Due: In class, Friday October 14, 2022	**Name(s)** (*Each* student must submit; list anyone you worked with)	**PSU ID (abc123)**
	Student submitting:	
	Worked with:	
	Worked with:	

ME 420
Fall Semester, 2022
Homework Set # 6

Professor J. M. Cimbala

For instructor or TA use only:		
Problem	**Score**	**Points**
1		15
2		35
3		50
Total:		**100**

1. (15 pts) In class we discussed the “more elegant” method to generate an expression for the Mach number downstream of a stationary normal shock in an ideal gas. We had these two equations:

$$M_2^* = \frac{1}{M_1^*} \tag{1}$$

$$M^{*2} = \frac{M^2(\gamma+1)}{2+(\gamma-1)M^2} \tag{2}$$

Using *only* these two equations, prove the equation that all the textbooks give for Mach number downstream of a stationary normal shock in an ideal gas, namely,

$$M_2^2 = \frac{1+\frac{\gamma-1}{2}M_1^2}{\gamma M_1^2 - \frac{\gamma-1}{2}}$$

2. (35 pts) Consider again the Mollier diagrams (h vs. s) that you plotted in Homework Set 5, Problem 3. In that problem you plotted curves for both Fanno and Rayleigh and showed that they intersect at two points representing the normal shock. Now add a ***third*** curve to your plot based on the **Hugoniot equation** (consider only an ideal gas; use the ideal gas Hugoniot equation on the equation sheet). Using air as the gas at the same initial state as in the previous homework problem, plot ***all three curves on the same plot***. *Hint*: Start by creating a table of density ratios. If you do everything correctly, your Hugoniot curve should intersect at the same two points on the Mollier diagram as where the Fanno and Rayleigh curves intersect. How cool is that? Explain your procedure.

3. (50 pts) Consider the same geometry, tank pressure, tank temperature, etc. as Problem 2 of HW 5. The pressure at the nozzle exit is adjusted until there is a normal shock sitting at x = 0.900 m. Give all answers to four significant digits.

(**a**) Calculate Mach number M_1 before the normal shock and Mach number M_2 after the shock. Also calculate the following ratios across the shock: P_2/P_1, $P_{0,2}/P_{0,1}$, and T_2/T_1.

(**b**) Calculate the pressure (in kPa) and Mach number at the nozzle exit.

(**c**) Carefully plot Mach number and pressure as functions of axial distance x from the entrance to the exit of the converging-diverging nozzle, including through the normal shock.