hydrostatic forces acting on the vertical and horizontal plane surfaces as well as the weight of the liquid block are determined as

Horizontal force on vertical surface:

$$F_H = F_x = P_{\text{ave}} A = \rho g h_C A = \rho g (s + R/2) A$$
$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(4.2 + 0.8/2 \text{ m})(0.8 \text{ m} \times 1 \text{ m}) \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right)$$

$$= 36.1 \text{ kN}$$

The centroid of the projected vertical surface is located at a depth of  $s + R/2$  from the water surface.

Vertical force on horizontal surface (upward):

$$F_y = P_{\text{ave}} A = \rho g h_C A = \rho g h_{\text{bottom}} A$$

$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(5 \text{ m})(0.8 \text{ m} \times 1 \text{ m}) \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right)$$

$$= 39.2 \text{ kN}$$

The centroid of the projected horizontal surface is located at a depth of  $h_{\text{bottom}}$  from the water surface. ( $P$  is constant along this surface since it is at a constant depth.)

Weight of fluid block per m length (downward):

$$W = mg = \rho g V = \rho g (R^2 - \pi R^2/4)(1 \text{ m})$$

$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.8 \text{ m})^2 (1 - \pi/4)(1 \text{ m}) \left( \frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2} \right)$$

$$= 1.3 \text{ kN}$$

We subtract weight  $W$  since the shaded volume is *below* the curved surface.

Therefore, the net upward vertical force is

$$F_V = F_y - W = 39.2 - 1.3 = 37.9 \text{ kN}$$

Then the magnitude and direction of the hydrostatic force acting on the cylindrical surface become

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{36.1^2 + 37.9^2} = 52.3 \text{ kN}$$

$$\tan \theta = F_V/F_H = 37.9/36.1 = 1.05 \rightarrow \theta = 46.4^\circ$$

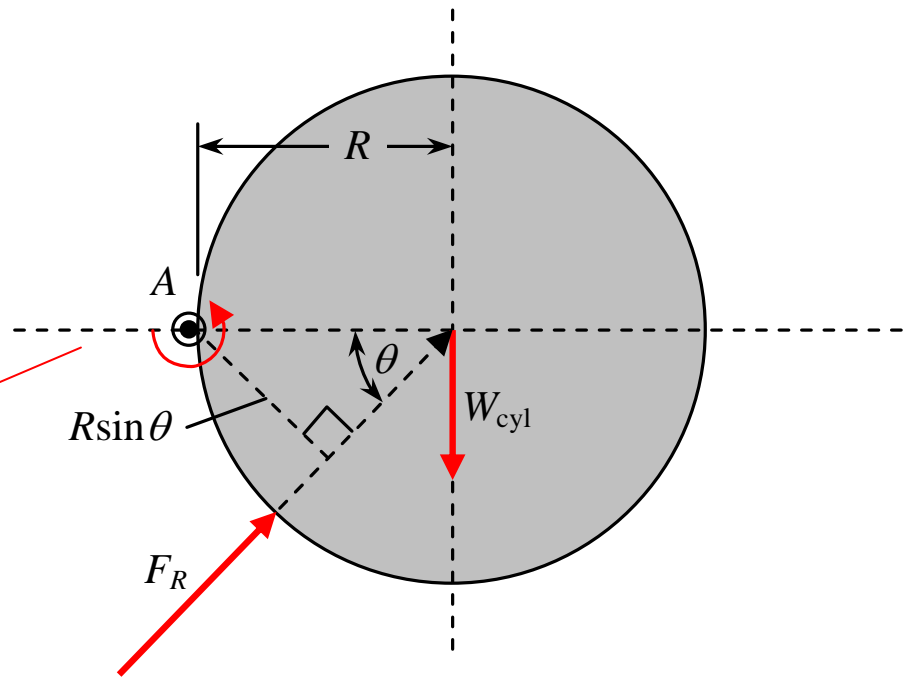
Therefore, the magnitude of the hydrostatic force acting on the cylinder is 52.3 kN per m length of the cylinder, and its line of action passes through the center of the cylinder making an angle  $46.4^\circ$  with the horizontal.

It turns out that for *cylindrical* surfaces (a circular arc shape), the resultant hydrostatic force acting on the surface always passes through the *center* of the circular arc.

(b) When the water level is 5 m high, the gate is about to open and thus the reaction force at the bottom of the cylinder is zero. Then the forces other than those at the hinge acting on the cylinder are its weight, acting through the center, and the hydrostatic force exerted by water. Taking a moment about point A at the location of the hinge and equating it to zero gives

$$F_R R \sin \theta - W_{\text{cyl}} R = 0 \rightarrow W_{\text{cyl}} = F_R \sin \theta = (52.3 \text{ kN}) \sin 46.4^\circ = \mathbf{37.9 \text{ kN}}$$

We take the moment about point A, using counterclockwise as positive. As shown in the sketch to the right, there are only two moments acting about point A: (1) the weight of the cylinder times its moment arm, which is the radius of the cylinder, and (2) the net hydrostatic force acting on the portion of the cylinder that is in contact with the water times its moment arm. Its force is  $F_R$  and its moment arm is  $R \sin \theta$  which is the perpendicular distance from A to the line of action of the force.



**Discussion** The weight of the cylinder per m length is determined to be 37.9 kN. It can be shown that this corresponds to a mass of 3863 kg per m length and to a density of  $1921 \text{ kg/m}^3$  for the material of the cylinder.