Today, we will:

- Continue to derive equations for compressible adiabatic flow in ducts
- Simplify to steady, adiabatic 1-D duct flow
- Do an example problem – steady, adiabatic 1-D duct flow
- Begin to discuss ***isentropic***, adiabatic duct flow & derive equations for ducts of variable area

Last lecture, we wrote the First Law of Thermodynamics in terms of the Reynolds Transport Theorem (RTT):

$$\boxed{\dot{Q}_{\text{net in}} + \dot{W}_{\text{net in}} = \frac{dE_{\text{system}}}{dt} = \frac{d}{dt}\int_{CV} e\rho dV + \int_{CS} e\rho\left(\vec{V}\cdot\vec{n}\right)dA} \quad (3)$$

∴ we analyzed the pressure work term

* NOW APPLY TO FLOW IN A DIVERGING DUCT

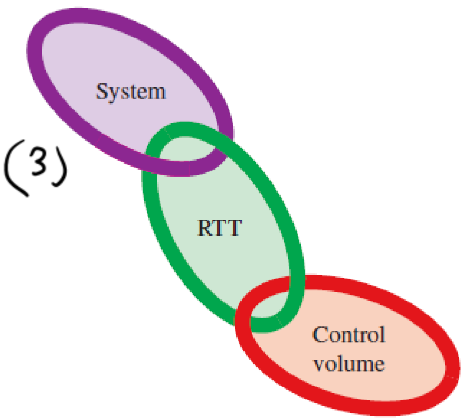
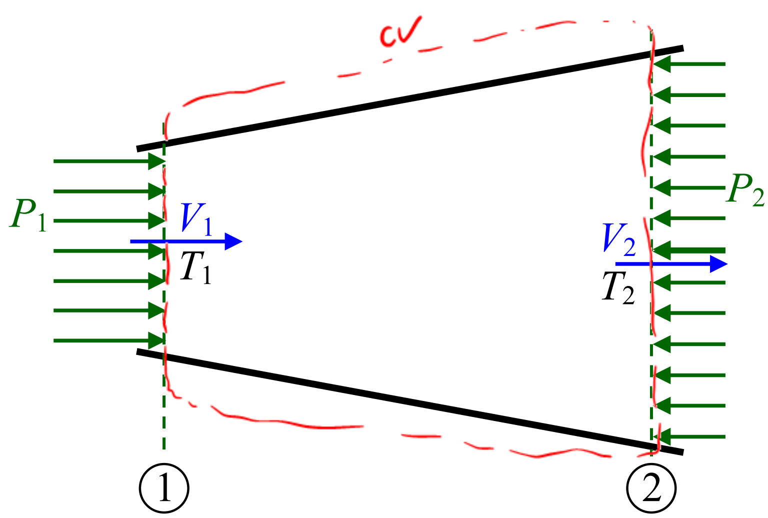


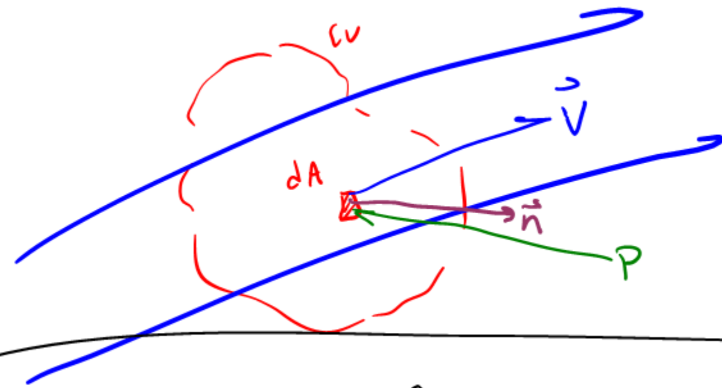
FIGURE 4–55
The *Reynolds transport theorem* (RTT) provides a link between the system approach and the control volume approach.

For flow through a duct as sketched,



$$\dot{W}_{\text{pressure, net in}} = P_1A_1V_1 - P_2A_2V_2$$

More general CV



$$\boxed{\dot W_{pressure,\ net\ in} = -\int_{CS} p\left(\vec V\cdot\vec n\right)dA = -\int_{CS}\frac{p}{\rho}\rho\left(\vec V\cdot\vec n\right)dA} \quad (4)$$

Plug (4) into (3)

<u>A&A</u> <u>ADIABATIC</u> → $\dot Q = 0$, <u>STEADY</u>

$$-\int_{CS}\frac{p}{\rho}\rho(\vec V\cdot\vec n)\,dA = \int_{CS} e\rho(\vec V\cdot\vec n)\,dA$$

or

$$\boxed{\int_{CS}\left(e+\frac{p}{\rho}\right)\rho(\vec V\cdot\vec n)\,dA = 0}$$

recall $\boxed{e = u + \frac{V^2}{2}}$ (ignore p.e.)

$$\int_{CS}\left(u+\frac{p}{\rho}+\frac{V^2}{2}\right)\rho(\vec V\cdot\vec n)\,dA = 0$$

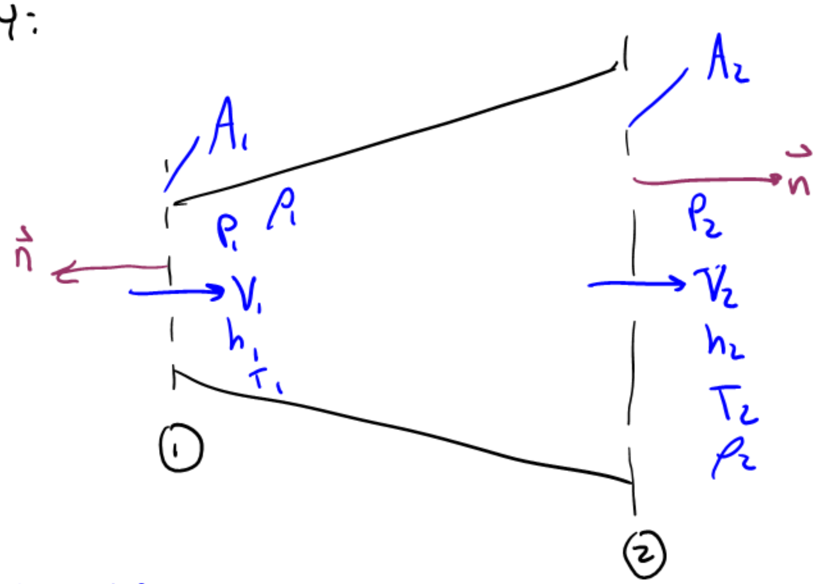
recall $\boxed{h = u + \frac{p}{\rho}}$

$$\boxed{\int_{CS}\left(h+\frac{V^2}{2}\right)\rho(\vec V\cdot\vec n)\,dA = 0}$$

★ For steady, adiabatic w/o shaft work & neglecting gravity effects (5)

[even for non-ideal gas]

APPLY:



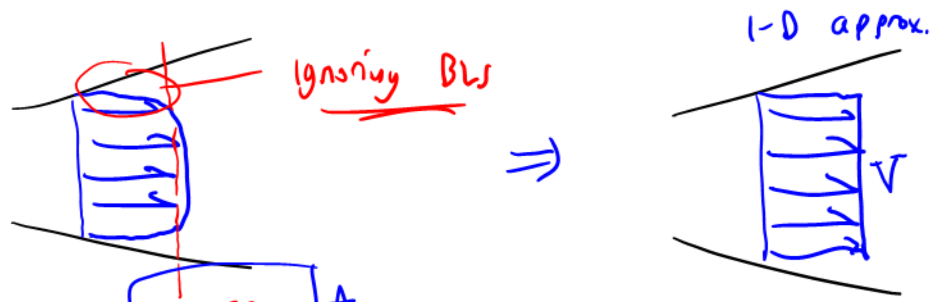
Further simplification: Only 1 inlet & 1 outlet

$$(5) \Rightarrow \quad -\underbrace{\left(h_1+\frac{V_1^2}{2}\right)\underbrace{\rho_1 V_1 A_1}_{\dot m_1 = \dot m}}_{\text{INLET}} + \underbrace{\left(h_2+\frac{V_2^2}{2}\right)\underbrace{\rho_2 V_2 A_2}_{\dot m_2 = \dot m}}_{\text{OUTLET}} = 0$$

$$\boxed{\dot m_1 = \dot m_2 = \dot m}$$ (cons of mass)

$$\boxed{h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}}$$

Simplified eq for steady adiabatic 1-D duct flow



$$\boxed{V = V_{avg}}$$

Define h_0 = stagnation specific enthalpy $\boxed{h_0 = h + \frac{V^2}{2}}$

h_o = specific enthalpy of a fluid brought to rest adiabatically

$\therefore$ $\boxed{h_{o_1} = h_{o_2}}$ ☆

Steady, adiabatic one-D duct flow

1 = inlet 2 = outlet of our CV

This eq. applies even if the flow is non-isentropic

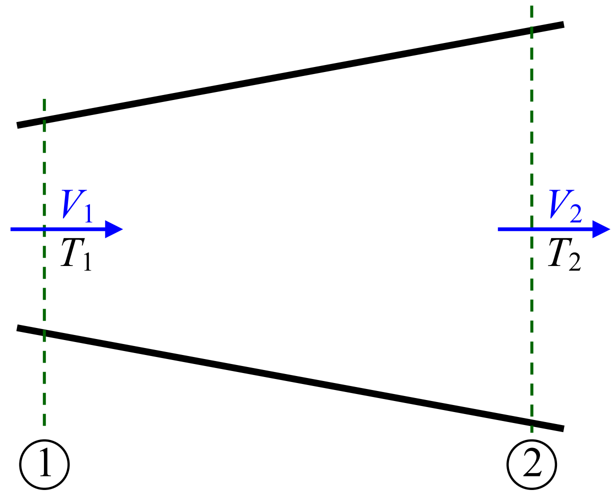
can have irreversibilities

eg, friction, shock waves

Example: Steady adiabatic duct flow

Given: Air flows steadily in an expanding duct. The duct is insulated. We measure:

- $V_1 = 453.1$ m/s
- $T_1 = 389.2$ K
- $T_2 = 279.4$ K



To do:

(**a**) Calculate V_2.

(**b**) Calculate M_1 and M_2. Is this flow subsonic or supersonic?

Solution:

To be completed in class.

Assumptions and Approximations:

1) steady & adiabatic
2) Quasi 1-D flow $\quad V_1$ = avg. speed @ 1
$\quad V_2$ = -- -- -- 2
3) ideal gas

$$\boxed{h_{0_1} = h_{0_2}} \Rightarrow h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

ideal gas $\rightarrow \boxed{h = c_p T}$

$$c_p T_1 + \frac{V_1^2}{2} = c_p T_2 + \frac{V_2^2}{2}$$

$$\boxed{\therefore \quad V_2 = \sqrt{V_1^2 + 2c_p(T_1 - T_2)}}$$

$\star$ ans in variables

#'s

$$V_2 = \sqrt{\left(453.1 \frac{m}{s}\right)^2 + 2\left(1004.5 \frac{m^2}{s^2 \cdot \cancel{K}}\right)(389.2 - 279.4)\cancel{K}} = \underline{652.601} \frac{m}{s}$$

$$\rightarrow \boxed{V_2 = 652.6 \ m/s}$$

Since $V_2 > V_1$, $\rho_2 < \rho_1$ to conserve mass

$$\dot{m} = \underset{\downarrow}{\rho}\underset{\uparrow}{V}\underset{\uparrow}{A} = \text{const}$$

(b) Sub or super sonic?

$$M_1 = \frac{V_1}{a_1} = \frac{V_1}{\sqrt{\gamma R T_1}} = 1.1458 \qquad \boxed{M_1 > 1 \quad \text{Supersonic}}$$

$$M_2 = \frac{V_2}{a_2} = \boxed{M_2 = 1.948} \quad \text{Supersonic}$$

$M \uparrow$ & $V \uparrow$ as $A \uparrow$ in supersonic duct flow

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ISENTROPIC, ADIABATIC DUCT FLOW

Eqs. $\quad h_{0_1} = h_{0_2} \Rightarrow \boxed{h + \frac{V^2}{2} = \text{constant}}$

differentiate: $\quad dh + \frac{\cancel{2}VdV}{\cancel{2}} = 0$

$$\boxed{dh = -VdV} \quad (1)$$

Approx. as isentropic $(ds = 0)$

recall, $\quad T\cancel{ds} = dh - \frac{1}{\rho}dP \Rightarrow \boxed{dh = \frac{dP}{\rho}} \quad (2)$

equate (1) & (2) $\rightarrow \boxed{-VdV = \frac{dP}{\rho}} \quad (3)$

Cons. of mass $\dot{m} = \rho V A = \text{const}$

Differentiate: $d\dot{m} = 0$

$$0 = \rho A\, dV + \rho V\, dA + VA\, d\rho$$

$\div \rho V A$ & rearrange:

$$\boxed{\frac{dA}{A} + \frac{dV}{V} + \frac{d\rho}{\rho} = 0} \quad (4)$$

Plug (3) into (4) & rearrange:

$$\frac{dA}{A} = -\frac{dp}{\rho V^2} + \frac{d\rho}{\rho} = 0$$

$$\boxed{\frac{dA}{A} = \frac{dp}{\rho}\left(\frac{1}{V^2} - \frac{d\rho}{dp}\right)} \quad (5)$$

Recall, defn of speed of sound $a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s$

means s = fixed = Isentropic

$$\boxed{\frac{1}{a^2} = \left(\frac{\partial \rho}{\partial p}\right)_s} = \frac{d\rho}{dp} \quad \text{since } s = \text{const}$$

Eq (5) becomes

$$\frac{dA}{A} = \frac{dp}{\rho}\left(\frac{1}{V^2} - \frac{1}{a^2}\right)$$

$$= \frac{dp}{\rho V^2}\left(1 - \underbrace{\frac{V^2}{a^2}}_{M^2}\right)$$

$$\boxed{\frac{dA}{A} = \frac{dp}{\rho V^2}\left(1 - M^2\right)}$$

Now use Eq (3) again, $\frac{dp}{\rho} = -V\,dV$

$$\frac{dA}{A} = \frac{-V\,dV}{V^2}\left(1 - M^2\right)$$

$$= \frac{dV}{V}\left(1 - M^2\right)$$

rearrange:

$$\boxed{\frac{dV}{V} = \frac{1}{M^2 - 1}\frac{dA}{A}}$$

Valid for steady, isentropic, adiabatic 1-D duct flow