The Taxonomy of Fluid Dynamics

The Cauchy Equation

- Valid for any fluid
- $$\boxed{\frac{\partial\left(\rho\vec{V}\right)}{\partial t}+\vec{\nabla}\cdot\left(\rho\vec{V}\vec{V}\right)=\rho\vec{g}+\vec{\nabla}\cdot\sigma_{ij}}$$

The Navier-Stokes Equation

- Valid for *Newtonian* fluid only
- $$\boxed{\rho\frac{D\vec{V}}{Dt}=-\vec{\nabla}P+\rho\vec{g}+\mu\nabla^2\vec{V}}$$

Approximations of the Navier-Stokes equation:

Creeping Flow

- Linear
- Density drops out
- Very *low* Re
- $\boxed{\vec{\nabla}P=\mu\nabla^2\vec{V}}$

Inviscid Flow

- Nonlinear
- Viscous terms drop out
- Bernoulli equation valid along streamlines
- Euler equation
- $$\boxed{\rho\frac{D\vec{V}}{Dt}=-\vec{\nabla}P+\rho\vec{g}}$$

Irrotational Flow

- Linear
- Viscous terms drop out
- Bernoulli equation valid everywhere
- Superposition valid
- Potential flow solutions
- $$\boxed{\begin{array}{l}\vec{\zeta}=\vec{\nabla}\times\vec{V}=0\\ \vec{V}=\vec{\nabla}\phi,\quad \nabla^2\phi=0\end{array}}$$

Boundary Layer

- Nonlinear
- Some viscous terms drop out
- Very *high* Re
- $$\boxed{\begin{array}{l}\dfrac{\partial u}{\partial x}+\dfrac{\partial v}{\partial y}=0\\[2ex] u\dfrac{\partial u}{\partial x}+v\dfrac{\partial u}{\partial y}=U\dfrac{dU}{dx}+\nu\dfrac{\partial^2 u}{\partial y^2}\end{array}}$$