Operational Amplifiers (Op-Amps)

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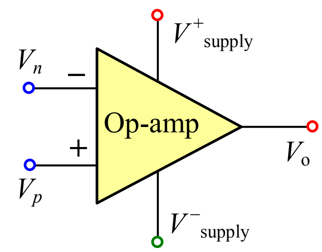
Latest revision: 22 October 2013

Introduction

- An ***operational amplifier*** (abbreviated ***op-amp***) is an ***integrated circuit*** (***IC***) that amplifies the signal across its input terminals. Op-amps are *analog*, not *digital*, devices, but they are also used in digital instruments.
- Op-amps are widely used in the electronics industry, and are thus rather inexpensive – the ones used in the lab are about $0.25 each!
- In this learning module, no details are given about the internal structure of the op-amp. Rather, we summarize many useful applications of op-amps.

Description of op-amps



- A triangle is used as the universal symbol for an op-amp (in schematic circuit diagrams), as shown to the right.
- The ***supply voltage*** terminals are at the top and bottom of the schematic diagram. Supply voltage is necessary because the op-amp requires power to run its internal circuitry. Both a positive and negative supply voltage are required, typically +/– 15 V. In other words, V^+_{supply} = 15 V, and V^-_{supply} = –15 V.
- In applications, any + and – voltage between about 10 to 20 V can be used for the supply voltage, depending on the manufacturer's specifications. (*Note*: We don't usually draw the supply voltages on circuit diagrams, but they must be connected or the op-amp will not work!)
- The ***signal input*** terminals are on the left – a ***positive input*** terminal V_p and a ***negative input*** terminal, V_n. However, the actual input voltages do *not* need to be positive and negative for inputs V_p and V_n, respectively.
- In fact, the V_p input is usually referred to as the ***noninverting input*** instead of the positive input, and the V_n input as the ***inverting input*** instead of the negative input, respectively.
- The connection for the ***output voltage*** V_o is on the right (pointed) side of the op-amp, as sketched above.

Ideal versus actual op-amps

- An ***ideal op-amp*** has ***infinite input impedance***, and therefore it has no effect on the input voltage. This is called ***no input loading***.
- An *actual* op-amp has *very high*, though not infinite, input impedance (typically millions of ohms), so that it has little effect on the input voltage. This is called ***minimal input loading***.
- A direct result of the high input impedance is that we may assume *negligible current flowing into (or out of) either op-amp input, V_p or V_n*. This result helps us to analyze op-amp circuits, as discussed below.
- An ideal op-amp has ***zero output impedance***, so that whatever is done to the output signal farther downstream in the circuit has no affect on the output voltage V_o. This is called ***no output loading***.
- An actual op-amp has very low, though not zero, output impedance (typically a few Ω), so that what is done downstream of the op-amp has little effect on the output voltage. This is called ***minimal output loading***.
- An ideal op-amp has infinite ***gain***, g (Note that a lower case g is used here for the op-amp gain so as not to be confused with G, the gain of amplifier or filter *circuits*.) Op-amp gain g is called ***open loop gain***.
- An actual op-amp has a very high, though not infinite, open loop gain; g is typically in the 10^5 to 10^6 range for the off-the-shelf op-amps that are commonly used in circuits.
- In the analysis of most of the circuits discussed below, the op-amps are approximated as ideal. The performance of an actual (real) op-amp is similar, but not exactly the same as that of an ideal op-amp.

Open-loop versus closed-loop configurations

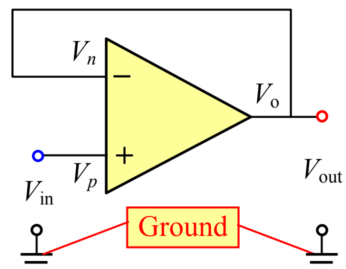
- In an ***open-loop configuration***, as in the above schematic diagram, $V_o = g\left(V_p - V_n\right)$.
- In other words, the output voltage V_o is a factor of g times the input voltage difference, $V_p - V_n$. This might be useful if the incoming signal is extremely small (microvolts), and in need of high amplification.
- In practice, however, circuits are most often built with a ***feedback loop*** (***closed-loop configuration***), which forces the noninverting input and the inverting input to be nearly equal to each other, $V_p \approx V_n$.
- Without a feedback loop, the op-amp can easily ***saturate*** since g is large. ***Saturation*** means that the output voltage clips at some maximum value, typically a volt or so lower than supply voltage V^+_{supply}.
- Likewise, saturation can occur at the low end as well, when the output voltage clips at some minimum value, typically a volt or so greater than supply voltage V^-_{supply}.

- **Example**:
  Given: The gain of an op-amp is 1 million ($g = 1 \times 10^6$). The high supply voltage V^+_{supply} is 15.0 V. The op-amp saturates at 13.9 V.
  To do: Calculate the input voltage difference ($V_p - V_n$) that will cause saturation when the op-amp is operated in an open-loop configuration.
  Solution: From the open-loop gain equation, $\left(V_p - V_n\right) = \dfrac{V_o}{g} = \dfrac{13.9\text{ V}}{1\times 10^6} = 1.39 \times 10^{-5}\text{ V} = 13.9\ \mu\text{V}$.
  Thus, **Any voltage difference greater than 13.9 μV will saturate the op-amp**.
  Discussion: Note how easily an op-amp is saturated. This is why the open-loop configuration is rarely used.

Closed-loop op-amp circuits

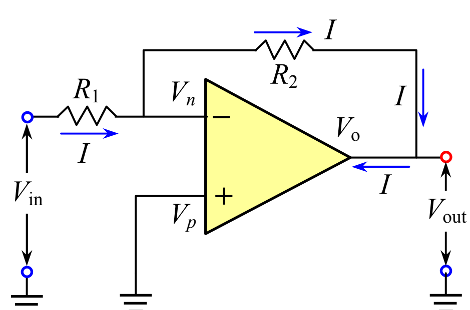
- Almost all practical circuits utilize op-amps in a *closed-loop configuration* to avoid op-amp saturation.
- In all of the examples below, there is ***feedback*** from the output terminal of the op-amp to one of the input terminals. (This is what makes the configuration a *closed-loop* configuration.)
- In the analyses to follow, it is assumed that $V_n \approx V_p$ because of the feedback loop, and because we approximate the op-amps as nearly ideal. This will be better understood as these circuits are analyzed.
- *Note*: In all the schematic diagrams to follow, the supply voltages V^+_{supply} and V^-_{supply} are *not* shown, but you must remember to wire these to the op-amp, or it will not work!

Buffer (also called a ***voltage follower***)



- **Purpose**: To provide high impedance for a voltage signal.
- **Schematic**:
  - A ***circuit diagram*** or ***schematic*** of a buffer is shown to the right.
  - Note the symbol for ***ground*** at the bottom of the schematic diagram.
  - In a buffer, the output is fed directly back into the inverting input terminal to provide the feedback loop.
  - The voltage signal is fed directly into the noninverting input terminal.
- **Analysis**:
  - Since the op-amp output is connected directly to the inverting input, $V_{\text{out}} = V_o = V_n$.
  - If the op-amp were *ideal*, $V_n = V_p = V_{\text{in}}$, and, $\boxed{V_{\text{out}} = V_{\text{in}}}$. For a *real* op-amp, $V_n \approx V_p = V_{\text{in}}$, and $\boxed{V_{\text{out}} \approx V_{\text{in}}}$.
- **Discussion**:
  - What have we accomplished? At first it appears that *nothing has been accomplished at all*, since the output voltage is simply equal to the input voltage! Why do we even need a buffer in the first place?
  - Actually, a buffer is very important when the input signal needs to be ***isolated*** from the rest of the circuit.
  - Because of the high input impedance of the op-amp, the buffer causes V_{in} to be insensitive to anything that happens *downstream* in the circuit.
  - For example, suppose the signal must be connected to something that draws a *current*. Without the buffer, V_{in} would be affected by the current draw. With the op-amp buffer in place, however, the current draw has no effect on input signal V_{in}.
  - Because of the low output impedance of the op-amp, the output voltage V_{out} is likewise not affected significantly by the current draw; V_{out} remains nearly equal to V_{in} regardless of the downstream circuit.

Inverting amplifier

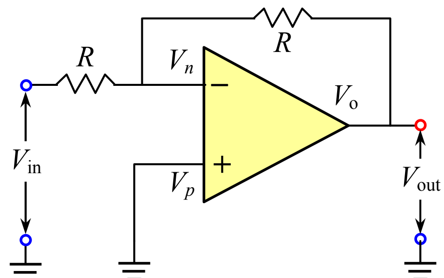


- **Purpose**: To multiply (and invert) a voltage signal by some factor.
- **Schematic**:
  - A schematic of an inverting amplifier is shown to the right.
  - The input signal comes into the *negative* (inverting) input of the op-amp, after first passing through resistor R_1.
  - Since current flows from the input (at voltage V_{in}) through resistor R_1, and then to the op-amp input (at zero voltage), the input impedance of this op-amp circuit is about the same as R_1.
  - The resistors are typically around 10 kΩ to 100 kΩ.
  - The output impedance of this op-amp circuit is on the order of one ohm.
- **Analysis**:
  - Since negligible current flows into the op-amp at input V_n, the *same* current (I) flowing through the first resistor also flows through the second resistor, as shown.

- Since we assume that V_{out} is measured by a *voltmeter, oscilloscope, or data acquisition system with very high input impedance*, all of this current (I) must flow *into* the output terminal of the op-amp, as shown.
- The sign of the current is assumed, as sketched in the diagram. If V_{in} is positive, this assumption is correct. If V_{in} is negative, the current is of opposite sign to that shown.
- Using Ohm's law, $V_{in} - IR_1 = V_n$.
- But, for ideal op-amp behavior, $V_n = V_p = 0$, since V_p is *grounded*. [For a real op-amp, $V_n \approx V_p = 0$.]
- Thus, solving for the current, $I = V_{in} / R_1$.
- In the feedback portion of the circuit, $V_o = V_n - IR_2 = 0 - IR_2 = -(V_{in} / R_1)R_2$.
- Or, finally, $\boxed{V_{out} = V_o = -\frac{R_2}{R_1}V_{in} = GV_{in}}$, where the ***gain G*** of any amplifier is defined as $\boxed{G = \frac{V_{out}}{V_{in}}}$.
- The gain of our *inverting* amplifier turns out to be the negative ratio of R_2 to R_1, i.e., $\boxed{G = \frac{V_{out}}{V_{in}} = -\frac{R_2}{R_1}}$.

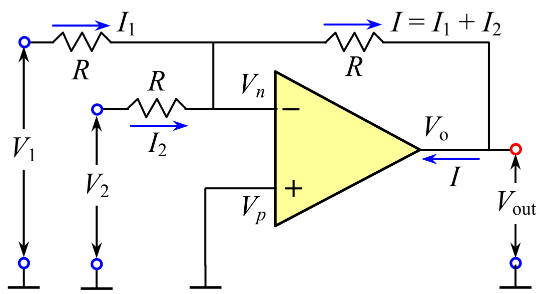
- **Discussion**:
  - Resistors R_1 and R_2 can be chosen to achieve any desired gain. If R_2 is greater than R_1, the signal is ***amplified*** (and inverted)
  - The magnitude of the gain does not necessarily have to be greater than unity. An inverting amplifier can actually ***attenuate*** (and invert) a signal if R_2 is less than R_1.

Inverter



- **Purpose**: Invert (change the sign) of a signal without amplification or attenuation.
- **Schematic**:
  - A schematic diagram of an inverter is shown to the right.
  - The schematic diagram is identical to that above for the inverting amplifier. The only difference is that R_1 and R_2 have the same value, and are indicated simply as R.
- **Analysis**:
  - The analysis is the same as above.
  - For identical R_1 and R_2, the gain is $G = -R_2/R_1 = -1$. Thus, $\boxed{V_{out} = V_o = -V_{in}}$.
- **Discussion**:
  - An inverter simply changes the sign of a signal. It also acts as a *buffer*, so all of the advantages of the buffer listed above apply here as well. We could instead call the inverter an ***inverting buffer***.

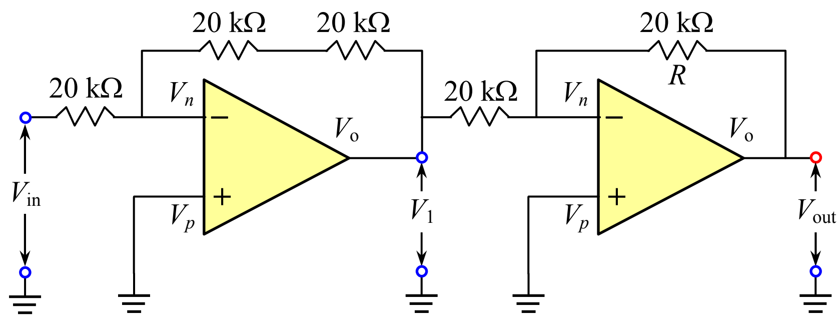
Inverting summer



- **Purpose**: To add (and invert) two voltage signals.
- **Schematic**:
  - A schematic diagram is shown to the right.
  - All three resistors in this circuit have the same value, typically around 10 kΩ to 100 kΩ.
  - The actual value of R is not critical. However, in typical circuits, resistors lower than 1 kΩ are generally *not* used because they waste power unnecessarily.
  - Likewise, resistors higher than 1 MΩ are generally *not* used because they can lead to stray capacitance effects, details of which are beyond the scope of the present discussion.
- **Analysis**:
  - We examine the input (left) side of the op-amp first (using Ohm's law): $V_1 - I_1R = V_2 - I_2R = V_n$.
  - But, if the op-amp is nearly ideal, $V_n \approx V_p = 0$ (since V_p is *grounded*). Thus, $V_1 = I_1R$ and $V_2 = I_2R$.
  - Now we examine the output (right) side, again using Ohm's law, and utilizing the fact that negligible current flows into the op-amp at input V_n (hence, the current flowing through the top right resistor is approximately equal to $I_1 + I_2$). Therefore, $V_o = V_n - (I_1 + I_2)R = 0 - I_1R - I_2R = -V_1 - V_2$.
  - Or, finally, $\boxed{V_{out} = V_o = -(V_1 + V_2)}$.
- **Discussion**:
  - The two input voltages V_1 and V_2 have been added, but the output is the *negative* of the sum – hence the name *inverting summer*.

- This is a very effective and *safe* way to perform a summation of two voltages. The main advantage is due again to the high input impedance of the op-amp – voltages V_1 and V_2 are *isolated* in this summing circuit. That is, they are not affected by what happens *downstream* of the op-amp or by *each other*.
- The inversion (negative sign) of the signal is not a problem. If the negative sign needs to be removed, it can be done so with a simple inverter, as discussed above.

- **Example**:
  Given: Some op-amps and a bunch of 20-kohm resistors are available in the lab.
  To do: Show how these components can be used to *double the voltage of an input signal*. Use *inverting* circuits, and draw the circuit diagram.
  Solution:
  - We use an inverting amplifier to produce the gain. Then we use an inverter in series to remove the negative sign produced by the inverting amplifier.
  - If R_1 is selected as 20 kΩ, R_2 must be 40 kΩ to achieve amplification by a factor of two. Two resistors in series will do the job nicely.
  - The available resistors can also be used for the inverter.
  - The schematic diagram is shown to the right.

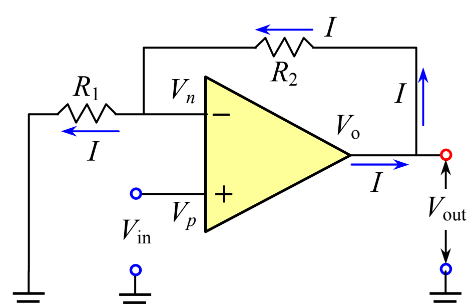


  - The left half of the circuit is the ***inverting amplifier***, with a gain of $G = -\frac{R_2}{R_1} = -\frac{40\text{ k}\Omega}{20\text{ k}\Omega} = -2$.
  - The output voltage from the first op-amp is labeled V_1, and $V_1 = GV_{in} = -2V_{in}$.
  - The right half of the circuit is the ***inverter***, with $V_{out} = -V_1$. Thus $\boxed{V_{out} = 2V_{in}}$ as desired.

  Discussion: The above circuit requires 2 op-amps and 5 resistors. If we use a noninverting amplifier instead, as discussed next, we can achieve the same result with just 1 op-amp and 2 resistors.

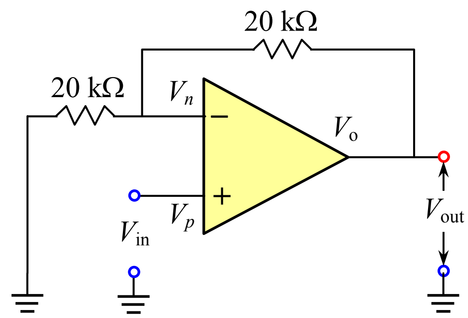
Noninverting amplifier

- **Purpose**: To amplify a voltage signal *without* inverting.
- **Schematic**:



  - A schematic of a noninverting amplifier is shown to the right.
  - As seen, the noninverting amplifier is similar to the inverting amplifier except that the input signal comes into the *positive* (noninverting) input of the op-amp instead of into the negative (inverting) input.
  - The input impedance of this op-amp circuit is on the order of hundreds of megaohms (the input impedance of a noninverting amplifier is equal to the internal impedance of the op-amp itself).
  - The output impedance of this op-amp circuit is on the order of one ohm.
- **Analysis**:
  - Since negligible current flows into the op-amp at input V_n, the *same* current (I) flowing through the first resistor also flows through the second resistor, as shown.
  - Assuming that V_{out} is measured by a *voltmeter*, *oscilloscope*, *data acquisition system*, etc. with *very high input impedance*, *all* the current (I) must flow *out of* the output terminal of the op-amp, as shown.
  - The sign of the current is assumed, as sketched in the diagram. If V_{in} is positive, this assumption is correct. If V_{in} is negative, the current is of opposite sign to that shown.
  - For ideal op-amp behavior, $V_n = V_p = V_{in}$ (since V_{in} is wired to V_p). [For a real op-amp, $V_n \approx V_p = V_{in}$.]
  - Using Ohm's law, $V_n - IR_1 = V_{in} - IR_1 = 0$ since the left-most wire of the circuit is *grounded*.
  - Thus, solving for the current, $I = V_{in}/R_1$.
  - In the feedback portion of the circuit, $V_o = V_n + IR_2 = V_{in} + IR_2 = V_{in} + (V_{in}/R_1)R_2$.
  - Finally, $\boxed{V_{out} = V_o = \left(1 + \frac{R_2}{R_1}\right)V_{in} = GV_{in}}$, where the ***gain*** of a *noninverting* amplifier is $\boxed{G = \frac{V_{out}}{V_{in}} = 1 + \frac{R_2}{R_1}}$.

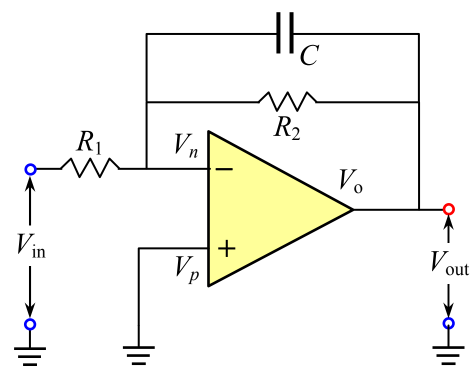
- **Example**:
  Given: Some op-amps and a bunch of 20-kohm resistors are available in the lab.
  To do: Show how these components can be used to *double the voltage of an input signal*. Use *noninverting* circuits, and draw the circuit diagram.
  Solution:
  - We use a noninverting amplifier to produce the gain.
  - If R_1 is selected as 20 kΩ, R_2 must also be 20 kΩ to achieve amplification by a factor of two.
  - The schematic diagram is shown to the right.
  - The gain of this ***noninverting amplifier*** is
    $$G = 1 + \frac{R_2}{R_1} = 1 + \frac{20\text{ k}\Omega}{20\text{ k}\Omega} = 2\,.$$
  - Thus, $V_{out} = GV_{in}$, or $\boxed{V_{out} = 2V_{in}}$ as desired.



  Discussion: The above circuit requires only 1 op-amp and 2 resistors. Compare this to the previous example using inverting circuits, where we needed 2 op-amps and 5 resistors to achieve the same result.

First-order, active, low-pass, inverting filter

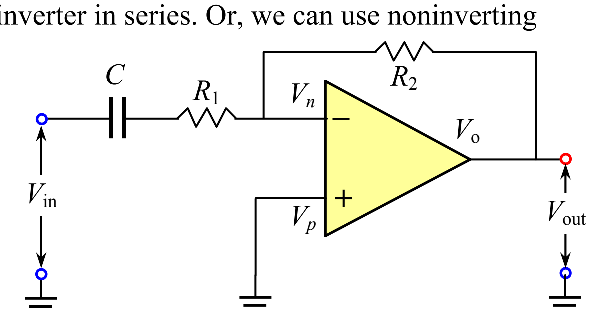
- **Purpose**: To low-pass filter (and invert) a voltage signal.
- **Schematic**:
  - The schematic is shown to the right.
  - As seen, a first-order, active, low-pass, inverting filter is simply an inverting amplifier *with a capacitor added in parallel to the resistor in the feedback loop*.
  - You are invited to compare this filter circuit with that of a simple *passive* first-order RC low-pass filter circuit.



- **Analysis**:
  - We consider the simplest case in which the resistor values are chosen to be identical ($R_1 = R_2 = R$); there is *no amplification*.
  - At *low frequencies*, the capacitor acts like an *open switch*, and thus does not contribute anything. The circuit is then *the same as an inverter*, with $V_{out} \approx -V_{in}$, and low frequencies pass unaffected (except that the signal is *inverted*).
  - At *high frequencies*, the capacitor acts like a *closed switch*, rendering R_2 useless, and forcing V_o to nearly equal V_n. However, V_p is grounded, so $V_{out} = V_o \approx V_n \approx V_p = 0$. In other words, high frequencies are attenuated significantly – the output voltage is nearly zero.
  - It turns out that at *intermediate frequencies*, this circuit behaves as a ***first-order Butterworth low-pass filter***. The equation for the gain G of the filter is the same as that defined previously for a simple RC filter, except for the negative sign. The phase shift is also the same as that defined previously.
  - The ***cutoff frequency*** of this active filter is in fact the same as that for a simple, passive, first-order, low-pass RC filter, $\boxed{f_{cutoff} = \frac{\omega_{cutoff}}{2\pi} = \frac{1}{2\pi R_2 C}}$. Notice that R_2 is used here rather than R_1.
- **Discussion**:
  - If a simple RC circuit is used for filtering, the filter is called a ***passive filter***.
  - When an op-amp with feedback is used, as in the present case, the filter is called an ***active filter***.
  - As with an inverting amplifier, values of R_2 and R_1 can be selected such that there is *amplification* (or *attenuation*) of the signal as well.
  - If the negative sign is a problem, we can add a simple inverter in series. Or, we can use noninverting circuitry instead.

First-order, active, high-pass, inverting filter

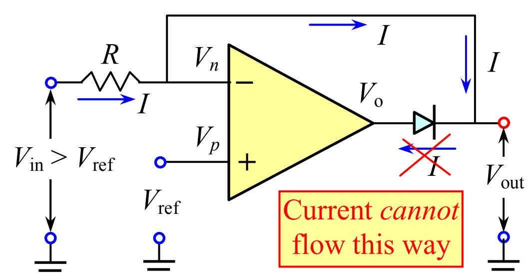
- **Purpose**: To high-pass filter (and invert) a voltage signal.
- **Schematic**:
  - The schematic is shown to the right.
  - The first-order, active, high-pass, inverting filter is similar to the first-order, active, low-pass, inverting filter discussed above, except that the capacitor is



placed in *series* with the *input* resistor instead of in *parallel* with the *feedback* resistor.

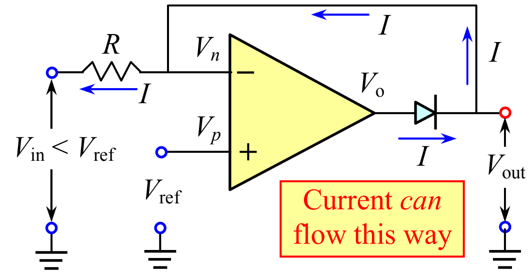
- **Analysis**:
  - If the resistor values are chosen to be identical ($R_1 = R_2 = R$), there is *no amplification*.
  - At *low frequencies*, the capacitor acts like an *open switch*, so V_{in} is effectively isolated from the output.
  - Another way to look at this case is that at low frequencies, no current can flow through the capacitor or through resistor R_1. Thus, $V_{out} = V_n \approx V_p = 0$ since V_p is *grounded*; low frequencies are attenuated.
  - At *high frequencies*, the capacitor acts like a *closed switch*, having no effect on the circuit. The circuit then becomes the same as an inverter, with $V_{out} = -V_{in}$; high frequencies pass unaffected (except that the signal is *inverted*).
  - It turns out that at *intermediate frequencies*, this circuit behaves as a ***first-order Butterworth high-pass filter***. The equation for the gain G of the filter is the same as that defined previously for a simple RC filter, except for the negative sign. The phase shift is also the same as that defined previously.
  - The ***cutoff frequency*** of this active filter is in fact the same as that for a simple, passive, first-order, low-pass RC filter, $f_{cutoff} = \frac{\omega_{cutoff}}{2\pi} = \frac{1}{2\pi R_1 C}$. Notice that R_1 is used here rather than R_2.
- **Discussion**:
  - Since an op-amp with a feedback loop is used in the present case, the filter is an ***active filter***.
  - As with an inverting amplifier, values of R_2 and R_1 can be selected such that there is *amplification* (or *attenuation*) of the signal as well.
  - If the negative sign is a problem, we can add a simple inverter in series. Or, we can use noninverting circuitry instead.

Low-voltage clipping circuit



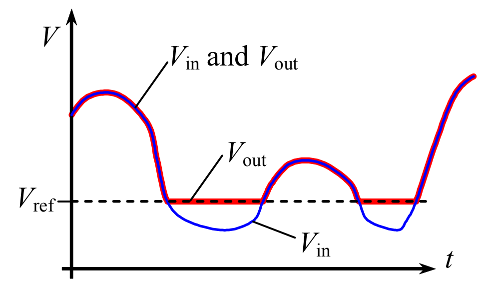
- **Purpose**: To clip voltages below some reference voltage, V_{ref}.
- **Schematic**:
  - The schematic is shown to the right.
  - The noninverting input of the op-amp is connected to the ***reference voltage*** V_{ref}.
  - The component just to the right of the op-amp is called a ***switching diode***.
  - A switching diode allows current to flow *in one direction only* (in the direction of the small triangle – to the right in the orientation shown here), and blocks currents from flowing the opposite way.
- **Analysis**:
  - **If $V_{in} > V_{ref}$**
    - Current tries to flow through resistor R in the direction indicated on the above diagram.
    - No current can flow into the left side (input side) of the op-amp, because of its high input impedance.
    - Likewise, no current can flow downstream from (to the right of) V_{out} because it is assumed that V_{out} either goes into another high-impedance component, such as another op-amp, or is being measured with a high impedance voltmeter, oscilloscope, digital data acquisition system, etc.
    - Thus, the only path for the current to flow is *through the feedback loop* and back into the output (right side) of the op-amp as shown.
    - However, *current cannot flow through the switching diode in the direction indicated on the above diagram*! Hence, *no current can flow at all*. Since $I = 0$, $V_n = V_{in}$ by Ohm's law when $I = 0$.
    - But $V_{out} = V_n$. So, finally, $V_{out} = V_{in}$ when $V_{in} > V_{ref}$.
    - In other words, ***the low-voltage clipping circuit has no effect on voltages greater than V_{ref}***.
    - Note that for $V_{in} > V_{ref}$, $V_p = V_{ref}$, while $V_n = V_{in}$. Thus, $V_n \neq V_p$ for this case, and the op-amp is *saturated*. (This is because the feedback loop, though present, is interrupted by the switching diode.)
  - **If $V_{in} < V_{ref}$**
    - $V_p = V_{ref}$, $V_n = V_p = V_{ref} > V_{in}$, and current tries to flow through resistor R in the direction *opposite* of that indicated on the above diagram.
    - Current tries to flow from the output of the op-amp, through the switching diode, and through the feedback loop.

- In this case, current *can* flow through the switching diode in the direction indicated in the diagram to the right.
- Ideally, there is no voltage drop across the diode; thus, $V_o = V_n$.
- Since the current cannot flow into the inverting input terminal (V_n) of the op-amp, current *must* flow through the resistor in the direction shown here.
- Hence, by Ohm's law, $V_n - IR = V_{in}$. (Since $V_n > V_{in}$, current flows to the left through resistor R as sketched in the circuit diagram to the right.)



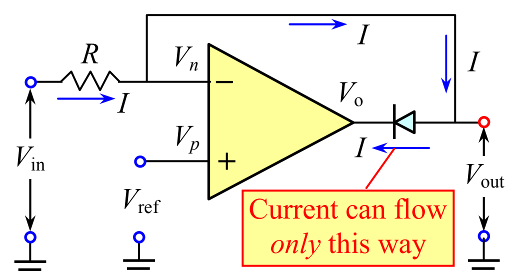
- But $V_n \approx V_p = V_{ref}$, and $V_{out} = V_o = V_n$. So, finally, $\boxed{V_{out} = V_{ref} \text{ when } V_{in} < V_{ref}}$.
- In other words, ***the low-voltage clipping circuit clips (to V_{ref}) voltages that are less than V_{ref}***.

- **Discussion**:
  - $V_{out} = V_{in}$ as long as V_{in} is greater than V_{ref}. However, if V_{in} drops *below* V_{ref}, $V_{out} = V_{ref}$. This is what is meant by low-voltage clipping.
  - The sketch to the right indicates the clipping.
  - Notice that the output follows the input exactly, but every time the input signal drops below V_{ref}, it gets clipped to V_{ref}.

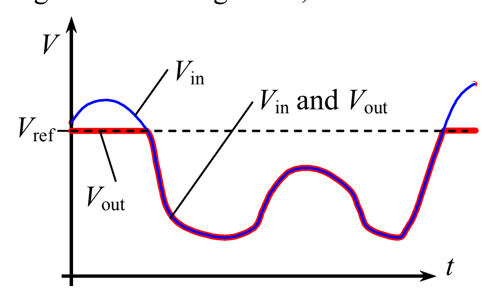


High-voltage clipping circuit

- **Purpose**: To clip voltages above some reference voltage, V_{ref}.
- **Schematic**:
  - The schematic is shown to the right.
  - The high-voltage clipping circuit is identical to the low-voltage clipping circuit, except the switching diode is turned around the opposite way.



- **Analysis**:
  - The analysis is similar, but opposite to that above, and we leave out the details.
  - Results: $\boxed{V_{out} = V_{ref} \text{ when } V_{in} > V_{ref}}$; current *can* flow through the switching diode in the direction indicated on the schematic. ***The high-voltage clipping circuit clips (to V_{ref}) voltages greater than V_{ref}***.
  - $\boxed{V_{out} = V_{in} \text{ when } V_{in} < V_{ref}}$; current tries to flow the wrong way through the switching diode, but cannot. ***The high-voltage clipping circuit has no effect on voltages smaller than V_{ref}***.
- **Discussion**:
  - High-voltage clipping is sometimes necessary to protect electronic components from voltages that are too high.
  - For the same input voltage signal as above, the sketch to the right indicates the clipping.
  - Every time the input signal rises above V_{ref}, it gets clipped.



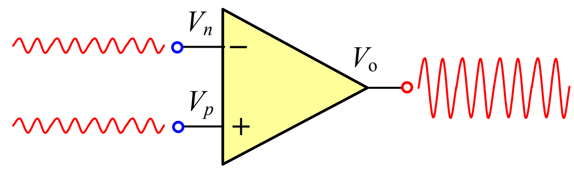
  - To construct a circuit that clips both high *and* low voltages, we connect a low-voltage clipping circuit and a high-voltage clipping circuit ***in series***. In such a case, the reference voltage for the low-voltage clip must be smaller than that for the high-voltage clip.

Miscellaneous properties of op-amp circuits

- It is not the intent of this learning module to discuss the internal circuitry of an op-amp.
- However, there are several aspects of actual op-amps that cause problems in circuits – problems that would not exist for ideal op-amps – due to limitations of the internal circuitry of real op-amps. Among these are:
  - Saturation effects (as previously mentioned)
  - Input loading effects (as also previously mentioned)
  - Common-mode rejection ratio (CMRR) effects
  - Gain-bandwidth product (GBP) effects
- The latter two are discussed in detail below.

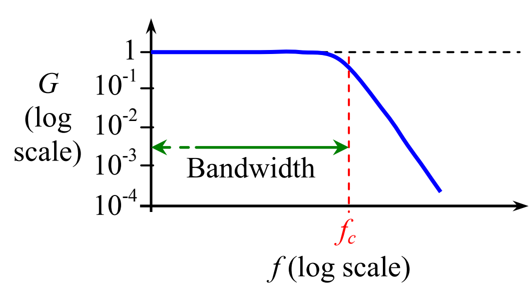
Common-mode rejection ratio (CMRR)

- **Definition**: ***Common-mode rejection ratio*** (CMRR) is defined as $\text{CMRR} = 20\log_{10}\frac{g}{G_{CM}}$, where
  - g is the ***open-loop voltage gain*** of the op-amp as discussed previously. Another name for g is ***differential voltage gain***. In other words, g is the gain in voltage when the input to V_p and V_n are *different*.
  - G_{CM} is the ***common-mode voltage gain***. In other words, G_{CM} is the gain in voltage when the input to V_p and V_n are *the same*.
  - The units of CMRR are decibels (dB). A large value of CMRR implies that G_{CM} is small compared to g, which is desirable for rejection of common-mode noise. Modern op-amps have values of CMRR greater than 100 dB.
- **Analysis**:



  - Common-mode gain occurs when the *same* noise (typically high frequency, small amplitude noise) is present in *both* V_p and V_n.
  - For example, consider identical noise at both input terminals of the op-amp, as sketched to the right.
  - Since $V_o = g\left(V_p - V_n\right)$, we expect V_o to be exactly zero since $V_p = V_n$ at any instant in time.
  - However, it turns out that the op-amp (due to its internal circuitry – beyond the scope of the present discussion) actually *amplifies* the common-mode noise by factor G_{CM} as sketched.
  - Since noise typically appears in *both* inputs (common-mode input), while the signal typically is wired to only one input (differential-mode input), CMRR needs to be large to ensure a large signal-to-noise ratio at the output of the op-amp.
- **Application**:
  - Which is better – inverting or noninverting op-amp amplifiers?
  - It turns out that there are two competing effects to consider:
    - Input loading: Recall that input loading problems occur when the input impedance R_i of an electronic device is not high enough.
      - For a ***noninverting op-amp amplifier***, R_i is determined by the internal circuitry of the op-amp, and is generally quite large (of order 1 MΩ).
      - For an ***inverting op-amp amplifier***, R_i is largely independent of the internal circuitry of the op-amp, and is instead approximately equal to R_1 (typically of order 10 kΩ to 100 kΩ).
    - Thus, *if input loading is of primary concern, noninverting amplifiers should be used*.
    - Noise rejection: As mentioned above, noise generally appears in both input terminals of the op-amp.
      - For a ***noninverting op-amp amplifier***, it turns out that in many applications, the noise amplitude is *nearly the same* in the V_p and V_n input terminals, and thus *common-mode gain is more of a problem in these circuits*.
      - For an ***inverting op-amp amplifier***, it turns out that the noise amplitude is generally greater in the V_n input terminal than in the V_p input terminal (since the V_p input terminal is grounded), and thus *common-mode gain is less of a problem in these circuits*.
    - Thus, *if noise reduction and signal-to-noise issues are of primary concern, inverting amplifiers should be used*.

Gain-bandwidth product (GBP)



- **Definition**: ***Gain-bandwidth product*** (GBP), also called simply ***bandwidth*** is defined as $\text{GBP} = G_{\text{theory}} \cdot f_c$, where
  - G_{theory} is the ***theoretical gain*** of the op-amp amplifier as discussed previously, and given the symbol G.
  - f_c is the ***internal cutoff frequency*** of the op-amp itself.
  - For a given op-amp, GBP is a constant – it is one of the specifications supplied by the op-amp manufacturer. The GBP of modern op-amps is typically on the order of 1.0 MHz, and is often listed as simply ***bandwidth***.
  - The units of GBP are the same as those of frequency.
  - Without going into details, the inner circuitry of an op-amp acts like a first-order low-pass filter (with cutoff frequency f_c) when very high frequency inputs are applied.

- So, the actual gain of the op-amp amplifier drops off at high frequencies, just like a low-pass filter, as sketched above for the case in which G_{theory} is 1.
- The range of frequencies from 0 Hz (DC) to internal cutoff frequency f_c is called the ***bandwidth***. In some electronics literature, the bandwidth is defined simply as f_c itself, $\text{bandwidth} = f_c$ when $G_{\text{theory}} = 1$.
- Equations for the internal low-pass filtering effects of the op-amp are the *same* as those for first-order low-pass Butterworth filters except that f_c rather than f_{cutoff} is the cutoff frequency. E.g., the gain G_{GBP} due to internal low-pass filtering effects is $G_{\text{GBP}} = \dfrac{1}{\sqrt{1+(f/f_c)^2}}$, where f is the input signal frequency.
- GBP effects are related to the op-amp's ***slew rate***, defined as *the rate of change in output voltage when the input is a step change of voltage*. The units of slew rate are typically volts per microsecond (V/µs).

- **Analysis**:
  - We now compare GBP effects for inverting and noninverting op-amp amplifiers.
  - **Noninverting op-amp amplifier**:
    - The theoretical gain (no GBP effects) of a noninverting op-amp amplifier is $G_{\text{theory}} = 1 + R_2/R_1$.
    - We define $\text{GBP}_{\text{noninverting}}$ as the ***noninverting gain-bandwidth product*** of the op-amp. *Note*: This is the same as the GBP value supplied by the op-amp manufacturer in their list of specifications.
    - The equation for GBP is written as $\text{GBP}_{\text{noninverting}} = G_{\text{theory}} \cdot f_c$, from which the internal cutoff frequency can be calculated (for known values of $\text{GBP}_{\text{noninverting}}$ and G_{theory}).
    - The *actual* gain of a noninverting op-amp amplifier is thus *less* than the theoretical gain, namely, $G = G_{\text{theory}} G_{\text{GBP, noninverting}} = \left(1 + \dfrac{R_2}{R_1}\right)\dfrac{1}{\sqrt{1+(f/f_c)^2}}$ where $f_c = \dfrac{\text{GBP}_{\text{noninverting}}}{G_{\text{theory}}}$.
    - Consider for example a 741 op-amp with $\text{GBP}_{\text{noninverting}} = \text{GBP}_{\text{manufacturer spec}} = 1.0$ MHz. For a theoretical gain of 10, the internal cutoff frequency of the op-amp is thus $f_c = \text{GBP}_{\text{noninverting}}/G_{\text{theory}}$ = (1.0 MHz)/10 = 0.10 MHz = 100,000 Hz. But **f_c is not a constant – it depends on G_{theory}**.
    - As we increase the theoretical gain, f_c must decrease. For this example op-amp, when $G_{\text{theory}} = 100$, f_c = 10,000 Hz, and when $G_{\text{theory}} = 1000$, f_c = 1,000 Hz. In other words, ***the larger the theoretical gain of the amplifier, the lower the internal cutoff frequency (bandwidth) of the amplifier***.
  - **Inverting op-amp amplifier**:
    - The theoretical gain (no GBP effects) of an inverting op-amp amplifier is $G_{\text{theory}} = -R_2/R_1$.
    - We define $\text{GBP}_{\text{inverting}}$ as the ***inverting gain-bandwidth product*** of the op-amp (a negative quantity), $\text{GBP}_{\text{inverting}} = -\dfrac{R_2}{R_1+R_2}\text{GBP}_{\text{noninverting}}$. *Note*: This is *not* the same as the GBP value supplied by the op-amp manufacturer in their list of specifications.
    - The equation for GBP is written as $\text{GBP}_{\text{inverting}} = G_{\text{theory}} \cdot f_c$, from which the internal cutoff frequency is determined (for known values of $\text{GBP}_{\text{inverting}}$ and G_{theory}).
    - The *actual* gain of an inverting op-amp amplifier is thus *less* than the theoretical gain, namely, $G = G_{\text{theory}} G_{\text{GBP, inverting}} = -\dfrac{R_2}{R_1}\dfrac{1}{\sqrt{1+(f/f_c)^2}}$ where $f_c = \dfrac{\text{GBP}_{\text{inverting}}}{G_{\text{theory}}}$.
    - As with a noninverting op-amp amplifier, when we increase the theoretical gain of an inverting op-amp amplifier, f_c must decrease. Again, as above, ***the larger the theoretical gain of the amplifier, the lower the internal cutoff frequency (bandwidth) of the amplifier***.
  - For either case (inverting or noninverting amplifier), we see, therefore, that it is futile to try to amplify a very high frequency signal by a very large gain using only one op-amp – the internal low-pass filtering effects quickly overpower the theoretical gain of the amplifier.
  - Solution? ***Use two (or more) op-amp amplifiers in series.*** Some examples will be shown in class.