Linear Velocity Measurement

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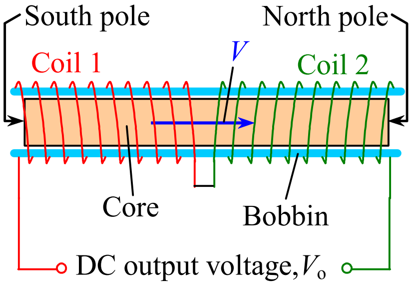
Introduction

- ***Velocity*** is a vector that consists of a magnitude (***speed***) and a direction.
- ***Linear velocity*** is defined as *the rate of change of the position vector with time* at an instant in time.
- Technically, velocity is always a *vector*, while speed is always a *scalar*. However, most people (erroneously) do not distinguish properly between velocity and speed.
- In this learning module, we discuss various ways to measure the velocity of solid objects and flowing fluids. We are concerned here only with *linear* velocity, not *angular* velocity.
- Two words are used interchangeably to describe the measurement of velocity: ***anemometry*** and ***velocimetry***.

Velocity of solid objects

- There are several instruments used to measure the linear velocity of a solid object, and we highlight several of them here.

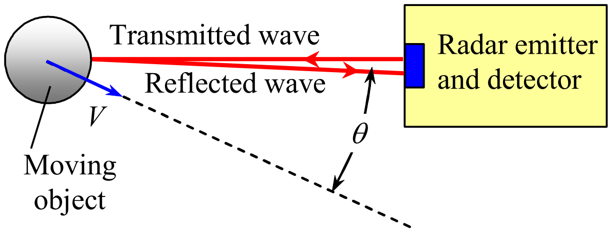
Linear velocity transducer (LVT)



- A ***linear velocity transducer*** (***LVT***) is an inductive device that is similar in principle to the linear variable displacement transducer (LVDT) discussed previously.
- It utilizes the link between electricity and magnetism as found by H. A. Lorentz, namely, if a magnetic field moves near an electrical wire, current flows through the wire.
- Whereas an LVDT measures *displacement*, an LVT measures *speed*.
- An LVT consists of a rod called the ***core*** (a permanent magnet), and two electrical ***coils***, as sketched to the right.
- The core slides inside a hollow cylindrical tube called a ***bobbin***, as sketched. A DC voltage is generated when the core moves.
- Since the two coils are wrapped with *opposite polarity*, and since the magnet also has two poles (north and south), the south pole induces a voltage primarily in coil 1, and the north pole primarily in core 2.
- It turns out that the net voltage is proportional to the speed of the core, and is relatively independent of position within some limited range near the center (typically about ±15% to either side of center).
- Although the range is limited, LVTs are used in some types of machinery, like milling machines.
- In Spring semester 2008, two former M E 345 students, James Weyand and Collin Julius, used a velocity coil to measure the muzzle speed of an air cannon [see picture to the right].

Doppler radar velocity measurement

- You are familiar with the ***Doppler shift*** for sound waves. Namely, a noise (such as a car horn) moving towards you (or away from you) has an apparently higher (or lower) pitch or frequency, since the wavelength of the sound that reaches your ears is compressed (or stretched) due to the relative motion.
- ***Doppler radar*** works by the same principle, but with *radio waves* instead of sound waves. When radio waves strike a moving object, the frequency of the reflected radio waves is altered in a similar manner as the sound waves.
- A radar-Doppler velocimeter is sketched to the right. Here is how it works:
  - Radio waves of wavelength λ are transmitted (incident waves) towards a moving object.
  - The object moves with velocity V at angle θ relative to the radar unit, as sketched.
  - The radio waves reflect off the moving object, and are sensed by a radio wave detector (receiver), that is also mounted on the radar unit.



  - The detector measures the frequency of the reflected radar beam, and the unit compares the frequency of the transmitted and reflected beams.
  - Some trigonometry reveals that the ***Doppler frequency shift*** Δf_D, defined as the change in frequency of the radar beam, is $\boxed{\Delta f_D = \frac{2V\cos\theta}{\lambda}}$.
- Doppler radar units are used by police to measure the speed of automobiles, and they are also used in professional sports to measure the speed of baseballs, etc.
- Laser light can be used in place of radio waves. Then, the device is called a ***laser Doppler velocimeter***.

- **Example**:
  Given: Don is driving 67.0 mph in a 55 mph zone. A police officer nails him with a radar gun that uses a frequency of 10,000 MHz. Angle θ is 10° at the moment of the reading.
  To do: Calculate the Doppler frequency shift.
  Solution:
  - For any kind of electromagnetic wave, wavelength λ and frequency f are related by $\boxed{f\lambda = c}$, where c is the speed of light, which is 2.9979×10^8 m/s in a vacuum; we approximate this same c in air.
  - Thus, $\lambda = c/f$, and the above equation for the Doppler frequency shift is

$$\Delta f_D = \frac{2V\cos\theta}{c} f = \frac{2(67.0\text{ mile/hr})(\cos 10^\circ)}{2.9979\times10^8\text{ m/s}}(10{,}000\text{ Mhz})\left(\frac{1609.3\text{ m}}{1\text{ mile}}\right)\left(\frac{1\text{ hr}}{3600\text{ s}}\right) = 0.00196777\text{ MHz}$$

  - Or, since there are 10^6 Hz in one MHz, $\boxed{\Delta f_D = 1970\text{ Hz}}$ to three significant digits.

  Discussion: For small angles, $\cos\theta$ is nearly 1, but as the car gets closer to the officer, the effect of the angle becomes more and more important, and is difficult to take into account. That is why police officers like to use radar guns on a straight line, nearly facing oncoming traffic.

Velocity measurement using displacement and acceleration sensors

- Velocity is often not measured directly, but rather by calculation from measurement of displacement or acceleration.

- ***Displacement sensors***
  - In a previous learning module, we discussed methods to measure displacement (distance traveled), using a ***displacement sensor***.
  - By fundamental definition, *velocity is the time derivative of displacement*, $\boxed{V(t) = dx(t)/dt}$.
  - So, theoretically, we could calculate velocity by taking the time derivative of displacement measurements from a displacement sensor (potentiometer, LVDT, laser displacement meter, etc.).
  - However, there is an inherent problem with this technique – namely, *the process of differentiation of a signal* ***amplifies*** *the noise in the system* (decreases the signal-to-noise ratio).
  - Thus, velocity measurement by differentiation of displacement data is generally not a wise choice unless the displacement sensor has an extremely high signal-to-noise ratio.
  - However, displacement *can* be used to very accurately measure *average* velocity.
  - For example, a runner or race car driver can easily calculate his or her average speed by timing (with a stopwatch) how long it takes to travel some known distance (see photo to the right).

  - In such cases, we are not interested in measuring velocity as a function of time, but rather only the *average* velocity over a particular period of time.
  - Optical sensors, such as ***photodetectors***, are often used to indicate the time at which a moving object crosses the optical path of the sensor.
  - With two such sensors located at a known distance apart, the average speed of the object is easily calculated as $\boxed{V_{\text{avg}} = \Delta x/\Delta t}$.
  - This technique is often used for measuring the velocity of very fast-moving objects like bullets.

- ***Acceleration sensors***
  - In some instruments, an ***accelerometer sensor*** is available – it measures acceleration as a function of time.
  - By fundamental definition, *velocity is the time integral of acceleration*, $\boxed{V(t) = V_0 + \int_{t_0}^{t} a(t)dt}$, where V_0 is the velocity at time t_0, and we integrate from time t_0 to some later time t.
  - Unlike differentiation, *the process of integration **decreases** the noise in the system* (increases the signal-to-noise ratio).
  - Thus, velocity measurement by integration of acceleration data is generally a wise choice.

Velocity of a flowing fluid

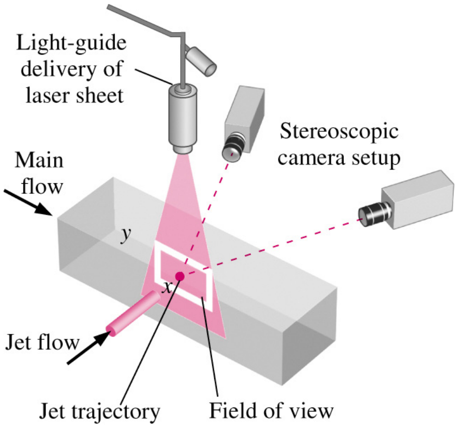
- The velocity of a flowing fluid is more difficult to measure than that of a solid object for several reasons:
  - The fluid is often not visible (air) or not transparent (e.g., oil, milk).
  - The fluid is usually not moving as a solid body, but rather, individual fluid particles move *relative to each other* – velocity is a function of spatial location within the fluid flow.
  - Velocity probes placed in the flow often disturb the very flow we are trying to measure.
  - The flow is often unsteady.
- Nevertheless, engineers have invented many devices that accurately measure fluid flow velocity. Some of these are discussed here.
- As you may recall from your study of fluid mechanics, there are two ways to describe fluid motion:
  - In the ***Lagrangian method***, we follow the movement of individual fluid particles – we describe the location (position), velocity, and acceleration of a particular fluid particle as a function of time.
  - In the ***Eulerian method***, we define a ***control volume***, and describe the velocity *field* and acceleration *field* as functions of space and time within that control volume.

Lagrangian velocity measurements

- Lagrangian velocity measurements consist of following or tracking a fluid particle that is "marked" or identified in some way (dye in liquids, smoke in air, tiny soap bubbles in air, tiny particles in water – it is assumed that the tiny particles move with the fluid; this is called ***seeding*** the flow). Some call this ***time-of-flight velocity measurement***.
- The velocity of the fluid particle is calculated by differentiation of position, $\boxed{V(t) = dx(t)/dt}$.
- A simple example is throwing a floating ball into a river and timing how long it takes to move a certain distance, thus inferring the velocity of the water surface.
- An interesting application of the Lagrangian velocity measurement technique is the monitoring of glacier velocity, as in the photo to the right. Markers are placed on the glacier, and then their movement is tracked in time, either by surveying equipment, or nowadays by global positioning systems.

Particle image velocimetry

- A high-tech velocity measurement device for fluid flows is called ***particle image velocimetry*** (***PIV***). Here is a brief description of how PIV works:
  - The fluid is seeded with tiny particles that are so small that they move with the fluid.
  - A double-pulse laser illuminates a region of flow under study, and a digital camera (sometimes two separate cameras) records two images – timed with the two flashes (pulses) of laser light. Illuminated particles appear as bright spots on the photographs because of the flashes of laser light.
  - The displacement of illuminated particles is then determined by analyzing (interrogating) the two digital photographs with sophisticated image processing software:



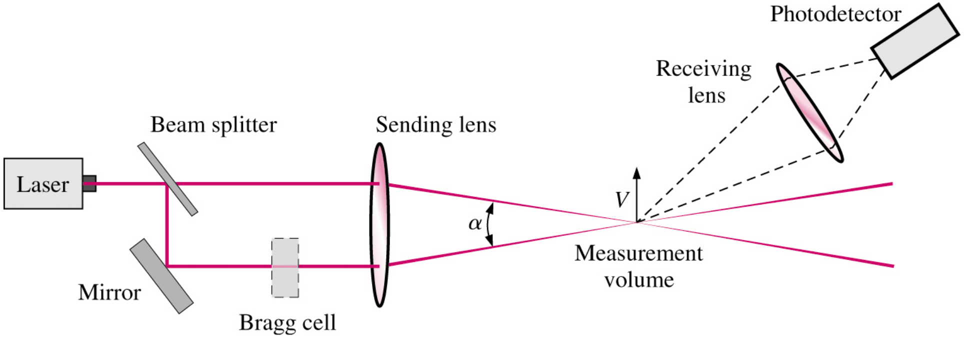
    - Distance Δs between the two bright spots is measured, and the *speed* is determined by $\boxed{V = \Delta s / \Delta t}$, where Δt is the (known) time interval between laser pulses.
    - The *direction* of the particle movement is determined by image processing, and therefore the velocity of the illuminated particle is calculated.
    - There are two basic types of PIV:
      - ***Standard PIV*** (two rapid laser flashes with a very small Δt followed by a long pause before the next series of laser flashes)
      - ***Cinema*** or ***cinemagraphic PIV*** (one laser flash per camera frame with no pause, like a video or movie, but typically a longer Δt)
  - There are both 2-D and 3-D PIV systems:
    - 2-D systems use one camera and measure flow velocity in a plane illuminated by a laser light sheet, as shown here (from Georgia Tech).
    - 3-D systems use *two* cameras (***stereoscopic photography***) to measure the velocity in the plane of the laser light sheet, and also in the direction normal to the plane of the light sheet, as sketched above.
- Although PIV is fundamentally Lagrangian, the final result is a velocity *field*, which is an Eulerian result.

Eulerian velocity measurements

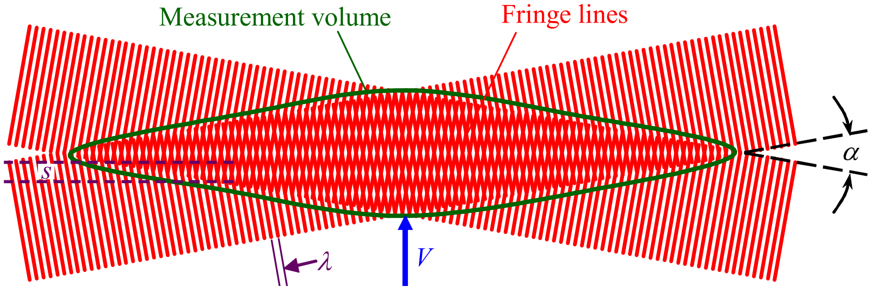
- Eulerian velocity measurements involve a probe or sensor of some kind sitting in a fluid flow.
- With Eulerian techniques, instead of tracking individual marked fluid particles, we measure the velocity of the fluid that happens to be flowing past the sensor at the time of measurement.
- If the sensor is at a fixed location in the fluid flow, it measures the velocity at that point as a function of time.
- If the sensor is traversed (moved around) in the flow field, and the flow field is steady, we can map the steady (or time-averaged) velocity field as a function of spatial location.
- There are various Eulerian velocity measurement devices, some of which are described here.

Laser Doppler velocimetry

- An optical technique involving a laser is ***laser Doppler velocimetry*** (***LDV***), also called ***laser velocimetry*** (***LV***) or ***laser Doppler anemometry*** (***LDA***).
- Like PIV, LDV measures the velocity of small seed particles as they move through the fluid.
- However, whereas PIV is a Lagrangian technique, following the motion of individual particles, LDV is an Eulerian technique, since the velocity is measured at a fixed point in the flow.



- A basic single velocity component ***dual-beam LDV system*** is sketched above and described below:
  - The laser beam is split by a half-silvered mirror called a ***beam splitter*** into two parallel laser beams of equal intensity.
  - Both beams pass through a converging lens that focuses the beams at a "point" in the flow (actually a small volume called the ***measurement volume*** or the ***focal volume***).
  - When the two beams cross, the waves interfere with each other, creating a bright and dark fringe pattern, as sketched to the right.



  - Laser light is scattered by small seed particles that pass through the measurement volume. The scattered light intensity is bright, then dark, then bright, etc. as the particle moves through the fringe pattern (constructive and destructive interference, as seen in the sketch above).
  - The scattered laser light is collected by a receiving lens and photodetector.
  - The photodetector converts fluctuations in light intensity into a fluctuating voltage signal that is proportional to the amplitude of the scattered light.
  - Finally, a signal processor determines the frequency f of the voltage signal, and thus the frequency f of the scattered light.
  - Mathematically, the spacing s between fringe lines is obtained from trigonometry, $\boxed{s = \dfrac{\lambda}{2\sin(\alpha/2)}}$, where λ is the wavelength of the laser light and α is the angle between the two beams, as sketched above.
  - The speed of the seed particle turns out to be linearly proportional to the frequency of the fluctuating light intensity, and is given by $\boxed{V = fs = \dfrac{f\lambda}{2\sin(\alpha/2)}}$.
- In practice, velocity is measured in more than one direction simultaneously.
  - 2-D LDV systems employ two separate laser beams of two different wavelengths (or colors), and the respective fringe patterns are in two (usually mutually orthogonal) directions.
  - 3-D LDV systems employ three separate laser beams (three colors) and are used to measure the three-dimensional velocity at the location of the measurement volume.
- Laser *Doppler* velocimetry is somewhat of a misnomer, since we do not actually calculate a Doppler shift. Rather, we simply measure the frequency scattered by particles passing through the focal volume.
- Finally, a ***Bragg cell*** is used on one of the beams to shift its frequency slightly, causing the fringe pattern to move, making it possible to distinguish between positive and negative velocities through the focal volume.

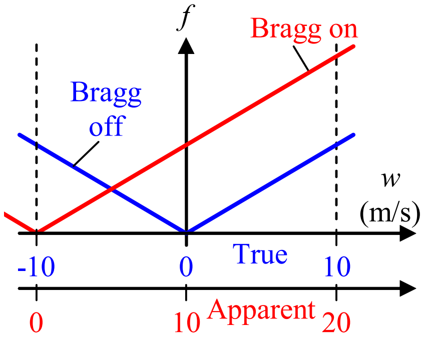
- **Example**:

  Given: An LDV is used to measure velocity component w in the z direction. w varies from -10 to 10 m/s.

  To do: Discuss why Bragg shifting is necessary in this situation. At what speed and in what direction should the fringe pattern in the measurement volume be shifted so that we can distinguish between positive and negative values of w?

  Solution:

  - Without any Bragg shifting, the fringe pattern is *stationary*, and it is impossible to distinguish between negative (w = -10 m/s) and positive (w = +10 m/s) components of velocity (blue line in plot below).
  - So, we adjust the Bragg shift such that the fringe pattern moves *down* at $\boxed{w_{\text{fringe}} = -10 \text{ m/s}}$, effectively shifting the frequency vs. velocity plot *to the left* by 10 m/s (red line in plot below) Thus,
    - When w = -10 m/s, seed particles in the flow move at the *same speed as the fringes*; light scatters from the measurement volume at zero scatter frequency when measured by the photodetector, corresponding to an apparent speed of 0 m/s.
    - When w = 0 m/s, the seed particles are stationary, but the fringes are still moving at -10 m/s, so light is scattered at a frequency corresponding to an apparent speed of 10 m/s.
    - When w = +10 m/s, the seed particles are moving *against* the motion of the fringes, so light is scattered at a frequency corresponding to an apparent speed of 20 m/s.

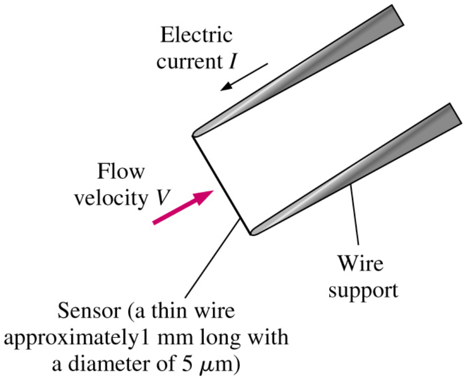


  - To correct for the moving fringes, we subtract 10 m/s from the apparent speed to get the true speed.
  - If w goes higher than 10 m/s, the true speed can still be determined. However, if w is less than -10 m/s, the correction no longer works, since it is impossible to detect a negative scatter frequency with the photodetector.

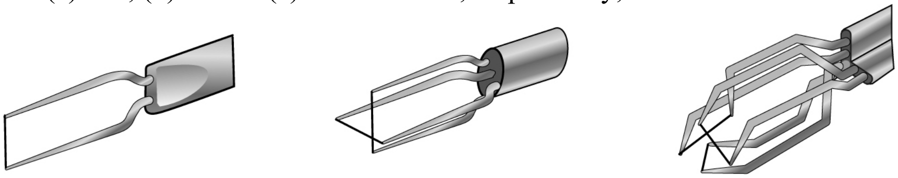
  Discussion: Modern LDV systems enable the user to dial in the minimum and maximum expected speeds, and the Bragg shifting is calculated and applied based on user input.

Thermal (hot-wire and hot-film) anemometry

- A ***thermal anemometer*** works under the principle that the rate of convective heat transfer from a hot object to the surrounding fluid increases as the speed of the fluid flowing around the object increases.



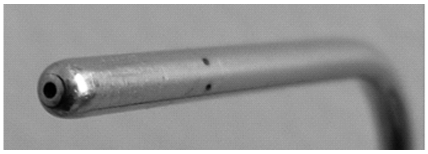
- A simple example is blowing at your spoon full of soup to cool it down – the rate of convection heat transfer (and thus the rate of cooling of the soup) is increased by blowing cooler air over it.
- Thermal anemometers consist of an electrically heated sensor, as sketched to the right, along with appropriate electronics.
- Thermal anemometers have extremely small sensors, and thus they can be used to measure the instantaneous velocity at a point in the flow without appreciably disturbing the flow.
- The time response of the sensor is fast, also due to its small size.
- There are two primary types of thermal anemometer probes:
    - If the sensing element is a ***wire*** (typically a few microns in diameter and about a millimeter long), it is called a ***hot-wire probe***, and the measurement system is called a ***hot-wire anemometer***.
    - If the sensing element is a ***thin metallic film*** (typically less than 0.1 micron thick), it is called a ***hot-film probe***, and the measurement system is called a ***hot-film anemometer***. The film is mounted on a ceramic cylindrical support that is much larger than a hot wire, typically about 50 microns in diameter.
- Hot-film sensors have poorer spatial resolution and frequency response because of their larger size, but are much more rugged compared to hot-wire sensors. Because of this, hot films are commonly used in water and other liquids, while hot wires are more commonly used in air and other gas flows.
- The most common thermal anemometer system is the ***constant-temperature anemometer*** (***CTA***).
- As its name implies, the sensor is heated (by electric current) to a constant temperature (typically about 200°C).
- The sensor wants to cool down as fluid velocity increases, but electronic controls (a Wheatstone bridge) maintain the temperature by varying the electric current I that passes through the sensor.
- The higher the velocity, the higher the rate of heat transfer required to keep the sensor at its fixed temperature, and therefore the higher the current passing through the sensor.
- The electrical power supplied to the sensor by electrical resistance heating is $\boxed{\dot{W}_{\text{electric}} = I^2 R_{\text{sensor}} = E^2 / R_{\text{sensor}}}$, where R_{sensor} is the electrical resistance of the sensor (hot wire or hot film), and E is the voltage across the sensor.
- Assuming that all of this supplied electrical power heats the wire, the power balance results in ***King's law***, $\boxed{E^2 = a + bV^n}$, where constants a, b, and n are calibrated for a given sensor.
- Finally, the voltage is measured, and is calibrated as a function of velocity across the probe.
- Thermal anemometers can be used to measure one, two, or three velocity components simultaneously by using probes with (a) one, (b) two or (c) three sensors, respectively, as sketched below.



(*a*)   (*b*)   (*c*)

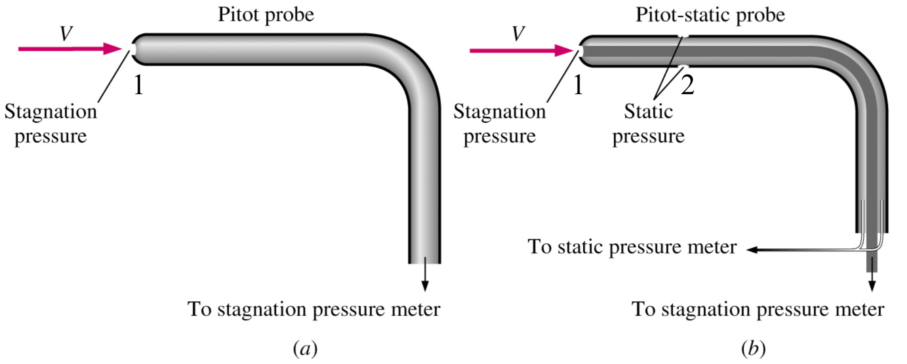
Pitot and Pitot-static probes

- A less sophisticated Eulerian velocity measurement technique involves probes that infer velocity by measuring pressure.
- A ***Pitot probe*** is just a tube with a pressure tap at the stagnation point that measures stagnation pressure.

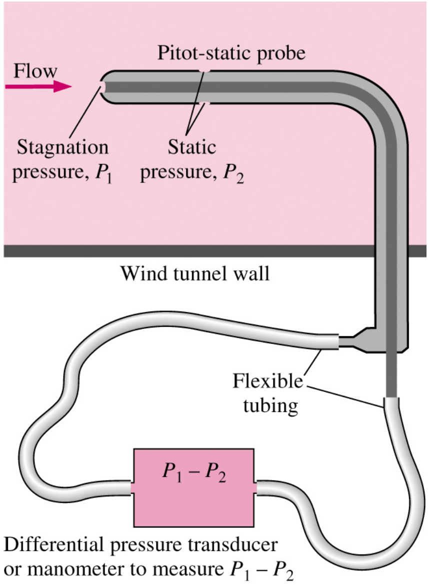


- A ***Pitot-static probe*** has both a stagnation pressure tap and several circumferential static pressure taps, and it measures both stagnation and static pressures.
- A close-up photograph of a Pitot-static probe is shown to the right.
- Some authors call Pitot-static probes ***Pitot-Darcy probes***, since Darcy was actually the first to combine the two pressure measurements into one single probe.
- Fluid velocity is inferred from the Pitot probe measurement of stagnation pressure by assuming that the static pressure is known (or measured with a separate pressure tap or probe).

- When using a Pitot-static probe either *both* pressures are measured, or the pressure *difference* is measured.
- We consider here incompressible flow, and assume that the probe is aligned into the flow, as sketched here for both the (a) Pitot probe and the (b) Pitot-static probe.



- At sufficiently high velocities, the Bernoulli equation is a reasonable approximation (viscous effects are neglected), and thus $\boxed{\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2}$, where ρ is the fluid density, location 1 is the stagnation point, location 2 is the average location of the static pressure taps in the probe, and z is vertical elevation.
- Since location 1 is a stagnation point, the velocity there (V_1) is zero.
- The probe is assumed to be small compared to the length scale associated with changes in flow velocity – we assume that velocity V_2 (just outside the boundary layer above the static pressure holes in the probe) is equal to velocity V upstream of the probe, which we are trying to measure.
- Since locations 1 and 2 are in close proximity, we neglect changes in elevation. (In fact, if the probe is *horizontal*, as in the sketch to the right, the *average* elevation of all the static pressure holes at location 2 is identical to that at the stagnation point.)
- Thus, with $V_1 = 0$, $V_2 = V$, and $z_1 = z_2$, the Bernoulli equation reduces to the ***Pitot formula***, $\boxed{V=\sqrt{\frac{2\left(P_1-P_2\right)}{\rho}}}$.
- The two pressures are measured by some kind of pressure sensor, as discussed in a previous learning module. In the above sketch, the difference between stagnation pressure P_1 and static pressure P_2 is measured by a differential pressure transducer – a U-tube manometer or other pressure sensor can be used instead.
- A typical application is sketched to the right – measurement of velocity in a wind tunnel.



- Some advantages of the Pitot-static probe are:
  - It is simple to use
  - It is inexpensive compared to some of the other more sophisticated velocity measurement techniques.
  - It is highly reliable since it has no moving parts.
- Some disadvantages of this velocity measurement technique include:
  - The probe must be aligned reasonably straight into the flow or else significant errors may result.
  - The probe disturbs the flow, unlike the LDV.
  - Pitot-static probes are not very responsive to fluctuating velocity fields. In other words, they have a poor frequency response compared to LDV or hot-wire anemometry.

- Nevertheless, Pitot-static probes have been used successfully for decades to measure flow velocity in both gases and liquids. For example, the next time you fly on an airplane, look for a Pitot or Pitot-static probe under the wing or on the fuselage. Ice build-up on the Pitot probes is believed to have contributed to the crash of Air France Flight 447 on June 1, 2009 – an Airbus A330-200.
- In many cases, the static pressure is atmospheric or nearly atmospheric, and thus only the *stagnation* pressure measurement is needed to determine the velocity. This can be accomplished with a Pitot tube. An example is shown here – a Pitot gauge designed to measure fire hydrant velocity, along with a photo of it being used.

Rotating velocimeters

- In many practical applications, it is not necessary to use *small* sensors to measure flow velocity. For example, weather stations measure the velocity of the wind, and the velocimeters can be rather large compared to Pitot-static probes or the measurement volume of an LDV system.
- ***Rotating velocimeters*** are devices that infer velocity by measuring the rotation rate of a rotating turbine, and there are several varieties.
- The most common rotating velocimeter used by weather stations is the ***cup anemometer***, as shown to the right.
- The one shown here has three cups, but four-cup anemometers are also common.
- Cup anemometers work on the principle that the open (concave) side of the cup experiences a higher drag force than does the closed (convex) part. Thus, when the wind blows, the shaft rotates in a preferred direction.
- To infer the wind speed, some type of mechanism converts shaft rotation speed into a voltage, a current, or a series of pulses that is calibrated to display the wind speed in the desired units (mi/hr, m/s, km/hr, knots, etc.).
- Some ***handheld cup anemometers*** are also commercially available, as shown to the right.

- There are some obvious disadvantages of cup anemometers:
  - They are quite large compared to the other "point" measurement devices discussed previously – cup anemometers measure the average wind speed over the volume occupied by the cups.
  - Most cup anemometers are designed only for horizontal flows, although this is usually not a problem with wind speed measurements.
  - They do not measure the *direction* of the wind. Three-cup anemometers are therefore usually coupled with a weather vane device that measures the wind direction independently of it speed.
- Another very popular type of rotating velocimeter is the ***turbine anemometer***, also called a ***vane anemometer***. This device employs a rotating turbine to infer velocity.
- A turbine anemometer works on the same principle as the cup anemometer, except that the turbine looks like a small ducted window fan (running backwards), as seen in the pictures below right.
- The more accurate turbine anemometers usually have a separate turbine portion and electronics portion, as shown to the near right, but the "all in one" units are more convenient to use (see picture to the far right).

- A temperature sensor (thermistor or thermocouple) is typically included with a turbine anemometer so that both air temperature and air speed are measured simultaneously. Such devices are called ***thermoanemometers***.
- Turbine anemometers are smaller and more convenient to use than cup anemometers, and they can be turned in any direction to face the air flow – even vertically if necessary.
- Turbine anemometers are often used by HVAC (heating, ventilation, and air conditioning) engineers to measure the air flow in ducts and out of air supply diffusers, etc.
- The major disadvantage is that the measured velocity is integrated over the face area of the turbine, which is quite large compared to the measurement size of a Pitot-static probe, hot wire, or LDV measurement volume.

Electromagnetic velocimeters

- A device called an ***electromagnetic velocimeter*** can be used to measure fluid velocity in *conducting* fluids.
- The principle of operation is that when a magnetic field is applied through an electrically conducting fluid, motion of the fluid induces a voltage across two electrodes placed in the flow.
- The output voltage is greatest for highly conducting fluids such as liquid metals.
- Seawater has fairly high conductance, and electromagnetic velocimeters are often used in oceanography experiments, where they are called ***electromagnetic current meters***.
- The strength of the output voltage increases with speed, and if properly calibrated, can infer fluid velocity.