Due: In class, Friday October 28, 2022	**Name(s)** (*Each* student must submit; list anyone you worked with) **PSU ID (abc123)** Student submitting: Worked with: Worked with:

ME 420

Fall Semester, 2022

Homework Set # 8

Professor J. M. Cimbala

For instructor or TA use only:		
Problem	**Score**	**Points**
1		xx
2		75
Total:		**100**

1. (25 pts) Consider a normal shock in the moving shock frame of reference (FOR) – the FOR in which the gas in front of the shock is stagnant and the shock passes by at speed V_s. We mentioned in class that the Hugoniot equation across a shock holds regardless of the FOR. We also mentioned that sometimes people who study moving shocks prefer to calculate properties based on the pressure ratio P_2/P_1 rather than on shock Mach number M_s or shock speed V_s. In Parts (a) and (b) below, begin with the Hugoniot equation for an ideal gas (see HW 6, Problem 1).

(**a**) For an ideal gas, show that the density ratio across the shock is $\dfrac{\rho_{as}}{\rho_1} = \dfrac{\rho_2}{\rho_1} = \dfrac{1+\dfrac{\gamma+1}{\gamma-1}\dfrac{P_2}{P_1}}{\dfrac{\gamma+1}{\gamma-1}+\dfrac{P_2}{P_1}}$.

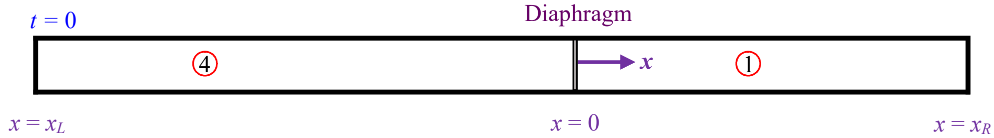
(**b**) For an ideal gas, show that the temperature ratio across the shock is $\dfrac{T_{as}}{T_1} = \dfrac{T_2}{T_1} = \dfrac{P_2}{P_1}\dfrac{\dfrac{\gamma+1}{\gamma-1}+\dfrac{P_2}{P_1}}{1+\dfrac{\gamma+1}{\gamma-1}\dfrac{P_2}{P_1}}$.

(**c**) For an ideal gas, generate an expression for V_{as} (the speed of the gas after the shock, following the shock) as a function of P_2/P_1, γ, and shock speed V_s only.

(**d**) Consider the example we discussed in class of a moving normal shock from a blast wave. In that example, the shock speed was 880 m/s into STP air. Pretend that you do not know the shock speed, but only the pressure ratio $P_2/P_1 = 7.375$ and that the air in front of the moving shock is at STP. Use your equations from Parts (a) through (c) above to calculate ρ_2/ρ_1, T_2/T_1, and V_{as} across this moving shock. If all your equations and calculations are correct, you should get the same values that we got in class (which were based on V_s rather than on P_2/P_1). Finally, use the equation given in class for V_s, namely $V_s = a_1\sqrt{1+\dfrac{\gamma+1}{2\gamma}\left(\dfrac{P_2}{P_1}-1\right)}$ to calculate the shock speed for this given pressure ratio. Again, it should match the value in the class example, and serves as a validation of our equations and calculations.

Note: There is another page. →

2. (75 pts) *Note*: You are strongly encouraged to set up the problem in Excel, EES, Matlab, or other software of your choice. First plug in all the values from the class example and make sure that your answers agree with those given in class for that problem. Once you are confident in your analysis, re-run for the values given here. A shock tube is set up with the following properties and two different gases:



- Right (low pressure) side: Argon: P_1 = 101.3 kPa, T_1 = 320.15 K; look up γ_1, R_1, etc. as needed.
- Left (high pressure) side: Air: P_4 = 1250 kPa, T_4 = 500 K, γ_4 = 1.40, R_4 = 287. J/(kg K).
- x_L = -12 m, x_R = 18 m.

At t = 0 the diaphragm ruptures and instantaneously "disappears."

(**a**) Predict the following properties and velocities on the left and right sides of the shock tube after rupture: P_2, P_3, T_2, T_3, M_s, V_s, V_{CS}.

(**b**) Calculate the speed of the leading and trailing waves of the expansion fan. The leading wave always moves to the left, but the trailing wave can move to the left or right, depending on the initial properties of the two gases. Which way (right or left) does the trailing wave travel for this case? Accurately plot (with software) the *x*-*t* diagram (expansion fan, contact surface, and shock wave) from t = 0 up to time t_R when the shock hits the right wall ($x = x_R$).

(**c**) Calculate the Mach number M_{sR} of the shock wave and its speed V_{sR} after it reflects off the right wall. Also calculate temperature T_5 and pressure P_5 of the stagnant gas after the reflected shock has passed by. Use the same numbering convention we used in class.

(**d**) Keeping everything else fixed, vary P_4 to see what happens. Discuss briefly: Does the shock strength increase or decrease as P_4 increases? Calculate the value of initial pressure P_4 that causes the trailing wave of the expansion fan to sit at x = 0 (in other words, it moves neither to the left or to the right). Give the results (P_2, P_3, T_2, …) as in Part (b) and compare to Part (b). Accurately plot (with software) the *x*-*t* diagram for this case.