Today, we will:

- Continue to discuss the ideal gas law: specific heats, ratio of specific heats
- Do an example problem – ideal gas
- Discuss entropy and the *Tds* equations
- Discuss isentropic relations for an ideal gas, and do an example problem

on M&p:

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MORE THERMO REVIEW:

• Specific Heats

$$\boxed{c_v = \left(\frac{\partial u}{\partial T}\right)_v}$$ = spec. heat at constant volume

specific internal energy

fix spec. volume

$$\boxed{c_p = \left(\frac{\partial h}{\partial T}\right)_p}$$ = spec. heat @ constant pressure

• IDEAL GAS

c_p = constant
c_v = "

$$\boxed{\begin{array}{l} u = c_v T \\ h = c_p T \end{array}}$$

$$\boxed{c_p - c_v = R}$$

• Ratio of specific heats = $\boxed{\gamma = \frac{c_p}{c_v}}$ $\{\gamma\} = \{1\}$

Some authors use k instead of γ

Thermo Fluids

• Eg [AIR]

$c_p - c_v = R$

$\frac{c_p}{c_v} - 1 = \frac{R}{c_v}$ $\Rightarrow$ $\boxed{c_v = \frac{R}{\gamma - 1}}$

γ

Similarly

$$\boxed{c_p = \frac{R\gamma}{\gamma - 1}}$$

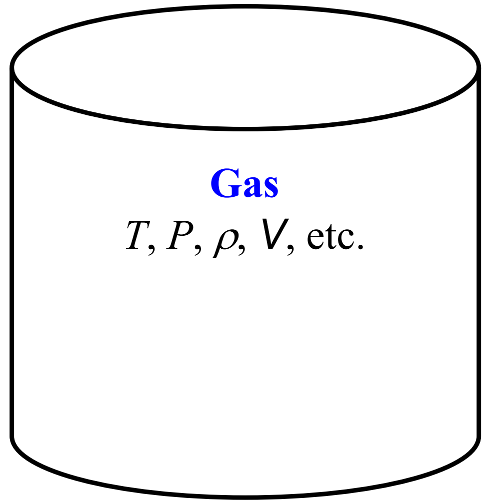
Air: $R = 0.2870 \ \frac{kJ}{kg\,K}$

$\therefore$ $c_p = 1.0045 \ \frac{kJ}{kg\cdot K}$ $c_v = 0.7175 \ \frac{kJ}{kg\,K}$

Example: Ideal gas properties and calculations

<u>**Given**</u>: A sample gas (not air) is in a pressurized container. The gas is assumed to behave as an ideal gas. The following properties are measured:

- P = 208.4 kPa
- T = 43.7°C $\longrightarrow = (43.7 + 273.15)\ \text{K}$
- ρ = 1.436 kg/m^3
- c_v = 1.238 kJ/(kg K)



(a) <u>**To do**</u>: Calculate R, c_P, and γ for this gas. Give R in units of J/(kg K).

<u>**Solution**</u>:

<u>**Assumptions and Approximations**</u>: "A & A"

1. The gas is an ideal gas.

To be completed in class.

$$\cdot\ P = \rho R T \rightarrow$$

unity conversion factor (=1)

$$R = \frac{P}{\rho T} = \frac{208.4\ \text{kPa}}{\left(1.436\ \frac{\text{kg}}{\text{m}^3}\right)\left(43.7 + 273.15\right)\text{K}}\left(\frac{\text{kN}}{\text{m}^2\cdot\text{kPa}}\right)\left(\frac{\text{J}}{\text{N}\cdot\text{m}}\right)\left(\frac{1000\ \text{N}}{\text{kN}}\right)$$

$$R = 458.0254\ \frac{\text{J}}{\text{kg}\cdot\text{K}} \qquad \boxed{R = 458.0\ \frac{\text{J}}{\text{kg}\cdot\text{K}}}$$

$$\cdot\ c_p - c_v = R \rightarrow c_p = R + c_v = 458.0254\ \frac{\text{J}}{\text{kg}\cdot\text{K}} + 1.238\ \frac{\cancel{\text{kJ}}}{\text{kg K}}\left(\frac{1000\ \text{J}}{\cancel{\text{kJ}}}\right)$$

$$c_p = 1696.025\ \frac{\text{J}}{\text{kg}\cdot\text{K}} \qquad \boxed{c_p = 1696.\ \frac{\text{J}}{\text{kg}\cdot\text{K}}}$$

$$\cdot\ \gamma = \frac{c_p}{c_v} = 1.36997 \qquad \boxed{\gamma = 1.370}$$

$$\cdot\ \text{Verify: } c_v = \frac{R}{\gamma - 1}\ ? \quad c_v = \frac{458.0254\ \frac{\text{J}}{\text{kg}\cdot\text{K}}}{1.36997 - 1} = 1238.0\ \frac{\text{J}}{\text{kg}\cdot\text{K}}\ \checkmark$$

(b) Calc M

Sol'n: $\boxed{R = \frac{R_u}{M}} \rightarrow M = \frac{R_u}{R} = \frac{8.314 \frac{\cancel{kJ}}{kmol \cdot \cancel{K}}}{458.0254 \frac{\cancel{J}}{kg\,\cancel{K}}} \left(\frac{1000 \cancel{J}}{\cancel{kJ}} \right) =$

$M = 18.1518\ kg/kmol$

$$\boxed{M = 18.15 \frac{kg}{kmol}}$$

ENTROPY : THE Tds EQUATIONS

Recall, from thermo, For any gas (not just ideal gas)

$$\boxed{\begin{aligned} T\,ds &= du + P\,dv \quad (1) \\ T\,ds &= dh - v\,dP \quad (2) \end{aligned}}$$

s = specific entropy

For an ideal gas

$\boxed{du = c_v\,dT}$ $\boxed{P = \frac{RT}{v}}$

(1) → $T\,ds = du + P\,dv$

$$ds = c_v \frac{dT}{T} + \frac{R\cancel{T}}{v} \frac{1}{\cancel{T}} dv$$

$$ds = c_v \frac{dT}{T} + R \frac{dv}{v}$$

Integrate from ① to ②

$$\boxed{s_2 - s_1 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1}} \quad (i)$$

ideal gas

Similarly, (2) →

$$\boxed{s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}} \quad (2i)$$

★ ISENTROPIC RELATIONS for an ideal gas

negligible irreversibilities

$$s_2 - s_1 = 0 \quad \rightarrow \quad 0 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}$$

start w/ (2i)

$$\boxed{\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}}$$

★ Isentropic flow of an ideal gas

Similarly start with (1i)

$$0 = c_v \ln \frac{T_2}{T_1} + R \ln \frac{\rho_1}{\rho_2}$$

$$\ln \frac{\rho_2}{\rho_1} = -\ln \frac{\rho_1}{\rho_2} = \frac{c_v}{R} \ln \frac{T_2}{T_1} = \ln \left[\left(\frac{T_2}{T_1}\right)^{\frac{c_v}{R}}\right]$$

$e^{(\,)}$ both sides

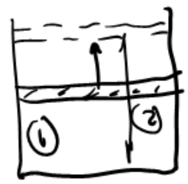
$$\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\frac{c_v}{R}} \qquad \frac{c_v}{R} = \frac{1}{\gamma - 1}$$

Isentropic flow of an ideal gas ⟹ $\therefore \boxed{\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}}$ ★

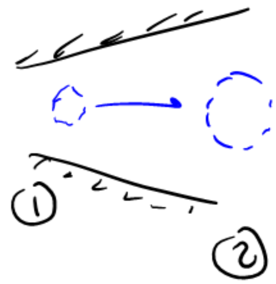
Example: Isentropic expansion

Given: Air is very carefully and slowly expanded isentropically from state 1 to state 2. The following are measured:

- $P_1 = 289.3$ kPa
- $T_1 = 69.7$ºC
- $T_2 = 33.2$ºC



or



To do: Calculate P_2.

in kPa

Solution:

Assumptions and Approximations:

1. The air is an ideal gas.
2. The process is isentropic.

To be completed in class.

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}} \Rightarrow P_2 = P_1\left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$$

Plug in #s →

$$P_2 = (289.3\text{ kPa})\left(\frac{(33.2 + 273.15)\cancel{K}}{(69.7 + 273.15)\cancel{K}}\right)^{\frac{1.40}{1.40-1}}$$

$$P_2 = 195.095 \text{ kPa}$$

$$\boxed{P_2 = 195.1 \text{ kPa}}$$