| Due: <br> In class, Friday December 2, 2022 | Name(s) (Each student must submit; list anyone you worked with) PSU ID (abc123) <br> Student submitting: <br> Worked with: <br> Worked with: |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| ME 420 <br> Fall Semester, 2022 <br> Homework Set \# 10 |  | For instructor or TA use only: |  |  |
|  |  | Problem | Score | Points |
|  |  | 1 |  | 15 |
|  |  | 2 |  | 50 |
|  |  | 3 |  | 35 |
|  | Professor J. M. Cimbala | Total: |  | 100 |

1. ( 15 pts ) Air enters a $6.5-\mathrm{cm}$ diameter, $64.5-\mathrm{m}$ long round pipe at 240 K and 360 kPa , with inlet Mach number $=0.15$. The Darcy friction factor is 0.0266 for this flow.
(a) Calculate $\left(f L^{*} / D_{h}\right)_{2}$ at the outlet of the pipe. Once you have a numerical value for this quantity, solve implicitly for $M_{2}$.
(b) Calculate the maximum possible length of this pipe such that the inlet conditions remain as given. Calculate $M$ at the duct outlet for this case (maximum length).
2. (50 pts) Note: You are strongly encouraged to set up this problem in Excel, EES, Matlab, or other software of your choice. First plug in all the values from the class example and make sure that your answers agree with those given in class for that problem. Once you are confident in your analysis, re-run for the values given here. Air enters a $7.5-\mathrm{cm}$ diameter, 154.1-m long nearly smooth
 round pipe at $350 \mathrm{~K}, 240 \mathrm{kPa}$, and $55.5 \mathrm{~m} / \mathrm{s}$. The average roughness height of the inside wall of the pipe is estimated to be only 0.0008 mm . The pipe is well-insulated, but we need to be concerned about friction in the pipe since it is so long. Use the same approximations and assumptions as we used in our class example.
(a) Calculate the Darcy friction factor at the pipe inlet using the Churchill equation. Use this $f$ for the next part of this problem, assuming it remains constant.
(b) Estimate the Mach number, temperature, pressure, and velocity at location 2 (the outlet). Also estimate the maximum possible extension of pipe length such that the inlet flow is not disrupted (call it $L_{\max }$ ). How much does the specific entropy change from the inlet to the outlet for the case of maximum pipe length? Give your answer in units of $\mathrm{J} /(\mathrm{kg} \mathrm{K})$.
(c) Calculate the Darcy friction factor at the end of the pipe. Is it necessary to refine your calculations with some kind of average $f$ ? Justify your answer.
(d) Once you are confident that your code is running correctly, run a range of pipe lengths from 0 m to $L_{\max }$ (or as close to $L_{\max }$ as you can get). Summarize your results qualitatively. What happens if you try to make the pipe longer than this? Explain.
(e) Plot $M_{2}$ as a function of pipe length $L$. Also plot your results on a $T$-s diagram. Note that inlet specific entropy is arbitrary since all we can calculate is a change of specific entropy. Therefore, for consistency so that everybody generates the same plot, let's arbitrarily "pick" $s_{1}=2500 \mathrm{~J} /(\mathrm{kg} \mathrm{K})$. On the same plot, plot the theoretical Fanno line for the given inlet conditions for this problem. [See a previous homework where we already did this, but for different inlet conditions.] If you do everything correctly, your two curves should fall on top of each other! If they do, you should be very () .
3. ( 35 pts ) Note: This will be useful in the next problem as well. Many times in this course we have encountered equations that are explicit for one variable but implicit for another. Most of the time, the implicit form occurs when solving for Mach number. The $\theta-\beta-M_{1}$ equation for oblique shocks is one such example. In a typical solution, we need to solve for $\beta$ implicitly for a given $M_{1}$ and a given $\theta$,

$$
\tan \theta=\frac{2 \cot \beta\left(M_{1}^{2} \sin ^{2} \beta-1\right)}{M_{1}^{2}[\gamma+\cos (2 \beta)]+2}
$$

(a) Using software of your choice (Excel, EES, Matlab, ...), generate an iteration scheme to solve for $\beta$ implicitly for a given $M_{1}$ and a given $\theta$. Report the software you used and your iteration scheme (Newton's method, trial and error, false position method, ...). Test your scheme using the example numbers shown in class.
(b) Apply your scheme for $M_{1}=2.38$ and $\theta=11.9^{\circ}$ (for air). Calculate $\beta$ for the weak oblique shock and $\beta$ for the strong oblique shock. Attach a printout of your computer program with your answers boxed or somehow highlighted. Also verify your results with the online Compressible Aerodynamics Calculator.


