| Due: <br> In class, Friday September 23, 2022 | Name(s) (Each student must submit; list anyone you worked with) PSU ID (abc123) <br> Student submitting: <br> Worked with: <br> Worked with: |  |  |  |
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| ME 420 <br> Fall Semester, 2022 <br> Homework Set \# 4 |  | For instructor or TA use only: |  |  |
|  |  | Problem | Score | Points |
|  |  | 1 |  | 20 |
|  |  | 2 |  | 20 |
|  |  | 3 |  | 60 |
|  | Professor J. M. Cimbala | Total: |  | 100 |

1. (20 pts) Generate a plot of Mach angle $\mu$ as a function of Mach number $M$ in the range $1<M<10$. Discuss how Mach angle varies with Mach number and predict its asymptotic value as $M \rightarrow \infty$.
2. (20 pts) For steady, incompressible, adiabatic flow with negligible irreversibilities and no gravitational effects, we have everybody's favorite fluids equation, the "Beloved Bernoulli equation," $\frac{P_{1}}{\rho_{1}}+\frac{V_{1}^{2}}{2}=\frac{P_{2}}{\rho_{2}}+\frac{V_{2}^{2}}{2}$, where 1 and 2 are any two points along a streamline. However, this equation is not valid when compressible effects are significant. Consider steady, compressible, adiabatic flow of an ideal gas along a streamline from 1 to 2. Begin with $h_{0_{1}}=h_{0_{2}}$ and generate a similar "compressible Bernoulli equation," $\frac{P_{1}}{\rho_{1}}+C \frac{V_{1}^{2}}{2}=\frac{P_{2}}{\rho_{2}}+C \frac{V_{2}^{2}}{2}$, where $C$ is a constant. Showing all your work, generate an expression for constant $C$, and write the final form of our "compressible Bernoulli equation." Hint: Apply the ideal gas law along with other relations we wrote for $h$ and $c_{p}$, etc., for an ideal gas. Write your final answer in the form of constant $C$ as a function only of $\gamma$, the ratio of specific heats.
3. (60 pts) Air flows from a large tank at 500 kPa and 400 K into an axisymmetric converging nozzle. The coordinates for radius $r$ as a function of $x$ for the nozzle are provided in an Excel file on the course website. Give all answers to three significant digits. For Parts (a) and (b), show all your work, including equations and all numbers, units, etc. Parts (c) through (e) can be done in Excel or Matlab or EES or other software of your choice, and all you need to turn in are your final plots and a printout of relevant portions of your spreadsheet or code.
(a) Calculate the pressure (in kPa ) at the nozzle exit for the case in which the flow is just barely choked at the exit.
(b) Consider the case in which the back pressure is not low enough to choke the flow. Namely, let $P_{e}=P_{b}=450 \mathrm{kPa}$. Calculate the Mach number $M_{e}$ at the exit plane for this condition. Calculate the critical area $A^{*}$ for this condition. Note: Be careful here. Since the flow is not choked, $A^{*}$ is not equal to the exit area of this nozzle.
(c) For the case in which $P_{b}=450 \mathrm{kPa}$ [same as Part (b)], carefully plot Mach number vs. axial distance $x$ (in meters) from the entrance to the exit of the converging nozzle. On a separate plot, plot pressure (in kPa) vs. axial distance $x$. Verify that your results for $M_{e}$ and $P_{e}$ at the exit plane agree with those of Part (b). Do not print out these plots since you will be adding to them.
(d) Repeat Part (c) for the following back pressures: $P_{b}=400,350,300$, and 250 kPa . Plot your results on the same plots as those of Part (c), with a clear legend to label the curves. Don't print out these plots yet since you will be adding to them.
(e) Finally, repeat for the back pressure calculated in Part (a) in which the flow is just barely choked at the exit plane. Again, plot your results on the same plots. In the end, you should print out a total of two plots: $M$ vs. $x$ and $P$ vs. $x$, each plot having six curves for the six different values of back pressure.

