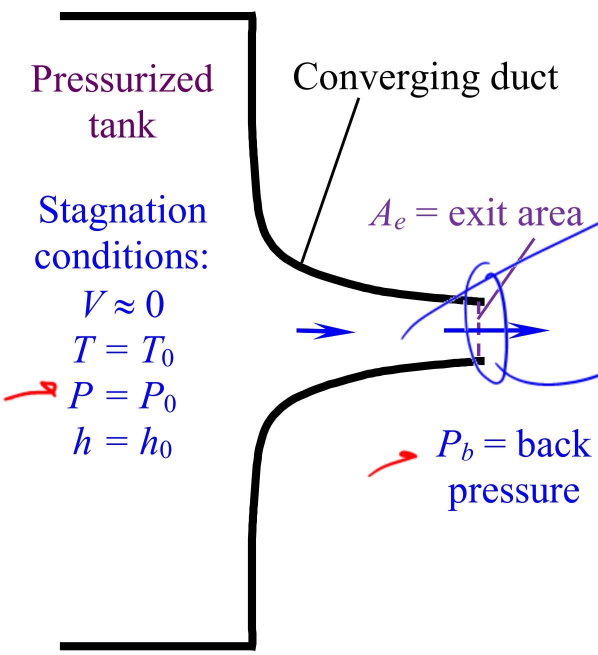
Today, we will:

- Continue discussing and defining critical (sonic) properties
- Continue discussing converging nozzles: mass flow rate, choking, etc.
- Derive equations for compressible flow through a converging nozzle
- Do an example problem – flow in a converging duct
- Do Candy Questions for Candy Friday

QUICK REVIEW:



$M < 1$ in nozzle

M_{exit} may be 1 or < 1 (depends on P_b)

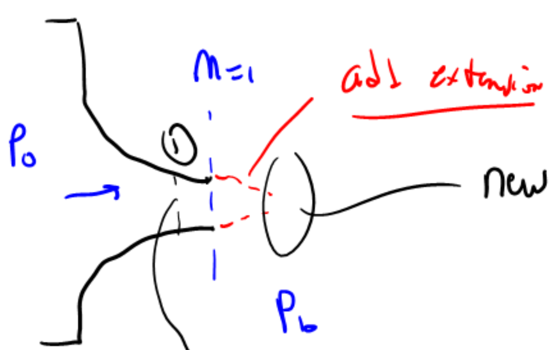
M_{exit} can never be supersonic

If $M_{exit} = 1$, we define critical or sonic

$A^* = A$ @ $M=1$

$T^* = T$ " "

$P^* = P$ " " ; etc



M @ new exit plane will = 1

new * condition here

$\dot{m}$ goes down by adding extension

$M < 1$ w/o extension $M_1 < 1$ w/extension $M_1 <$ it was w/o extension

- What if we increase P_0? $\rightarrow$ $\dot{m} \uparrow$

  But M still = 1 @ exit plane

- What if we increase T_0? $\rightarrow$ @ same P_0, $\rho_0 \downarrow$

  $\therefore$ $\dot{m}$ actually $\downarrow$

- CAUTION: Just because there is a throat (A = min.), no guarantee that flow is sonic there

It depends on $\frac{P_b}{P_0}$ $\left[\text{or } \frac{P_0}{P_b}\right]$

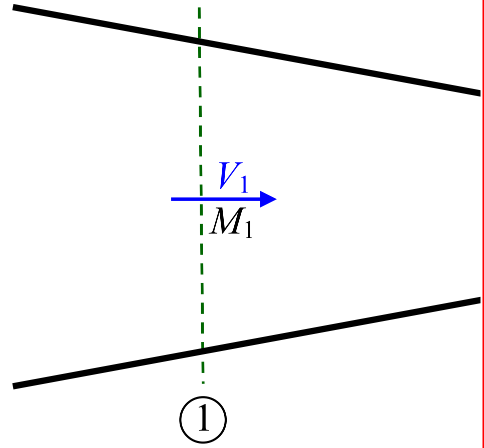
determines if flow is sonic @ throat or not

Example: Steady adiabatic duct flow

Given: Air flows steadily in converging duct. The duct is insulated. The following properties are known at location 1:

- $V_1 = 200.$ m/s
- $M_1 = 0.500$

(air, $\gamma = 1.40$)



To do: Calculate the critical speed of sound a^* at location 1 in this flow. in m/s

Solution: **To be completed in class.**

Assumptions and Approximations:
Steady, Ideal gas, 1-D approximation, Adiabatic, Isentropic

Equations that are useful:

$$\boxed{M^2 = \frac{2}{\frac{\gamma+1}{M^{*2}} - (\gamma - 1)}}, \quad \boxed{a^* = \sqrt{\gamma R T^*}}, \quad \boxed{M^* = V/a^*}$$

$$M_1^* = 0.53452$$

$$a_1^* = \frac{V_1}{M_1^*} = 374.166 \ \frac{m}{s}$$

$$\boxed{a^* = 374.\ m/s}$$

EQUATIONS FOR STEADY, 1-D, ADIABATIC, ISENTROPIC DUCT FLOW OF AN IDEAL GAS

$$\boxed{h_0 = h + \frac{V^2}{2} = \text{const everywhere}}$$

$$c_p T_0 = c_p T + \frac{V^2}{2}$$

$$\boxed{T_0 = T + \frac{V^2}{2c_p}} \qquad \text{recall, } \boxed{c_p = \frac{R\gamma}{\gamma - 1}}$$

$$T_0 = T + \frac{V^2(\gamma-1)\,T}{2\,\underbrace{\gamma R T}_{a^2}} \quad M^2 \quad \Rightarrow \quad \boxed{\frac{T_0}{T} = 1 + \frac{\gamma-1}{2}M^2} \quad (1)$$

Recall <u>isentropic relations</u> $\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$

let $P_2 = P_0$

let $P = P$ somewhere in nozzle

$$\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}}$$

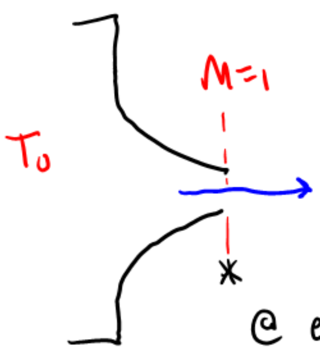
Eq(1)

$$\therefore \boxed{\frac{P_0}{P} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}} \quad ☆ \quad (2)$$

Similarly →

$$\boxed{\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{1}{\gamma-1}}} \quad ☆ \quad (3)$$

At sonic conditions (at the throat, if $\frac{P_b}{P_0}$ is small enough to achieve sonic conditions)



At $M=1$ → (1) $\frac{T_0}{T} = 1 + \frac{\gamma-1}{2}(1)^2$

@ exit, $T = T^*$, $P = P^*$, ∴ $\frac{T_0}{T^*} = \frac{\gamma+1}{2}$

or $\boxed{\frac{T^*}{T_0} = \frac{2}{\gamma+1}}$ = critical ratio of temp.

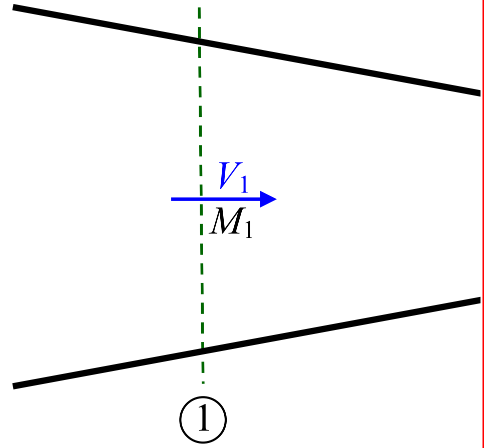
Similarly from Eq (2) & (3)

$$\boxed{\frac{P^*}{P_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}} \qquad \boxed{\frac{\rho^*}{\rho_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}}$$

critical ratios of P & ρ

RATIOS ARE KING FOR COMPRESSIBLE FLOW ☆

Example: Steady adiabatic duct flow (continued – alternate way to solve)



Given: Air flows steadily in converging duct. The duct is insulated. The following properties are known at location 1:

- $V_1 = 200.$ m/s
- $M_1 = 0.500$

To do: Calculate the critical speed of sound a^* at location 1 in this flow.

Assumptions and Approximations:
Steady, Ideal gas, 1-D approximation, Adiabatic, Isentropic

Solution: We used the M-M^* relationship → $\boxed{a^* = 374.\ \text{m/s.}}$
Now, we show an alternate method using *ratios*.
Equations that are useful:

$$\boxed{a^* = \sqrt{\gamma R T^*}},\ \boxed{\frac{T^*}{T_0} = \frac{2}{\gamma+1}},\ \boxed{\frac{T_0}{T} = 1 + \frac{\gamma-1}{2}M^2}$$

$$a^* = \sqrt{\underbrace{\gamma R T}_{a^2}\,\frac{T^*}{T_0}\,\frac{T_0}{T}} = a_1\sqrt{\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_1^2\right)}$$

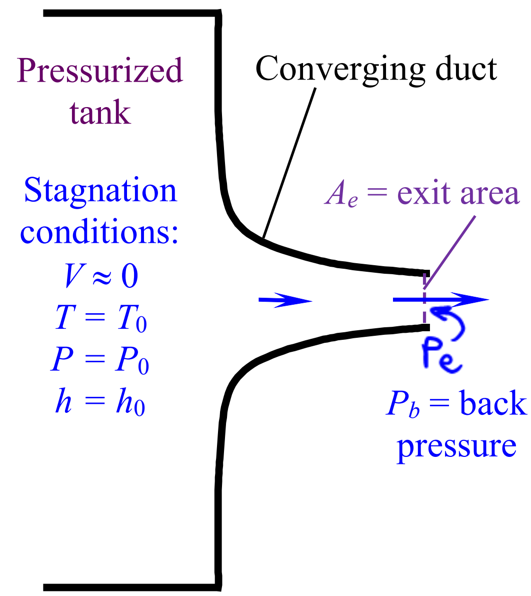
$$a^* = \frac{V_1}{M_1}\sqrt{\frac{2}{\gamma+1}\left(1+\frac{\gamma-1}{2}M_1^2\right)} = \boxed{a^* = 374.\ \text{m/s}}\ \checkmark$$

Ratios are useful ☆

Example: Converging nozzle flow

Given: Air flows steadily from a large pressurized tank at pressure P_0 into a converging duct that is open to another large tank at pressure P_b. The duct is insulated and we ignore friction. We measure:

- $P_0 = 158.0$ kPa → absolute P
- $T_0 = 520.0$ K → absolute T
- P_b = back pressure = 101.3 kPa
- A_e = exit area = 0.0130 m^2



To do:

(**a**) Is the flow at the exit subsonic, sonic, or supersonic?

(**b**) Calculate M_e and T_e for this case.

(**c**) As we lower back pressure, at what value of P_b does the nozzle exit become sonic?

(**d**) What happens if $P_b < P^*$?

Solution:

To be completed in class.

Assumptions and Approximations:

- Ideal gas (air)
- Adiabatic
- Isentropic
- 1-D flow
- Steady (quasi steady)

$$\frac{P^*}{P_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}}$$ → If flow is sonic @ throat, then P_b must be $< P^*$

$\gamma = 1.40$ (air)

$$\boxed{\frac{P^*}{P_0} = 0.5283}$$

$P^* = 0.5283\, P_0 = 0.5283\,(158.\text{ kPa}) = 83.47$ kPa

$P_b <$ this to get sonic flow

We have $P_b = 101.3$ kPa

(a) $\boxed{\text{This flow is subsonic}}$

$M < 1$ @ exit plane

$M_e = M$ @ exit plane

(b) Calc M_e & T_e

Note: P_e not always $= P_b$

Set $P_e = P_b$ since flow is subsonic @ exit

$$\frac{P_0}{P_e} = \left(1 + \underbrace{\frac{\gamma - 1}{2}}_{0.2} M_e^2\right)^{3.5} \rightarrow \text{Solve for } M_e$$

$M_e = 0.82286$

$\boxed{M_e = 0.823}$

$$\boxed{\frac{T_0}{T_e} = 1 + 0.2 M_e^2}$$

$$T_e = \frac{T_e}{T_0} T_0 = \left(1 + 0.2 M_e^2\right)^{-1} (520\ K)$$

$$\boxed{T_e = 458.\ K}\ \star$$

Comments:

- $T_e < T_0$ (air gets colder)

$T_0 = 247°C$

$T_e = 185°C$ (much colder!)

- P_b would need to be lower to make this flow sonic @ the throat
- $M_e = 0.823 \rightarrow$ close, but not sonic
- If change area @ exit, M_e would not change (only the mass flow rate would change)