Fully Developed Pipe Flow Equations

There are empirical equations available to use in place of the Moody chart. The most useful one (in fact, the equation with which the turbulent portion of the Moody chart is drawn) is:

The Colebrook equation

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \qquad \text{(turbulent flow)} \qquad \textbf{(8–50)}$$

Note: This is $\log_{10}$, not the natural log, ln.

Unfortunately, the Colebrook equation is ***implicit*** in *f* (since *f* appears on both sides of the equation), and the equation must be solved by iteration. An approximation to the Colebrook equation was created by Haaland, accurate to ±2% compared to the Colebrook equation:

The Haaland equation

$$\frac{1}{\sqrt{f}} \cong -1.8 \log\left[\frac{6.9}{\mathrm{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right] \qquad \textbf{(8–51)}$$

Also $\log_{10}$, not ln.

Finally, Swamee and Jain generated some approximations that can be used directly in place of the Colebrook equation when solving problems of certain types. These are also accurate to ±2% compared to iterative solutions using the Colebrook equation:

The Swamee and Jain equations

$$h_L = 1.07\frac{\dot{V}^2 L}{g D^5}\left\{\ln\left[\frac{\varepsilon}{3.7D} + 4.62\left(\frac{\nu D}{\dot{V}}\right)^{0.9}\right]\right\}^{-2} \qquad \begin{array}{c} 10^{-6} < \varepsilon/D < 10^{-2} \\ 3000 < \mathrm{Re} < 3\times 10^{8} \end{array} \qquad \textbf{(8–52)}$$

$$\dot{V} = -0.965\left(\frac{g D^5 h_L}{L}\right)^{0.5} \ln\left[\frac{\varepsilon}{3.7D} + \left(\frac{3.17\nu^2 L}{g D^3 h_L}\right)^{0.5}\right] \qquad \mathrm{Re} > 2000 \qquad \textbf{(8–53)}$$

$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{L\dot{V}^2}{g h_L}\right)^{4.75} + \nu\dot{V}^{9.4}\left(\frac{L}{g h_L}\right)^{5.2}\right]^{0.04} \qquad \begin{array}{c} 10^{-6} < \varepsilon/D < 10^{-2} \\ 5000 < \mathrm{Re} < 3\times 10^{8} \end{array} \qquad \textbf{(8–54)}$$

Note: The Swamee & Jain equations are not valid when *minor losses* are significant.