Today, we will:

- Continue to discuss isentropic relations for an ideal gas, and do some example problems
- Discuss speed of sound and Mach number and do some example problems
- Begin to discuss compressible adiabatic flow in ducts
- Do Candy Questions for Candy Friday

Recall, for an ideal gas, the *Tds* equations simplify to

$$\boxed{s_2 - s_1 = c_\upsilon \ln\frac{T_2}{T_1} + R\ln\frac{\upsilon_2}{\upsilon_1}} \text{ or } \boxed{s_2 - s_1 = c_\upsilon \ln\frac{T_2}{T_1} - R\ln\frac{\rho_2}{\rho_1}} \quad (1i)$$

and

$$\boxed{s_2 - s_1 = c_P \ln\frac{T_2}{T_1} - R\ln\frac{P_2}{P_1}} \quad (2i)$$

Isentropic relationships for an ideal gas:

Manipulation of Eq. (2i) yields $\boxed{\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}}$. (3i)

Similarly, manipulation of Eq. (1i) yields $\boxed{\frac{\rho_2}{\rho_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{\gamma-1}}}$. (4i)

[*Note*: These equations hold for **ideal gases** only. From now on, we will deal only with ideal gases, and I will drop the "i" from equation numbers.]

Example: Isentropic expansion

Given: Air flows nearly isentropically from location 1 to location 2. The following are measured:

- P_1 = 289.3 kPa
- T_1 = 69.7°C
- T_2 = 33.2°C

To do: Calculate P_2.

Solution:

Assumptions and Approximations:

1. The air is an ideal gas.
2. The flow is approximated as isentropic.

To be completed in class.

(Sorry, we did this in previous lecture)

SPEED OF SOUND & MACH

Notation: Anderson uses a = speed of sound, M = Mach number

CC c " M_a "

Def'n of a:

$$\boxed{a = \sqrt{\left(\frac{\partial P}{\partial \rho}\right)_s}}$$

= a measure of incompressibility of a fluid

gases → • Small a → small ΔP can lead to a large $\Delta \rho$ → highly compressible

liquids → • Large a → large . . . . small . . → highly incompressible

approx. $a \approx \infty$ for a liquid → incompressible

ideal gas

$$\boxed{a = \sqrt{\gamma R T}}$$

always use absolute T

e.g., air @ $T = 25°C$

$$a = \sqrt{(1.40)\left(0.2870 \frac{kJ}{kg\,K}\right)(25 + 273.15)K \left(\frac{kN \cdot m}{kJ}\right)\left(\frac{1000\ kg \cdot m}{s^2 \cdot kN}\right)}$$

$$\boxed{a = 346.\ \frac{m}{s}}$$

Can use instead $\boxed{R_{air} = 287.0\ \frac{J}{kg\,K}}$

or $\boxed{R_{air} = 287.0\ \frac{m^2}{s^2 K}}$

$$a = \sqrt{\gamma R T} = \sqrt{(1.40)\left(287.0 \frac{m^2}{s^2 \cancel{K}}\right)(298.15\ \cancel{K})} = \underline{346.\ \frac{m}{s}}\ ✓$$

MACH NUMBER

$$\boxed{M = \frac{V}{a}} \qquad \{M\} = \{1\} \qquad [M] = [1]$$

M is the most important parameter in compressible flow

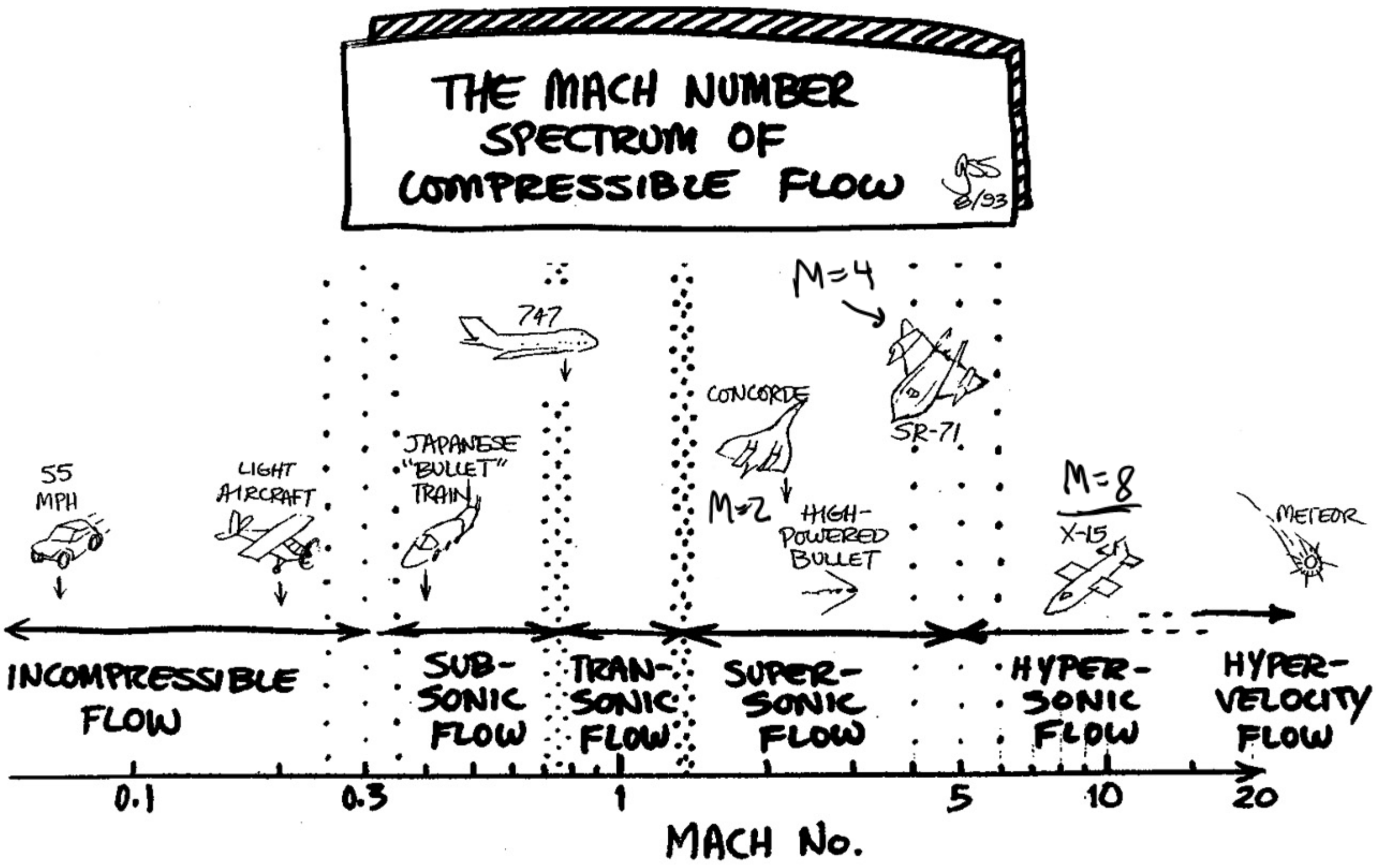
Anderson also uses M for molecular weight & mass

I use " " but m = mass

Flow regimes

- $M \lesssim 0.3$ → Nearly incompressible
- $M < 1$ → subsonic
- $M \approx 1$ → transonic
- $M = 1$ → sonic
- $M > 1$ → supersonic
- $M \gtrsim 5$ → hypersonic

Mach number regimes (sketch by Gary S. Settles):



Example: Speed of sound and significant digits

Given: An aircraft flies at speed $V = 423.1$ m/s through air at $T = 300.3$ K.

To do: Calculate the aircraft's Mach number to the correct number of significant digits.

Solution: Speed of sound $a = \sqrt{\gamma RT}$. Mach number $M = V/a$. The ratio of specific heats of air is $\gamma = 1.40$. The specific gas constant for air is $R_{air} = 287.0\ \mathrm{m^2/(s^2K)}$.

To be completed in class.

$$M = \frac{V}{a} = \boxed{\frac{V}{\sqrt{\gamma RT}}} = \frac{423.1\ \mathrm{m/s}}{\sqrt{(1.40)\left(287.0\ \frac{\mathrm{m^2}}{\mathrm{s^2K}}\right)(303.2\ \mathrm{K})}} = 1.21864$$

ans. in variable form

Final ans. $\rightarrow \boxed{M = 1.22}$ ✩

Example: Mach number and the incompressible approximation

Given: An aircraft flies at a high altitude where the pressure is P = 30.09 kPa and the density is ρ = 0.458 kg/m^3.

To do: Calculate how fast (in units of m/s to 3 significant digits) the aircraft can fly to stay within the "incompressible" approximation, commonly taken as the speed at which the Mach number is less than 0.3.

Solution: Speed of sound $a = \sqrt{\gamma RT}$. Mach number $M = V/a$. The ratio of specific heats of air is γ = 1.40. The specific gas constant for air is R_{air} = 287.0 m^2/(s^{2}K).

To be completed in class.

need T to get a $\rightarrow$ $P = \rho RT$ $\rightarrow$ $T = \frac{P}{\rho R}$

$a = \sqrt{\gamma RT}$ $\quad M = \frac{V}{a}$ $\rightarrow$ $V = M \cdot a$

$$V = M\sqrt{\gamma \cancel{R} \frac{P}{\rho \cancel{R}}} = \boxed{M\sqrt{\gamma \frac{P}{\rho}}}$$ ans. in var.

$$V = (0.3)\sqrt{(1.40)\frac{30{,}090 \text{ N/m}^2}{0.458 \text{ kg/m}^3}\left(\frac{\text{kg}\cdot\text{m}}{\text{s}^2\cdot\text{N}}\right)} = 90.984 \text{ m/s}$$

$$\boxed{V = 91.0 \text{ m/s}}$$

COMP. ADIABATIC FLOW IN A DUCT

review
- 1st law of therm
- RTT Reynolds Transport Theorem

RTT: (Fixed CV)

$$\boxed{\frac{dB_{sys}}{dt} = \frac{d}{dt}\int_{CV} b\rho\, d\forall + \int_{CS} b\rho(\vec{V}\cdot\vec{n})\,dA}$$

$\left[b = \frac{B}{m}\right]$

For cons. of energy   let $B = E$ = total energy

$b = \frac{E}{m} = e$ = specific total energy

$$\boxed{\frac{dE_{sys}}{dt} = \frac{d}{dt}\int_{CV} e\rho \, d\forall + \int_{CS} e\rho\left(\vec{V}\cdot\vec{n}\right)dA} \quad (1)$$

1st law of thermo is written for a system

$$\boxed{\dot{Q}_{net\ in} + \dot{W}_{net\ in} = \frac{dE_{sys}}{dt}} \quad (2)$$

(heat transf in)   (power in)

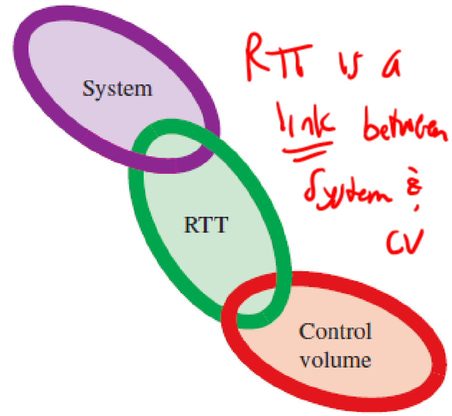
The First Law of Thermodynamics in terms of the Reynolds Transport Theorem (RTT):

$$\boxed{\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \frac{dE_{\text{system}}}{dt}} = \frac{d}{dt}\int_{CV} e\rho dV + \int_{CS} e\rho\left(\vec{V}\cdot\vec{n}\right)dA$$

1st law for a system

CV. form of the 1st law

(cons. of energy for a CV)



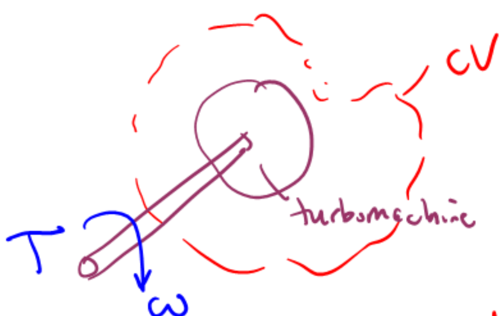
RTT is a link between system & CV

FIGURE 4–55
The *Reynolds transport theorem* (RTT) provides a link between the system approach and the control volume approach.

typically zero

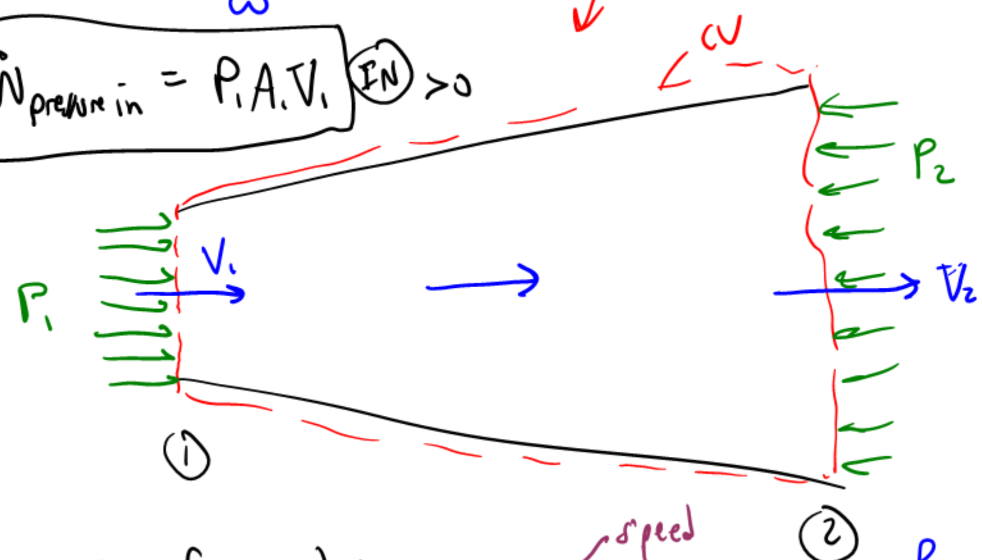
$$\dot{W}_{\text{net in}} = \cancel{\dot{W}_{\text{shaft net in}}} + \dot{W}_{\text{pressure net in}}$$

Power added to the CV (or system) by pressure work



$$\boxed{\dot{W}_{\text{pressure in}} = P_1A_1V_1} \text{ IN } > 0$$

$$\boxed{\dot{W}_{\text{pressure in}} = -P_2A_2V_2}$$



Work = force × distance

$$\text{power} = \frac{\text{work}}{\text{time}} = \text{force} \times \frac{\text{distance}}{\text{time}} = \frac{\text{force}}{\text{area}} \cdot \text{area} \cdot \text{speed}$$

speed

Pressure