

Due: In class, Friday September 16, 2022	Name(s) (Each student must submit; list anyone you worked with) PSU ID (abc123)
	Student submitting:
	Worked with:
	Worked with:

ME 420
Fall Semester, 2022
Homework Set # 3

Professor J. M. Cimbala

For instructor or TA use only:		
Problem	Score	Points
1		20
2		50
3		30
Total:		100

1. (20 pts) Air flows from a large tank at 600 kPa and 500 K into an axisymmetric converging nozzle. The exit area is 0.0079485 m².
- (a) Calculate the mass flow rate (in kg/s) for two cases: (1) subsonic flow with exit Mach number $M_e = 0.7$, and (2) choked flow.
- (b) You may have noticed that I have not included simplified equations for mass flow rate for air on the Equation Sheet. Why? Because these equations become “Engineering Equations,” which work only when you enter the values in specified units. I am not a big fan of Engineering Equations, but let’s create one for the choked mass flow rate of air. Specifically, generate an equation of the form below and calculate the appropriate constant.

$$\dot{m}_{\max} = \text{constant} \cdot P_0 A^* \sqrt{\frac{1}{T_0}}$$

Note: In this Engineering Equation, P_0 must be in kPa, A^* must be in m², and T_0 must be in K. It applies only to air. Calculate the constant to six significant digits.

- (c) What are the units of the constant in the above equation?
- (d) Verify your equation by plugging in the tank properties and exit area provided in the problem statement, and comparing with your choked result of Part (a).
2. (50 pts) *Note: This is a very important homework problem! We will encounter several implicit equations throughout this course, so it is critical that you know how to solve them confidently.* Consider steady, isentropic, one-D, adiabatic flow of an ideal gas through a duct of variable area. Choose the software that you want to use for calculations that involve solving the implicit equation for Mach number as a function of A/A^* as discussed in class. [I use EES and Excel, but you are welcome to use other software.] *Note: You will need to copy and paste this procedure into future homework problems, so make sure you do this part correctly.* For Parts (a) to (c), use the following values as your test case: $\gamma = 1.4$, $A = 0.0150 \text{ m}^2$, and $A^* = 0.00445 \text{ m}^2$. In all cases, use the implicit equation that was given in class,

$$\frac{A}{A^*} = \frac{1}{M} \left[\left(\frac{2}{\gamma + 1} \right) \left(1 + \frac{\gamma - 1}{2} M^2 \right) \right]^{\frac{\gamma + 1}{2(\gamma - 1)}}$$

and give your final answers to five (subsonic) and six (supersonic) significant digits.

- (a) **False Position Method**, also known as **“Smart” Trial and Error**: Set up an iteration scheme so that for given values of γ , A , and A^* , your software will calculate both the subsonic and supersonic roots of Mach number. For the test case values provided, calculate both the subsonic M and the supersonic M . Print the appropriate portion of your spreadsheet or program showing the results.
- (b) **Newton’s Method**: Repeat using Newton’s method. *Note:* You will need to carefully calculate the derivative using the product rule. Show all your algebra for full credit. If your scheme does not converge within 5 to 10 iterations, you probably have an incorrect derivative.
- (c) **Computer program**: Repeat using Matlab, EES, MathCad, Mathematica, or any other computer program you wish – trying to get the code to solve it for you without you having to iterate manually.
- (d) **Verification**: Verify your results for Parts (a) through (c) with the **Compressible Aerodynamics Calculator**, which is located at <http://www.dept.aoe.vt.edu/~devenpor/aoe3114/calc.html>. [It is also provided as a link on the course website.] All four of these methods should yield the same answers.

Note: There is another page. →

3. (30 pts) Consider a very large pressurized tank of air at $P_0 = 165$ kPa and $T_0 = 493.5$ K (these are stagnation values inside the tank where the air is still).
- (a) Calculate the critical temperature (in K) and critical pressure (in kPa) for this case.
 - (b) Repeat for *helium* at the same stagnation pressure and temperature as the air of Part (a).
 - (c) For both cases, there is a converging nozzle that lets the gas flow out of the tank. The back pressure outside of the converging nozzle is 99.65 kPa. Is the flow of air choked? Is the flow of helium choked?
 - (d) The area at the nozzle exit is 0.0130 m². For *both* gases, calculate the initial mass flow rate through the nozzle in kg/s.

