Due: In class, Friday **November 11, 2022**	**Name(s)** (*Each* student must submit; list anyone you worked with) **PSU ID (abc123)** Student submitting: Worked with: Worked with:

ME 420
Fall Semester, 2022
Homework Set # 9

Professor J. M. Cimbala

For instructor or TA use only:		
Problem	**Score**	**Points**
1		50
2		25
3		25
Total:		**100**

1. (50 pts) *Note*: You are strongly encouraged to set up the problem in software of your choice. First plug in all the values from the class example and make sure that your answers agree with those given in class for that problem. Once you are confident in your analysis, re-run for the values given here. Air and fuel enter a 13.5-cm diameter combustion chamber of a gas turbine engine at 505 K, 360 kPa, and 110 m/s. The fuel is burned between locations 1 and 2 in the tube, as sketched, and in the process, 5406 kW of heat is added. For simplicity, use the properties of air, ignoring the effects of the fuel on the gas properties. Use the same approximations and assumptions as we used in our Rayleigh flow class example.

(a) Calculate the Mach number at location 1 (the inlet). Estimate the Mach number, temperature, pressure, and velocity at location 2 (the outlet).

(b) Calculate the maximum possible rate of heat transfer $\dot{Q}_{\max}$ into this tube for the given inlet conditions. Discuss what would happen if we add more heat than this maximum value.

(c) In your computer code, play around with various values of the rate of heat transfer to see what happens. Specifically, try $\dot{Q} = 0$ and $\dot{Q} = \dot{Q}_{\max}$. If you have everything set up correctly, M_2, T_2, etc. should be the *same* as M_1, T_1, etc. when there is no heat transfer. When $\dot{Q} = \dot{Q}_{\max}$ you should find that M_2 approaches 1. *Note*: Depending on your software, you may have trouble getting exactly to the maximum. In that case, let $\dot{Q}$ be slightly less than $\dot{Q}_{\max}$ so that your solution converges. Discuss your results briefly. Do the results agree with what you expected?

(d) Once you are confident that your code is running correctly, run a *range* of $\dot{Q}$ from 0 to $\dot{Q}_{\max}$. Plot your results on a *T-s* diagram. Note that inlet specific entropy is arbitrary since all we can calculate is a *change* of specific entropy. Therefore, for consistency so that everybody generates the same plot, let's arbitrarily "pick" s_1 = 2850 J/(kg K). ***On the same plot***, plot the theoretical Rayleigh line for the given inlet conditions for this problem. [See HW 5 – we already did this, but for different inlet conditions.] If you do everything correctly, your two curves should fall on top of each other! If they do, you should be very ☺ .

2. (25 pts) *Note*: This is a simple Fanno flow problem, like the kind you might see on the final exam. I recommend that you use only the equation sheet for practice for the final exam. Air (γ = 1.40) enters a 6.5-cm diameter, 50-m long round pipe at 240 K and 360 kPa, with inlet Mach number = 0.13. The Darcy friction factor is 0.0216 for this flow.

(a) Calculate $(fL^*/D_h)_2$ at the outlet of the pipe. *Note*: Once you have a numerical value for this quantity, you could solve implicitly for M_2, but you are not asked to do that here since it involves iteration.

(b) Calculate the maximum possible length of this pipe such that the inlet conditions remain as given.

Note: There is another page. →

3. (25 pts) Consider Fanno flow of an ideal gas. The following expression was given in class,

$$\boxed{f\frac{(x_2 - x_1)}{D_h} = \left[-\frac{1}{\gamma M^2} - \frac{\gamma+1}{2\gamma}\ln\left(\frac{M^2}{1+\frac{(\gamma-1)}{2}M^2} \right) \right]_{M_1}^{M_2}}$$

This equation is valid for both the subsonic and the supersonic branches of Fanno flow.

(a) Set $M_1 = M$ to represent any general state or location in Fanno flow. Set $x_2 - x_1 = L^*$ and $M_2 = 1$ to represent the ***choked*** flow state or location in Fanno flow. Plug these into the above equation and show that

$$\boxed{\frac{fL^*}{D_h} = \frac{1-M^2}{\gamma M^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M^2}{2+(\gamma-1)M^2} \right)}$$

(b) Plot fL^*/D_h as a function of M for M ranging from 0.2 to 6.

(c) What is the maximum value of fL^*/D_h as M approaches infinity? You can get it from the plot, or better yet, use L'Hopital's rule to calculate it exactly.

(d) Discuss why conveyance of a gas through a long pipe is done with subsonic flow, and why nobody with knowledge of compressible flow would attempt to convey a gas through a very long pipe supersonically.