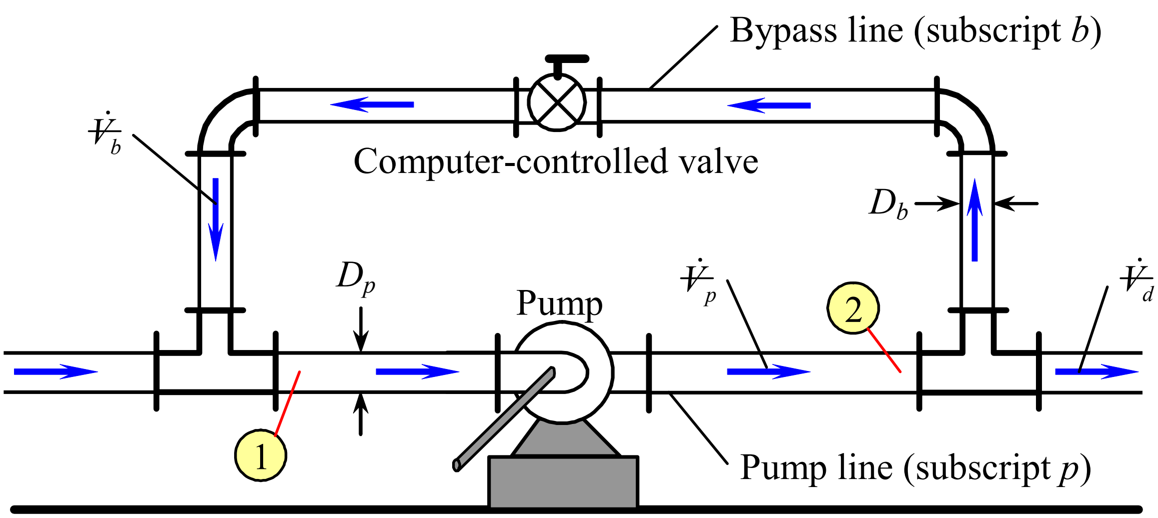
Example Problem – Parallel Pipe Network and Pump Bypass System

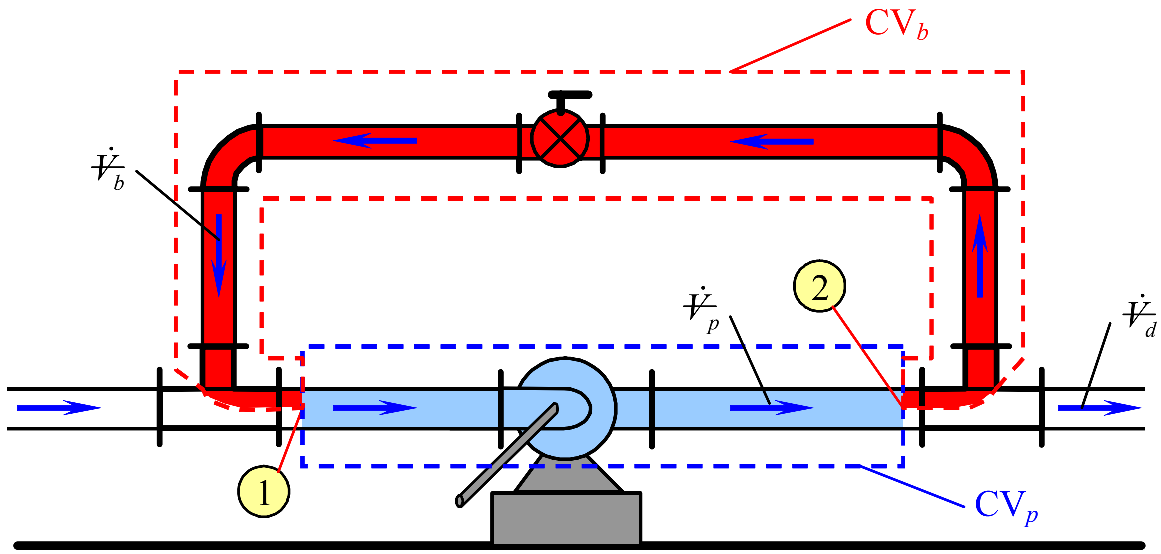
Given: In some applications (e.g., nuclear reactor cooling), it is critical that the volume flow rate of a fluid remains constant, regardless of head changes in the system downstream (within specified limits, of course). One method of ensuring a constant volume flow rate is to install an oversized pump to drive the flow. A bypass line is then installed *in parallel* with the pump so that some of the fluid (bypass volume flow rate $\dot{V}_b$) recirculates through the bypass line as shown. Based on feedback from a downstream volume flowmeter, the bypass valve is then adjusted by a computer to control both $\dot{V}_b$ and the volume flow rate through the pump $\dot{V}_p$ such that the volume flow rate $\dot{V}_d$ downstream of the system remains constant. In this particular case, water (ρ = 998 kg/m^3, μ = 1.00 × 10^{-3} kg/m·s) needs to be supplied at a constant downstream volume flow rate of $\dot{V}_d$ = 0.20 m^3/s. The pump's performance curve is given by the pump manufacturer as $\boxed{h_{\text{pump,u,supply}} = a\left(b - c\dot{V}_p^{\,2}\right)}$ where a = 100 m, b = 1.0, and c = 1.0 s^2/m^6. The units of $h_{\text{pump,u,supply}}$ are [m] and the units of $\dot{V}_p$ are [m^3/s]. The pump line has a diameter D_p = 0.50 m and length L_p = 3.0 m, while the bypass line has a diameter D_b = 0.50 m and length L_b = 5.0 m. All pipes have roughness ε = 0.002 m. Minor losses in the system include two flanged 90° bends (K_L = 0.20) and a gate valve (0.2 < K_L < ∞) in the bypass line, and two flanged tees (*line flow* K_L = 0.2, and *branch flow* K_L = 1.0).



To Do: Calculate and plot how volume flow rates $\dot{V}_p$, $\dot{V}_b$, and $\dot{V}_d$ vary with the minor loss coefficient of the valve as it goes from fully open ($K_{L,\text{ valve}}$ = 0.2) to fully closed ($K_{L,\text{ valve}} \to \infty$).

Solution:

- First, as always, we need to pick a control volume. In this case, we need *two* control volumes since there are two branches in the parallel pipe system. After careful thought (and experience), we decide that the most appropriate control volumes go between points (1) and (2) as labeled on the above sketch. On the next page, we re-draw the flow system including the two control volumes. In parallel pipe systems it is useful to imagine that the fluid in the bypass line (CV_b) is colored red, while that in the pump line (CV_p) is colored blue. This is an *artificial* separation of the fluid into the two branches since the fluid mixes because of turbulence. However, it is useful as an aid to drawing the control volumes. Note that CV_p has its inlet at (1) and its outlet at (2), while CV_b has its inlet at (2) and its outlet at (1).



- We apply conservation of mass at either tee:

$$\boxed{\dot{V}_p = \dot{V}_b + \dot{V}_d}$$

Note: This equation couples the two control volumes together. Other than this, we treat the two control volume separately in the analysis below.

- We apply the head form of the energy equation for CV$_p$ from inlet (1) to outlet (2), assuming that the flow at both (1) and (2) is fully developed turbulent pipe flow so that both velocity and kinetic energy correction factor are the same at (1) as at (2):

$z_1 = z_2$

$$\frac{P_1}{\rho g}+\alpha_1\cancel{\frac{V_1^2}{2g}}+\cancel{z_1}+h_{\text{pump,u}}=\frac{P_2}{\rho g}+\alpha_2\cancel{\frac{V_2^2}{2g}}+\cancel{z_2}+\cancel{h_{\text{turbine,e}}}+h_{L,p}$$

$V_1 = V_2$ and $\alpha_1 = \alpha_2$

We call this $h_{\text{pump,u,system}}$ since it is the required pump head for the given piping system.

Or, $$\boxed{h_{\text{pump,u,system}}=\frac{P_2-P_1}{\rho g}+h_{L,p}}.$$

- We next apply the head form of the energy equation for CV$_b$ from inlet (2) to outlet (1), recognizing again that $V_1 = V_2$ and $\alpha_1 = \alpha_2$:

$$\frac{P_2}{\rho g}+\alpha_2\cancel{\frac{V_2^2}{2g}}+\cancel{z_2}+\cancel{h_{\text{pump,u}}}=\frac{P_1}{\rho g}+\alpha_1\cancel{\frac{V_1^2}{2g}}+\cancel{z_1}+\cancel{h_{\text{turbine,e}}}+h_{L,b}$$

Or, $$\boxed{h_{L,b}=\frac{P_2-P_1}{\rho g}}.$$

Note: $P_2 - P_1$ is the same regardless of whether we are considering the pump line or the branch line.

- Combining the above two results, we get $\boxed{h_{\text{pump,u,system}} = h_{L,b} + h_{L,p}}$.
- We note that since the velocities (and therefore the Reynolds numbers) in the pump line and the bypass line are not the same, we therefore need *two* Colebrook equations to solve the problem, one for the pump line and one for the bypass line.
- The rest of this problem will be solved in class using EES.