Taguchi Orthogonal Arrays

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Introduction

- There are options for creating Taguchi arrays for the design of experiments, depending on how many times you choose to test each level of each parameter.
- For example, consider an experiment with 3 parameters and 3 levels of each parameter ($P = 3$ and $L = 3$), as discussed in a previous learning module. We showed two Taguchi arrays for this case:
  - a 6-run array for testing each level of each parameter *twice*
  - a 9-run array for testing each level of each parameter *three times*
- The 9-run array is more desirable (if cost and time permit) because for each level of any one parameter, *all three levels of the other parameters are tested*. Of course, either array here costs less to run than a full-factorial analysis, since the number of required runs for a full factorial analysis is $N = L^P = 3^3 = 27$.
- We call a Taguchi array an ***orthogonal array*** (some authors call it a ***full orthogonal array***) when *for each level of a particular parameter, all L levels of **each** of the (P-1) **other** parameters are tested at least once*.
- Sometimes, as P increases, it is necessary to test all levels of all parameters more than once in order to meet the rules for Taguchi arrays, as discussed previously. In such cases, there should be no *unnecessary* repeats.
- For example, consider the $P = 4$, $L = 3$ case on the next page. When parameter *a* is at level 1, parameter *b* is tested at levels 1, 2, and 3 (all levels). Similarly, parameters *c* and *d* are tested at levels 1, 2, and 3 (all L levels). The same thing holds when parameter *a* is at level 2 or level 3. The same thing holds for *all* of the parameters. Hence, we see that the definition of an orthogonal array holds for this case – for each level of a particular parameter, all L levels of each of the (P-1) other parameters are tested at least once – only once in this particular case. The required number of runs is therefore 3 ($L = 3$ levels) × 3 (each level of each parameter tested 3 times) = 9 required runs for this orthogonal array.
- Consider another example, the $P = 5$, $L = 3$ case on the next page. When parameter *a* is at level 1, parameter *b* is tested at level 1 twice, level 2 twice, and level 3 twice (twice for all 3 levels). Similarly, parameters *c* and *d* are tested twice at levels 1, 2, and 3 (all L levels). The same thing holds when parameter *a* is at level 2 or level 3. The same thing holds for *all* of the parameters. Hence, our definition of an orthogonal array also holds for this case – for each level of a particular parameter, all L levels of each of the (P-1) other parameters are tested at least once – actually *twice* in this particular case. The required number of runs is therefore 3 ($L = 3$ levels) × 6 (each level of each parameter tested 6 times) = 18 required runs for this orthogonal array.
- Orthogonal arrays are the "best" and most common type of Taguchi array, and ***you are encouraged to use orthogonal arrays whenever time and cost permit***. A table of Taguchi orthogonal arrays is provided below for values of P (number of parameters) ranging from 2 to 5, and L (number of levels) ranging from 2 to 5.

$P = \rightarrow$	2	3	4	5
$L = \downarrow$				

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Taguchi, $P = 2$, $L = 2$

Run #	a	b	X
1	1	1	X_1
2	1	2	X_2
3	2	1	X_3
4	2	2	X_4

Taguchi, $P = 3$, $L = 2$

Run #	a	b	c	X
1	1	1	1	X_1
2	1	2	2	X_2
3	2	1	2	X_3
4	2	2	1	X_4

Taguchi, $P = 4$, $L = 2$

Run #	a	b	c	d	X
1	1	1	1	1	X_1
2	1	1	1	2	X_2
3	1	2	2	1	X_3
4	1	2	2	2	X_4
5	2	1	2	1	X_5
6	2	1	2	2	X_6
7	2	2	1	1	X_7
8	2	2	1	2	X_8

Taguchi, $P = 5$, $L = 2$

Run #	a	b	c	d	e	X
1	1	1	1	1	1	X_1
2	1	1	1	2	2	X_2
3	1	2	2	1	1	X_3
4	1	2	2	2	2	X_4
5	2	1	2	1	2	X_5
6	2	1	2	2	1	X_6
7	2	2	1	1	2	X_7
8	2	2	1	2	1	X_8

$P = →$	2	3	4	5
$L = ↓$				

$L = 3$

Taguchi, $P = 2$, $L = 3$

Run #	a	b	X
1	1	1	X_1
2	1	2	X_2
3	1	3	X_3
4	2	1	X_4
5	2	2	X_5
6	2	3	X_6
7	3	1	X_7
8	3	2	X_8
9	3	3	X_9

Taguchi, $P = 3$, $L = 3$

Run #	a	b	c	X
1	1	1	1	X_1
2	1	2	2	X_2
3	1	3	3	X_3
4	2	1	2	X_4
5	2	2	3	X_5
6	2	3	1	X_6
7	3	1	3	X_7
8	3	2	1	X_8
9	3	3	2	X_9

Taguchi, $P = 4$, $L = 3$

Run #	a	b	c	d	X
1	1	1	1	1	X_1
2	1	2	2	2	X_2
3	1	3	3	3	X_3
4	2	1	2	3	X_4
5	2	2	3	1	X_5
6	2	3	1	2	X_6
7	3	1	3	2	X_7
8	3	2	1	3	X_8
9	3	3	2	1	X_9

Taguchi, $P = 5$, $L = 3$

Run #	a	b	c	d	e	X
1	1	1	1	1	1	X_1
2	1	2	2	2	2	X_2
3	1	3	3	3	3	X_3
4	2	1	1	2	2	X_4
5	2	2	2	3	3	X_5
6	2	3	3	1	1	X_6
7	3	1	2	1	3	X_7
8	3	2	3	2	1	X_8
9	3	3	1	3	2	X_9
10	1	1	3	3	2	X_{10}
11	1	2	1	1	3	X_{11}
12	1	3	2	2	1	X_{12}
13	2	1	2	3	1	X_{13}
14	2	2	3	1	2	X_{14}
15	2	3	1	2	3	X_{15}
16	3	1	3	2	3	X_{16}
17	3	2	1	3	1	X_{17}
18	3	3	2	1	2	X_{18}

$L = 4$

Taguchi, $P = 2$, $L = 4$

Run #	a	b	X
1	1	1	X_1
2	1	2	X_2
3	1	3	X_3
4	1	4	X_4
5	2	1	X_5
6	2	2	X_6
7	2	3	X_7
8	2	4	X_8
9	3	1	X_9
10	3	2	X_{10}
11	3	3	X_{11}
12	3	4	X_{12}
13	4	1	X_{13}
14	4	2	X_{14}
15	4	3	X_{15}
16	4	4	X_{16}

Taguchi, $P = 3$, $L = 4$

Run #	a	b	c	X
1	1	1	1	X_1
2	1	2	2	X_2
3	1	3	3	X_3
4	1	4	4	X_4
5	2	1	2	X_5
6	2	2	1	X_6
7	2	3	4	X_7
8	2	4	3	X_8
9	3	1	3	X_9
10	3	2	4	X_{10}
11	3	3	1	X_{11}
12	3	4	2	X_{12}
13	4	1	4	X_{13}
14	4	2	3	X_{14}
15	4	3	2	X_{15}
16	4	4	1	X_{16}

Taguchi, $P = 4$, $L = 4$

Run #	a	b	c	d	X
1	1	1	1	1	X_1
2	1	2	2	2	X_2
3	1	3	3	3	X_3
4	1	4	4	4	X_4
5	2	1	2	3	X_5
6	2	2	1	4	X_6
7	2	3	4	1	X_7
8	2	4	3	2	X_8
9	3	1	3	4	X_9
10	3	2	4	3	X_{10}
11	3	3	1	2	X_{11}
12	3	4	2	1	X_{12}
13	4	1	4	2	X_{13}
14	4	2	3	1	X_{14}
15	4	3	2	4	X_{15}
16	4	4	1	3	X_{16}

Taguchi, $P = 5$, $L = 4$

Run #	a	b	c	d	e	X
1	1	1	1	1	1	X_1
2	1	2	2	2	2	X_2
3	1	3	3	3	3	X_3
4	1	4	4	4	4	X_4
5	2	1	2	3	4	X_5
6	2	2	1	4	3	X_6
7	2	3	4	1	2	X_7
8	2	4	3	2	1	X_8
9	3	1	3	4	2	X_9
10	3	2	4	3	1	X_{10}
11	3	3	1	2	4	X_{11}
12	3	4	2	1	3	X_{12}
13	4	1	4	2	3	X_{13}
14	4	2	3	1	4	X_{14}
15	4	3	2	4	1	X_{15}
16	4	4	1	3	2	X_{16}

$P = \rightarrow$
$L = \downarrow$

$L = 5$

Taguchi, $P = 2$, $L = 5$

Run #	a	b	X
1	1	1	X_1
2	1	2	X_2
3	1	3	X_3
4	1	4	X_4
5	1	5	X_5
6	2	1	X_6
7	2	2	X_7
8	2	3	X_8
9	2	4	X_9
10	2	5	X_{10}
11	3	1	X_{11}
12	3	2	X_{12}
13	3	3	X_{13}
14	3	4	X_{14}
15	3	5	X_{15}
16	4	1	X_{16}
17	4	2	X_{17}
18	4	3	X_{18}
19	4	4	X_{19}
20	4	5	X_{20}
21	5	1	X_{21}
22	5	2	X_{22}
23	5	3	X_{23}
24	5	4	X_{24}
25	5	5	X_{25}

Taguchi, $P = 3$, $L = 5$

Run #	a	b	c	X
1	1	1	1	X_1
2	1	2	2	X_2
3	1	3	3	X_3
4	1	4	4	X_4
5	1	5	5	X_5
6	2	1	2	X_6
7	2	2	3	X_7
8	2	3	4	X_8
9	2	4	5	X_9
10	2	5	1	X_{10}
11	3	1	3	X_{11}
12	3	2	4	X_{12}
13	3	3	5	X_{13}
14	3	4	1	X_{14}
15	3	5	2	X_{15}
16	4	1	4	X_{16}
17	4	2	5	X_{17}
18	4	3	1	X_{18}
19	4	4	2	X_{19}
20	4	5	3	X_{20}
21	5	1	5	X_{21}
22	5	2	1	X_{22}
23	5	3	2	X_{23}
24	5	4	3	X_{24}
25	5	5	4	X_{25}

Taguchi, $P = 4$, $L = 5$

Run #	a	b	c	d	X
1	1	1	1	1	X_1
2	1	2	2	2	X_2
3	1	3	3	3	X_3
4	1	4	4	4	X_4
5	1	5	5	5	X_5
6	2	1	2	3	X_6
7	2	2	3	4	X_7
8	2	3	4	5	X_8
9	2	4	5	1	X_9
10	2	5	1	2	X_{10}
11	3	1	3	5	X_{11}
12	3	2	4	1	X_{12}
13	3	3	5	2	X_{13}
14	3	4	1	3	X_{14}
15	3	5	2	4	X_{15}
16	4	1	4	2	X_{16}
17	4	2	5	3	X_{17}
18	4	3	1	4	X_{18}
19	4	4	2	5	X_{19}
20	4	5	3	1	X_{20}
21	5	1	5	4	X_{21}
22	5	2	1	5	X_{22}
23	5	3	2	1	X_{23}
24	5	4	3	2	X_{24}
25	5	5	4	3	X_{25}

Taguchi, $P = 5$, $L = 5$

Run #	a	b	c	d	e	X
1	1	1	1	1	1	X_1
2	1	2	2	2	2	X_2
3	1	3	3	3	3	X_3
4	1	4	4	4	4	X_4
5	1	5	5	5	5	X_5
6	2	1	2	3	4	X_6
7	2	2	3	4	5	X_7
8	2	3	4	5	1	X_8
9	2	4	5	1	2	X_9
10	2	5	1	2	3	X_{10}
11	3	1	3	5	2	X_{11}
12	3	2	4	1	3	X_{12}
13	3	3	5	2	4	X_{13}
14	3	4	1	3	5	X_{14}
15	3	5	2	4	1	X_{15}
16	4	1	4	2	5	X_{16}
17	4	2	5	3	1	X_{17}
18	4	3	1	4	2	X_{18}
19	4	4	2	5	3	X_{19}
20	4	5	3	1	4	X_{20}
21	5	1	5	4	3	X_{21}
22	5	2	1	5	4	X_{22}
23	5	3	2	1	5	X_{23}
24	5	4	3	2	1	X_{24}
25	5	5	4	3	2	X_{25}

- **Bottom line**: Experimental test arrays are usually chosen based on a compromise between the *cost* of the experiments (cost includes the *time* required to run the experiments) and required *accuracy* of the results. Below is a hierarchy of how you should choose a test array:

Increasing accuracy →

Full Factorial Array:
If cost is not a big issue (in other words, you have enough time, and the runs are inexpensive and don't take too long), or if the accuracy of the results is critical, use a full factorial array.

Taguchi Orthogonal Array:
If the cost (including time) of a full factorial array analysis is high, and the accuracy of the results is not so critical, use an orthogonal Taguchi array.

Taguchi Non-Orthogonal Array:
If the cost is prohibitive (runs are extremely expensive or time consuming), and you can accept limited accuracy, use a non-orthogonal Taguchi array (but be sure to optimize it using the two rules given in the previous learning module for fractional factorial analysis).

Decreasing cost →