Equation Sheet for ME 420

For homework, in-class problems, exams, and future reference. Author: John M. Cimbala, Penn State University. Latest revision, 06 December 2022

General and conversions: $g = 9.807\ \frac{\text{m}}{\text{s}^2}$, $\frac{0.3048\ \text{m}}{1\ \text{ft}}$, $\frac{1\ \text{mile}}{1609.3\ \text{m}}$, $\frac{1\ \text{kPa}\cdot\text{m}^2}{1\ \text{kN}}$, $\frac{1\ \text{kN}\cdot\text{m}}{1\ \text{kJ}}$, $\frac{1\ \text{kW}\cdot\text{s}}{1\ \text{kJ}}$, $\frac{1\ \text{Btu}}{1.055056\ \text{kJ}}$, $\frac{1\ \text{kg}}{2.205\ \text{lbm}}$, $\frac{1\ \text{ton}}{2000\ \text{lbm}}$, $\frac{1\ \text{tonne (metric ton)}}{1000\ \text{kg}}$, $\frac{1\ \text{g}}{10^6\ \mu\text{g}}$, $\frac{1\ \text{m}}{10^6\ \mu\text{m}}$, $\frac{1\ \text{m}}{10^9\ \text{nm}}$, $V_{\text{sphere}} = \frac{4}{3}\pi (R_p)^3 = \frac{1}{6}\pi (D_p)^3$.

Molecular weights and mols: $m = nM$, $M_{\text{air}} = 28.97\ \text{g/mol}$, $M_{\text{water}} = 18.02\ \text{g/mol}$, Avagadro's number: 6.02214×10^{23}.

Air at SATP (25°C): $P_{\text{SATP}} = 101.325\ \text{kPa} = 760\ \text{mm Hg}$, $T_{\text{SATP}} = 25^\circ\text{C} = 298.15\ \text{K}$, $\rho = 1.184\ \text{kg/m}^3$, $\lambda = 0.06704\ \mu\text{m}$, $\mu = 1.849\times 10^{-5}\ \text{kg/(m s)}$.

Air at any *T*: $P = \rho R_{\text{air}} T$, $\lambda \approx \frac{\mu}{0.499}\sqrt{\frac{\pi}{8\rho P}}$, $\mu \approx \mu_s \left(\frac{T}{T_{s,0}}\right)^{3/2} \frac{T_{s,0} + T_s}{T + T_s}$, $T_{s,0} = 298.15$ K, $T_s = 110.4$ K, $\mu_s = 1.849\times 10^{-5}\ \frac{\text{kg}}{\text{m}\cdot\text{s}}$.

Ideal gas: $PV = nR_u T = mRT$, $R = \frac{R_u}{M}$, $P = \rho R T$, $R_u = 8.314\ \frac{\text{kJ}}{\text{kmol}\cdot\text{K}}$, $R_{\text{air}} = 0.2870\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 287.0\ \frac{\text{J}}{\text{kg}\cdot\text{K}} = 287.0\ \frac{\text{m}^2}{\text{s}^2\cdot\text{K}}$.

Mass flow rate in a duct: $\dot{m} = \rho V A =$ constant, where V is the magnitude of velocity averaged over cross-sectional area A.

Thermo review: $e = u + \frac{V^2}{2} + \cancel{gz}$, $\upsilon = \frac{1}{\rho}$, $h = u + P\upsilon = u + \frac{P}{\rho}$, $c_\upsilon = \left(\frac{\partial u}{\partial T}\right)_\upsilon$, $c_P = \left(\frac{\partial h}{\partial T}\right)_P$, $Tds = du + Pd\upsilon$, $Tds = dh - \upsilon dP$.

Ideal gas: $u = c_\upsilon T$, $h = c_P T$, $c_P - c_\upsilon = R$, $\gamma = \frac{c_P}{c_\upsilon}$, $c_\upsilon = \frac{R}{\gamma - 1}$, $c_P = \frac{R\gamma}{\gamma - 1}$, $s_2 - s_1 = c_P \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1} = c_\upsilon \ln\frac{T_2}{T_1} + R\ln\frac{\upsilon_2}{\upsilon_1}$.

Air: $\gamma = 1.40$, $c_\upsilon = 717.5\ \frac{\text{J}}{\text{kg}\cdot\text{K}} = 0.7175\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 717.5\ \frac{\text{m}^2}{\text{s}^2\cdot\text{K}}$, $c_P = 1004.5\ \frac{\text{J}}{\text{kg}\cdot\text{K}} = 1.0045\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}} = 1004.5\ \frac{\text{m}^2}{\text{s}^2\cdot\text{K}}$.

k sometimes instead of γ

Isentropic flow of ideal gas: $\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$, $\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}$. For air ($\gamma = 1.40$), $\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{3.5}$, $\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{2.5}$.

Ma sometimes instead of M

Mach number *M*, speed of sound *a*, Mach angle *μ*: $M = \frac{V}{a}$, $a = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_s}$ [ideal gas, $a = \sqrt{\gamma R T}$], $\mu = \sin^{-1}\frac{1}{M}$.

c sometimes instead of *a*

Steady, adiabatic duct flow: $\dot{m}_1 = \dot{m}_2$, $h_0 = h + \frac{V^2}{2}$, $h_{0,1} = h_{0,2}$, $\frac{dA}{A} + \frac{dV}{V} + \frac{d\rho}{\rho} = 0$, $\frac{dV}{V} = \frac{1}{M^2 - 1}\frac{dA}{A}$.

Critical or sonic values when $M = 1$ (at throat or at duct exit): $a^* = \sqrt{\gamma R T^*}$, $M^* = V/a^*$ = characteristic Mach number.

Adiabatic, isentropic, 1-D duct flow: $\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2}M^2$, $\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{1}{\gamma-1}}$, $\frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$, $\frac{a_0}{a} = \left(\frac{T_0}{T}\right)^{1/2}$,

$\frac{T_0}{T^*} = \frac{\gamma + 1}{2}$, $\frac{\rho_0}{\rho^*} = \left(\frac{T_0}{T^*}\right)^{\frac{1}{\gamma-1}}$, $\frac{P_0}{P^*} = \left(\frac{T_0}{T^*}\right)^{\frac{\gamma}{\gamma-1}}$, $\frac{a_0}{a^*} = \left(\frac{T_0}{T^*}\right)^{1/2}$, $\frac{A}{A^*} = \frac{1}{M}\left[\left(\frac{2}{\gamma + 1}\right)\left(1 + \frac{\gamma - 1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$, $M^2 = \frac{2}{\frac{\gamma + 1}{M^{*2}} - (\gamma - 1)}$.

Air (*γ*=1.4): $\frac{T_0}{T} = 1 + 0.2M^2$, $\frac{\rho_0}{\rho} = (1 + 0.2M^2)^{2.5} = \left(\frac{T_0}{T}\right)^{2.5}$, $\frac{P_0}{P} = (1 + 0.2M^2)^{3.5} = \left(\frac{T_0}{T}\right)^{3.5}$, $\frac{a_0}{a} = \left(\frac{T_0}{T}\right)^{1/2}$,

$\frac{T_0}{T^*} = \frac{\gamma + 1}{2} = 1.2$, $\frac{\rho_0}{\rho^*} = 1.577$, $\frac{P_0}{P^*} = 1.893$, $\frac{a_0}{a^*} = 1.095$, $\frac{A}{A^*} = \frac{1}{M}\frac{(1 + 0.2M^2)^3}{1.728}$, $M^2 = \frac{2}{(2.4/M^{*2}) - 0.4}$.

Mass flow rate: General case: $\dot{m} = P_0 A M \sqrt{\frac{\gamma}{R T_0}}\left(1 + \frac{\gamma - 1}{2}M^2\right)^{\frac{-(\gamma+1)}{2(\gamma-1)}}$, Choked case: $\dot{m} = \dot{m}_{\max} = P_0 A^* \sqrt{\frac{\gamma}{R T_0}}\left(\frac{\gamma + 1}{2}\right)^{\frac{-(\gamma+1)}{2(\gamma-1)}}$.

Exam 1 material ends here.

Normal shock equations (stationary, ideal gas): $h_{0,1} = h_{0,2}$, $T_{0,1} = T_{0,2}$, $\frac{T_2}{T_1} = \frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}$, $M_2^* = \frac{1}{M_1^*}$. The pressure ratio must satisfy both the Fanno curve $\frac{P_2}{P_1} = \frac{M_1}{M_2}\sqrt{\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}}$ and the Rayleigh curve $\frac{P_2}{P_1} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2}$. Equating these, solve for M_2^2: Method 1: $M_2^2 = \frac{-B-\sqrt{-(B+2A)}}{1+\gamma B}$ where $A = M_1^2\frac{1+\frac{\gamma-1}{2}M_1^2}{\left(1+\gamma M_1^2\right)^2}$, $B = 2\gamma A - 1$. Method 2: $M_2^2 = \frac{1+\frac{\gamma-1}{2}M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}}$.

Either method yields M_2 *after* the shock in terms of γ and M_1 *before* the shock. Other useful normal shock relations:

$\frac{P_2}{P_1} = \frac{2\gamma M_1^2 - \gamma + 1}{\gamma+1}$, $\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}$, $\frac{P_{02}}{P_{01}} = \frac{M_1}{M_2}\left[\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}} = \left[\frac{\frac{\gamma+1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_1^2}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{1-\gamma+2\gamma M_1^2}{\gamma+1}\right]^{\frac{-1}{\gamma-1}}$,

$\frac{P_1}{P_{0,2}} = \left[\frac{1-\gamma+2\gamma M_1^2}{\gamma+1}\right]^{\frac{1}{\gamma-1}}\left[\frac{\gamma+1}{2}M_1^2\right]^{\frac{-\gamma}{\gamma-1}}$. Hugoniot: $u_2 - u_1 = \frac{P_1+P_2}{2}(\upsilon_1 - \upsilon_2)$; Ideal gas Hugoniot: $\frac{P_2}{P_1} = \frac{\frac{\gamma+1}{\gamma-1}\frac{\rho_2}{\rho_1} - 1}{\frac{\gamma+1}{\gamma-1} - \frac{\rho_2}{\rho_1}}$.

Normal shock in one-D duct: $\frac{A^*_2}{A^*_1} = \frac{M_2}{M_1}\left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}} = \left[\frac{1+\frac{\gamma-1}{2}M_1^2}{\frac{\gamma+1}{2}M_1^2}\right]^{\frac{\gamma}{\gamma-1}}\left[\frac{1-\gamma+2\gamma M_1^2}{\gamma+1}\right]^{\frac{1}{\gamma-1}}$.

Air (γ=1.4): $h_{0,1} = h_{0,2}$, $T_{0,1} = T_{0,2}$, $\frac{T_2}{T_1} = \frac{1+0.2M_1^2}{1+0.2M_2^2}$, Fanno: $\frac{P_2}{P_1} = \frac{M_1}{M_2}\sqrt{\frac{1+0.2M_1^2}{1+0.2M_2^2}}$, Rayleigh: $\frac{P_2}{P_1} = \frac{1+1.4M_1^2}{1+1.4M_2^2}$.

$M_2^2 = \frac{1+0.2M_1^2}{1.4M_1^2 - 0.2}$, $\frac{P_2}{P_1} = \frac{2.8M_1^2 - 0.4}{2.4}$, $\frac{\rho_2}{\rho_1} = \frac{2.4M_1^2}{2+0.4M_1^2}$, $\frac{P_{02}}{P_{01}} = \frac{M_1}{M_2}\left[\frac{1+0.2M_2^2}{1+0.2M_1^2}\right]^3 = \left[\frac{1.2M_1^2}{1+0.2M_1^2}\right]^{3.5}\left[\frac{-0.4+2.8M_1^2}{2.4}\right]^{-2.5}$,

$\frac{P_1}{P_{02}} = \left[\frac{-0.4+2.8M_1^2}{2.4}\right]^{2.5}\left[1.2M_1^2\right]^{-3.5}$. Hugoniot: $\frac{P_2}{P_1} = \frac{6\frac{\rho_2}{\rho_1} - 1}{6 - \frac{\rho_2}{\rho_1}}$. One-D duct: $\frac{A^*_2}{A^*_1} = \left[\frac{1+0.2M_1^2}{1.2M_1^2}\right]^{3.5}\left[\frac{-0.4+2.8M_1^2}{2.4}\right]^{2.5}$.

Blast waves: $R\rho_{\text{atm}}^{1/5}E^{-1/5}t^{-2/5} = \text{constant}_1$, $V_s = \frac{dR}{dt} = \text{constant}_2 \cdot t^{-3/5}$.

Pitot probes: Incompressible: $V = \sqrt{2\frac{(P_0 - P)}{\rho}}$, Subsonic compressible: $V = \sqrt{\frac{2\gamma RT_1}{\gamma-1}\left[\left(\frac{P_{01}}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}$, Supersonic (the Rayliegh-Pitot formula): $\frac{P_1}{P_{02}} = \left(\frac{1-\gamma+2\gamma M_1^2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}\left(\frac{\gamma+1}{2}M_1^2\right)^{\frac{-\gamma}{\gamma-1}}$. For air, $\frac{P_1}{P_{02}} = \left(\frac{-0.4+2.8M_1^2}{2.4}\right)^{2.5}\left(1.2M_1^2\right)^{-3.5}$.

Moving normal shocks: Static property ratios $\left(\frac{T_2}{T_1}, \frac{P_2}{P_1}, \frac{\rho_2}{\rho_1}, \text{etc.}\right)$ are same as stationary shock, but stagnation property ratios $\left(\frac{T_{02}}{T_{01}}, \frac{P_{02}}{P_{01}}, \frac{\rho_{02}}{\rho_{01}}, \text{etc.}\right)$ are not. $M_1 = \frac{V_1}{a_1} = M_s = \frac{V_s}{a_1}$, $h_{0,as} = h_2 + \frac{V_{as}}{2}$, $\frac{V_1}{V_2} = \frac{\rho_2}{\rho_1} = \frac{V_s}{V_s - V_{as}}$, $V_{as} = \left(1 - \frac{V_2}{V_1}\right)V_s$.

Shock tubes: Here, 4 is the initially high-pressure region and 1 is the initially low-pressure region (ideal gas only).

Expansion fan: $\frac{a_3}{a_4} = 1 - \frac{\gamma_4 - 1}{2}\frac{V_3}{a_4}$, $\frac{T_3}{T_4} = \left(\frac{a_3}{a_4}\right)^2 = \left(1 - \frac{\gamma_4 - 1}{2}\frac{V_3}{a_4}\right)^2$, similarly, $\frac{P_3}{P_4} = \left(\frac{a_3}{a_4}\right)^{\frac{2\gamma_4}{\gamma_4 - 1}}$ and $\frac{\rho_3}{\rho_4} = \left(\frac{a_3}{a_4}\right)^{\frac{2}{\gamma_4 - 1}}$.

Shock tube: $V_2 = V_3 = V_{CS} = V_{as} = \frac{2a_1}{\gamma_1 + 1}\left(M_s - \frac{1}{M_s}\right)$, $P_2 = P_3$, $\frac{P_4}{P_1} = \frac{2\gamma_1 M_s^2 - (\gamma_1 - 1)}{\gamma_1 + 1}\left[1 - \frac{\gamma_4 - 1}{\gamma_1 + 1}\frac{a_1}{a_4}\left(M_s - \frac{1}{M_s}\right)\right]^{\frac{-2\gamma_4}{\gamma_4 - 1}}$.

Reflecting shock (2=before, 5=after): $\frac{V_{\text{before}}}{V_{\text{after}}} = \frac{V_{sR} + V_{CS}}{V_{sR}} = \frac{(\gamma_1 + 1)M_{SR}^2}{2 + (\gamma_1 - 1)M_{SR}^2}$, $M_{sR} = \frac{V_{\text{before}}}{a_{\text{before}}} = \frac{V_{sR} + V_{CS}}{a_{\text{before}}} = \frac{V_{sR} + V_{CS}}{\sqrt{\gamma_1 R_1 T_{\text{before}}}}$.

Rayleigh flow (Ideal gas): Cons. laws: $\rho_1 V_1 = \rho_2 V_2$, $P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2$, $q = \frac{\dot{Q}}{\dot{m}} = c_P(T_{02} - T_{01}) = c_P(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}$.

$\frac{P_2}{P_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}$, $\frac{P}{P^*} = \frac{1 + \gamma}{1 + \gamma M^2}$, $\frac{T_2}{T_1} = \left[\frac{M_2}{M_1}\frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}\right]^2$, $\frac{T}{T^*} = \left[\frac{(1 + \gamma)M}{1 + \gamma M^2}\right]^2$, $\frac{\rho^*}{\rho} = \frac{V}{V^*} = \frac{(1 + \gamma)M^2}{1 + \gamma M^2}$,

$\frac{T_0}{T_0^*} = \frac{\left[2 + (\gamma - 1)M^2\right](1 + \gamma)M^2}{\left[1 + \gamma M^2\right]^2}$, $\frac{P_0}{P_0^*} = \left[\frac{2 + (\gamma - 1)M^2}{1 + \gamma}\right]^{\frac{\gamma}{\gamma - 1}}\frac{(1 + \gamma)}{1 + \gamma M^2}$, $q_{\max} = \frac{\dot{Q}_{\max}}{\dot{m}} = c_P\left(T_0^* - T_{01}\right)$.

Air (γ=1.4): $\frac{P_2}{P_1} = \frac{1 + 1.4M_1^2}{1 + 1.4M_2^2}$, $\frac{P}{P^*} = \frac{2.4}{1 + 1.4M^2}$, $\frac{T_2}{T_1} = \left[\frac{M_2}{M_1}\frac{1 + 1.4M_1^2}{1 + 1.4M_2^2}\right]^2$, $\frac{T}{T^*} = \left[\frac{2.4M}{1 + 1.4M^2}\right]^2$, $\frac{\rho^*}{\rho} = \frac{V}{V^*} = \frac{(1 + 1.4)M^2}{1 + 1.4M^2}$,

Exam 2 material ends here.

$\frac{T_0}{T_0^*} = \frac{\left[2 + 0.4M^2\right](2.4)M^2}{\left[1 + 1.4M^2\right]^2}$, $\frac{P_0}{P_0^*} = \left[\frac{2 + 0.4M^2}{2.4}\right]^{3.5}\frac{2.4}{1 + 1.4M^2}$.

Fanno flow (Ideal gas): Cons. laws: $\rho_1 V_1 = \rho_2 V_2$, $P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2 + \frac{F_{\text{friction}}}{A}$, $T_{01} = T_{02}$ or $c_P T_1 + \frac{V_1^2}{2} = c_P T_2 + \frac{V_2^2}{2}$.

$f = \frac{8\tau_w}{\rho V^2}$ = Darcy friction factor from the **Moody chart** or **the Churchill equation**, $f = 8\left[\left(\frac{8}{\text{Re}}\right)^{12} + (A + B)^{-1.5}\right]^{1/12}$, where

$\text{Re} = \frac{\rho V D_h}{\mu} = \frac{V D_h}{\nu}$, $A = \left\{-2.457 \cdot \ln\left[\left(\frac{7}{\text{Re}}\right)^{0.9} + 0.27\frac{\varepsilon}{D_h}\right]\right\}^{16}$, $B = \left(\frac{37530}{\text{Re}}\right)^{16}$, $D_h = \frac{4A_c}{p}$ = **hydraulic diameter**, A_c is duct cross-sectional area, p is the perimeter. For solutions it is typical to use *ratios* at the ***critical reference state***:

$\frac{fL^*}{D_h} = \frac{1 - M^2}{\gamma M^2} + \frac{\gamma + 1}{2\gamma}\ln\left(\frac{(\gamma + 1)M^2}{2 + (\gamma - 1)M^2}\right)$, $\frac{fL}{D_h} = \left(\frac{fL^*}{D_h}\right)_1 - \left(\frac{fL^*}{D_h}\right)_2$, $\frac{T}{T^*} = \frac{\gamma + 1}{2 + (\gamma - 1)M^2}$, $\frac{\rho^*}{\rho} = \frac{V}{V^*} = M\sqrt{\frac{\gamma + 1}{2 + (\gamma - 1)M^2}}$,

$\frac{P}{P^*} = \frac{1}{M}\sqrt{\frac{\gamma + 1}{2 + (\gamma - 1)M^2}}$, $\frac{P_0}{P_0^*} = \frac{\rho_0}{\rho_0^*} = \frac{1}{M}\left[\frac{2 + (\gamma - 1)M^2}{\gamma + 1}\right]^{\frac{\gamma + 1}{2(\gamma - 1)}}$. **For air ($\gamma$=1.4)**: $\frac{fL^*}{D_h} = \frac{1 - M^2}{1.4M^2} + \frac{2.4}{2.8}\ln\left(\frac{2.4M^2}{2 + 0.4M^2}\right)$,

$\frac{T}{T^*} = \frac{2.4}{2 + 0.4M^2}$, $\frac{\rho^*}{\rho} = \frac{V}{V^*} = M\sqrt{\frac{2.4}{2 + 0.4M^2}}$, $\frac{P}{P^*} = \frac{1}{M}\sqrt{\frac{2.4}{2 + 0.4M^2}}$, $\frac{P_0}{P_0^*} = \frac{\rho_0}{\rho_0^*} = \frac{1}{M}\left[\frac{2 + 0.4M^2}{2.4}\right]^3$.

Oblique shocks (Ideal gas): *θ–β–M* equation: $\tan\theta = \frac{2\cot\beta\left(M_1^2\sin^2\beta - 1\right)}{M_1^2\left[\gamma + \cos(2\beta)\right] + 2}$, $M_{1,n} = M_1\sin\beta$, $M_{2,n} = M_2\sin(\beta - \theta)$.

All else identical to normal shocks except replace M_1 with $M_{1,n}$ and replace M_2 with $M_{2,n}$. Repeat for reflected shocks.

Prandtl-Meyer expansion waves (Ideal gas): P–M function: $\nu(M) = \sqrt{\frac{\gamma + 1}{\gamma - 1}}\tan^{-1}\left[\sqrt{\frac{\gamma - 1}{\gamma + 1}\left(M^2 - 1\right)}\right] - \tan^{-1}\left[\sqrt{\left(M^2 - 1\right)}\right]$,

$\theta_2 = \nu(M_2) - \nu(M_1)$, then use isentropic relations. **Air (γ=1.4)**: $\nu(M) = \sqrt{6}\tan^{-1}\left[\sqrt{\frac{1}{6}\left(M^2 - 1\right)}\right] - \tan^{-1}\left[\sqrt{\left(M^2 - 1\right)}\right]$.