ME 33, Fluid Flow
Chapter 3: Examples of Archimedes Principle

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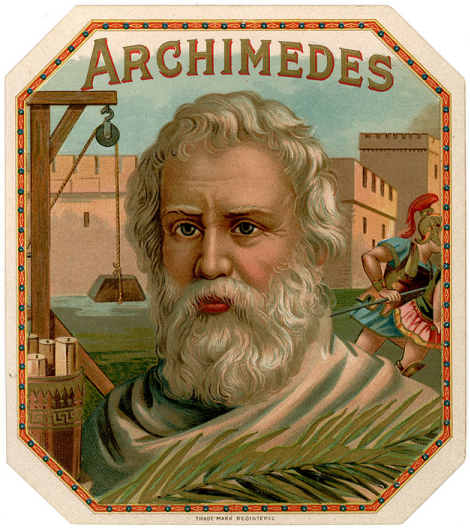
The Golden Crown of Hiero II, King of Syracuse



- Archimedes, 287-212 B.C.
- Hiero, 306-215 B.C.
- Hiero learned of a rumor where the goldsmith replaced some of the gold in his crown with silver. Hiero asked Archimedes to determine whether the crown was pure gold.
- Archimedes had to develop a nondestructive testing method!

The Golden Crown of Hiero II, King of Syracuse

- The weight of the crown and nugget are the same in air: $W_c = \rho_c V_c = W_n = \rho_n V_n$.
- If the crown is pure gold, $\rho_c = \rho_n$ which means that the volumes must be the same, $V_c = V_n$.
- In water, the buoyancy force is $B = \rho_{H_2O} V$.
- If the scale becomes unbalance, this implies that the $V_c \neq V_n$, which in turn means that the $\rho_c \neq \rho_n$.
- Goldsmith was shown to be a fraud!

Hydrostatic Bodyfat Testing

- What is the best way to measure body fat?
- Hydrostatic Bodyfat Testing using Archimedes Principle!
- Process
  - Measure body weight W. $W = \rho_{body} V$.
  - Get in tank, expel all air, and measure apparent weight W_a
  - Buoyancy force $B = W - W_a = \rho_{H_2O} V$. This permits computation of body volume.
  - Body density can be computed $\rho_{body} = \frac{W}{V}$.
  - Body fat can be computed from formulas.

Hydrostatic Bodyfat Testing

Table 1.4 Population-Specific Two-Component Model Conversion Formulas

	Population	Age	Gender	%BF	FFBd (g/cc)*
ETHNICITY	African American	9-17	Female	(5.24 / Db) – 4.82	1.088
		19-45	Male	(4.86 / Db) – 4.39	1.106
		24-79	Female	(4.86 / Db) – 4.39	1.106
	American Indian	18-62	Male	(4.97 / Db) – 4.52	1.099
		18-60	Female	(4.81 / Db) – 4.34	1.108
	Asian Japanese Native	18-48	Male	(4.97 / Db) – 4.52	1.099
			Female	(4.76 / Db) – 4.28	1.111
		61-78	Male	(4.87 / Db) – 4.41	1.105
			Female	(4.95 / Db) – 4.50	1.100
	Singaporean (Chinese, Indian, Malay)		Male	(4.94 / Db) – 4.48	1.102
			Female	(4.84 / Db) – 4.37	1.107
	Caucasian	8-12	Male	(5.27 / Db) – 4.85	1.086
			Female	(5.27 / Db) – 4.85	1.086
		13-17	Male	(5.12 / Db) – 4.69	1.092
			Female	(5.19 / Db) – 4.76	1.090
		18-59	Male	(4.95 / Db) – 4.50	1.100
			Female	(4.96 / Db) – 4.51	1.101
		60-90	Male	(4.97 / Db) – 4.52	1.099
			Female	(5.02 / Db) – 4.57	1.098
	Hispanic		Male	NA	NA
		20-40	Female	(4.87 / Db) – 4.41	1.105
ATHLETES	Resistance trained	24 ± 4	Male	(5.21 / Db) – 4.78	1.089
		35 ± 6	Female	(4.97 / Db) – 4.52	1.099
	Endurance trained	21 ± 2	Male	(5.03 / Db) – 4.59	1.097
		21 ± 4	Female	(4.95 / Db) – 4.50	1.100
	All sports	18-22	Male	(5.12 / Db) – 4.68	1.093
		18-22	Female	(4.97 / Db) – 4.52	1.099
CLINICAL POPULATIONS**	Anorexia nervosa	15-44	Female	(4.96 / Db) – 4.51	1.101
	Cirrhosis Childs A			(5.33 / Db) – 4.91	1.084
	Childs B			(5.48 / Db) – 5.08	1.078
	Childs C			(5.69 / Db) – 5.32	1.070
	Obesity	17-62	Female	(4.95 / Db) – 4.50	1.100
	Spinal cord injury (paraplegic/ quadriplegic)	18-73	Male	(4.67 / Db) – 4.18	1.116
		18-73	Female	(4.70 / Db) – 4.22	1.114

*FFBd = fat-free body density based on average values reported in selected research articles.
NA = no data available for this population subgroup.
**There are insufficient multicomponent model data to estimate the average FFBd of the following clinical populations: coronary artery disease, heart/lung transplants, chronic obstructive pulmonary disease, cystic fibrosis, diabetes mellitus, thyroid disease, HIV/AIDS, cancer, kidney failure (dialysis), multiple sclerosis, and muscular dystrophy.

For example

- women 8-12,
$$BF\% = \frac{5.27}{\rho_{body}} - 4.85.$$
- women 18-59,
$$BF\% = \frac{4.96}{\rho_{body}} - 4.51.$$

Hydrostatic Bodyfat Testing in Air?

- Same methodology as Hydrostatic testing in water.
- What are the ramifications of using air?
  - Density of air is 1/1000th of water.
  - Temperature dependence of air.
  - Measurement of small volumes.
  - Used by NCAA Wrestling (there is a BodPod on PSU campus)!