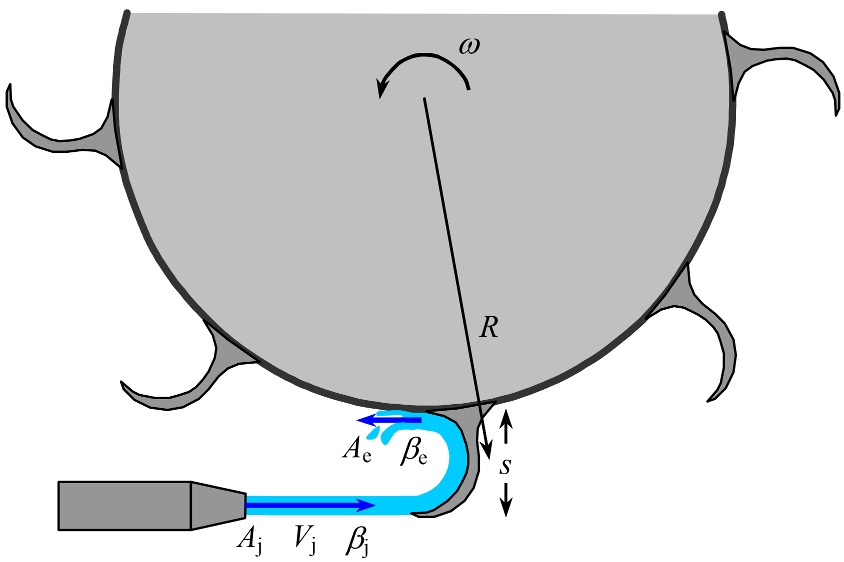
Moving Control Volume Example – Water Turbine

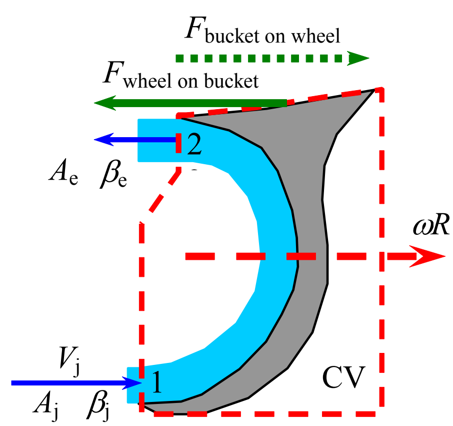
Given: An impulse turbine is driven by a high-speed water jet (average jet velocity V_j over jet area A_j, with momentum flux correction factor β_j) that impinges on turning buckets attached to a turbine wheel as shown. The turbine wheel rotates at angular velocity ω, and is horizontal; therefore, gravity effects are not important in this problem. (The view in the sketch is from the top.) The turning buckets turn the water approximately 180 degrees, and the water exits the bucket over exit cross-sectional area A_e with exit momentum flux correction factor β_e. Bucket dimension s is much smaller than turbine wheel radius R.



(a) **To do**: Calculate the force of the bucket on the turbine wheel, $F_{\text{bucket on wheel}}$, at the instant in time when the bucket is in the position shown.

Solution: We choose a control volume surrounding the bucket, cutting through the water jet at the inlet to the bucket, and cutting through the water exiting the bucket. Note that this is a *moving control volume*, moving to the right at speed ωR. We also cut through the welded joint between the bucket and the turbine wheel, where the force $F_{\text{bucket on wheel}}$ is to be calculated. Because of Newton's third law, the force acting on the control volume at this location is equal in magnitude, but opposite in direction, and we call it $F_{\text{bucket on wheel}}$.

Since the pressure through an incompressible jet exposed to atmospheric air is equal to P_{atm}, the pressure at the inlet (1) is equal to P_{atm}, and the pressure at the exit (2) is equal to P_{atm}.



(b) **To do**: Calculate the power delivered to the turbine wheel.

Solutions for parts (a) and (b) to be completed in class.