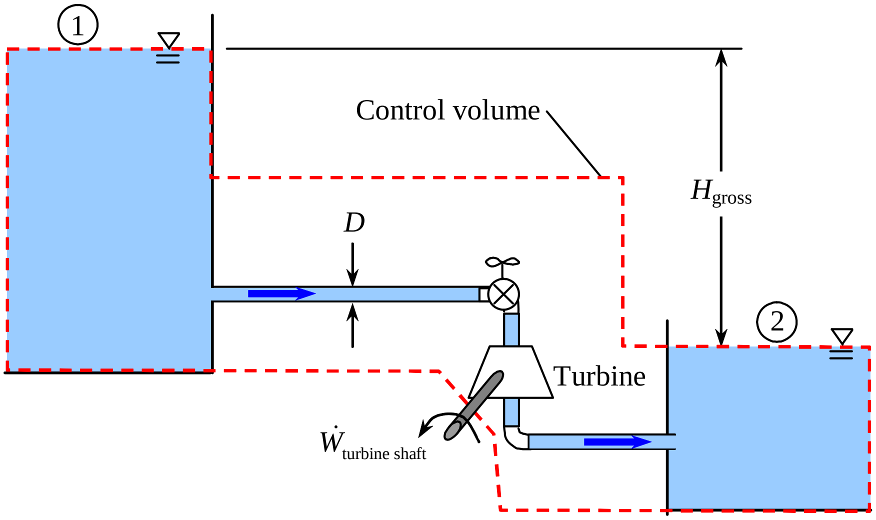
Example Problem – Calculation of Turbine Shaft Power

Given: Water (ρ = 998. kg/m^3, μ = 1.00 × 10^{-3} kg/m·s) flows from one large reservoir to another, and through a turbine as sketched. The elevation difference between the two reservoir surfaces is H_{gross} = 120.0 m. The pipe is 5.0 cm I.D. cast iron pipe. The total pipe length is 30.8 m. The entrance is slightly rounded; the exit is sharp. There is one regular flanged 90-degree elbow, and one fully open flanged angle valve. The turbine is 81% efficient. The volume flow rate through the turbine is 0.0045 m^3/s.



To do: Calculate the shaft power produced by the turbine in units of kilowatts.

Solution:

- First we draw a control volume, as shown by the dashed line. We cut through the surface of both reservoirs (inlet 1 and outlet 2), where we know that the velocity is nearly zero and the pressure is atmospheric. We also slice through the turbine shaft. The rest of the control volume simply surrounds the piping system.
- We apply the head form of the energy equation from the inlet (1) to the outlet (2):

$$\boxed{P_1 = P_2 = P_{\text{atm}}}$$

$$\frac{\cancel{P_1}}{\cancel{\rho g}} + \alpha_1 \frac{\cancel{V_1^2}}{\cancel{2g}} + z_1 + \cancel{h_{\text{pump,u}}} = \frac{\cancel{P_2}}{\cancel{\rho g}} + \alpha_2 \frac{\cancel{V_2^2}}{\cancel{2g}} + z_2 + h_{\text{turbine,e}} + h_L$$

$$\boxed{V_1 = V_2 \approx 0}$$

- But by definition of turbine efficiency, $\boxed{h_{\text{turbine, }e} = \frac{\dot{W}_{\text{turbine shaft}}}{\eta_{\text{turbine}}\dot{m}g}}$ where $\boxed{\dot{m} = \rho \dot{V}}$. Also, since the reference velocity is the same for all the major and minor losses (the pipe diameter is constant throughout), we may use the simplified version of the equation for h_L, i.e., Eq. 8-59: $\boxed{h_L = \frac{V^2}{2g}\left(f\frac{L}{D} + \sum K_L\right)}$. Therefore, we solve the energy equation for the desired unknown, namely, turbine shaft power, $\boxed{\dot{W}_{\text{turbine shaft}} = \eta_{\text{turbine}}\rho\dot{V}g\left[H_{\text{gross}} - \frac{V^2}{2g}\left(f\frac{L}{D} + \sum K_L\right)\right]}$. This is our answer in variable form, but we still need to calculate the values of some of the variables.

The rest of this problem will be solved in class.