Due: In class, Friday **September 9, 2022**	**Name(s)** (*Each* student must submit; list anyone you worked with) **PSU ID (abc123)**
	Student submitting:
	Worked with:
	Worked with:

ME 420
Fall Semester, 2022
Homework Set # 2

Professor J. M. Cimbala

For instructor or TA use only:		
Problem	**Score**	**Points**
1		15
2		20
3		15
4		50
Total:		**100**

1. (15 pts) We define the (square of) speed of sound as $a^2 = \left(\frac{\partial P}{\partial \rho}\right)_s$. In the class notes of Lecture 5, however, we used the *reciprocal* of this equation as part of a derivation, namely, $\frac{1}{a^2} = \left(\frac{\partial \rho}{\partial P}\right)_s$. In this problem, we ask the question, "Is this mathematically sound?"

(**a**) For a general function $y(x)$, prove by first principles (fundamental definition of derivative) that $\frac{dx}{dy} = \frac{1}{dy/dx}$.

(**b**) Test with a given function, $y(x) = 3x^2$. Does the equation of Part (a) work for this case? Show all your work.

2. (20 pts) In class we defined Mach number $M = \frac{V}{a}$ and characteristic Mach number $M^* = \frac{V}{a^*}$. On our equation sheet is given a relationship between these two kinds of Mach number, $M^2 = \frac{2}{\frac{\gamma+1}{M^{*2}} - (\gamma - 1)}$.

(**a**) Prove this equation, showing all your work.
(**b**) Write the inverse of this equation, namely, M^* as a function of M rather than the above, which is the other way around.
(**c**) For an arbitrary ideal gas (let γ remain a variable), write an expression for the maximum possible value of M^*. What is the maximum possible value of M^* for air?
(**d**) Repeat for helium instead of air.

3. (15 pts) In class, we derived expressions for the ratios of stagnation temperature and pressure to static temperature and pressure, respectively, for an ideal gas, $\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$ and $\frac{P_0}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma - 1}}$. We then used an ***isentropic*** relationship to get a similar expression for density, namely, $\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma - 1}}$. However, these expressions are valid at any ***point*** in the flow, even if the flow itself is not isentropic. So, we should be able to derive the above density ratio equation ***without invoking an isentropic relationship***. Do this. Namely, use only the first two equations above, along with the ideal gas equation, to derive the third equation above, showing all your work. *Note*: This will become important later on in the course – we can apply the above three equations at any point in the flow, even if the flow is not isentropic!

Note: There is another page. →

4. (50 pts) Consider steady, adiabatic, isentropic flow of air from a large tank through a converging nozzle. The temperature and pressure inside the large tank are 40.5ºC and 176.2 kPa, respectively. The exit area (at the outlet of the converging nozzle) is 0.0155 m^2. Give all final answers to 3 significant digits.

(**a**) Calculate the maximum back pressure P_b (air pressure outside of the exit of the converging nozzle) for which the flow in the nozzle is choked. Give your answer in kPa.

(**b**) Calculate the maximum possible mass flow rate through this nozzle in kg/s. What is the Mach number at the nozzle exit for this case?

(**c**) When the back pressure is atmospheric (P_b = 101.3 kPa), calculate the mass flow rate in kg/s. What is the Mach number at the nozzle exit for this case?

(**d**) Plot mass flow rate (kg/s) as a function of back pressure P_b, where P_b varies from 0 to P_0 (stagnation pressure in the tank).