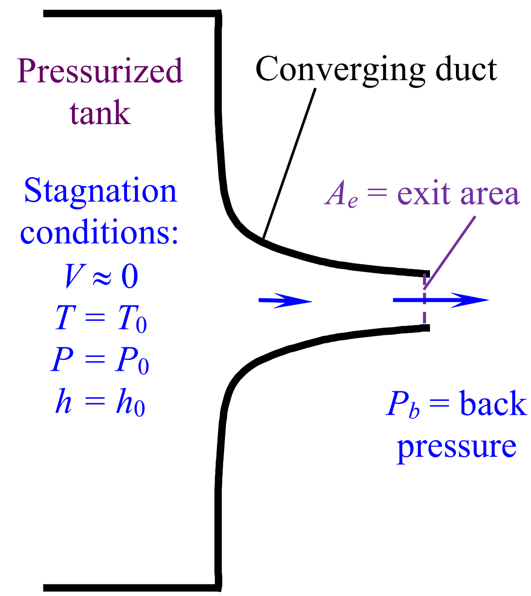
Today, we will:

- Finish the example problem we started last time – converging duct
- Discuss choked flow and choking in more detail
- Discuss how to calculate the mass flow rate for converging nozzle flow

Example: Converging duct flow

Given: Air flows steadily from a large pressurized tank at pressure P_0 into a converging duct that is open to another large tank at pressure P_b. The duct is insulated and we ignore friction. We measure:

- P_0 = 158.0 kPa
- T_0 = 520.0 K
- P_b = back pressure = 101.3 kPa
- A_e = exit area = 0.0130 m^2



To do:

(**a**) Is the flow at the exit subsonic, sonic, or supersonic?

(**b**) Calculate M_e and T_e for this case.

(**c**) As we lower back pressure, at what value of P_b does the nozzle exit become sonic?

(**d**) What happens if $P_b < P^*$?

Solution:

Assumptions and Approximations: steady, ideal gas, adiabatic, isentropic, 1-D approx.

Last time we did (a) and (b):

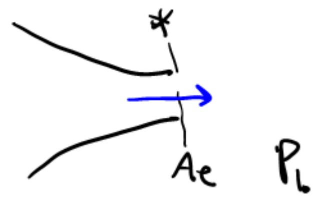
(a) Flow is subsonic

(b) M_e = 0.823 (subsonic) and T_e = 458. K

Comments:

1. Part (b) verifies Part (a) – namely that the Mach number at the exit is subsonic.
2. The back pressure would need to be lower than the given back pressure in order to make the flow sonic at the exit plane.
3. Notice how cold the exit temperature is. We wend from 520 K in the tank to 458 K at the exit plane, a drop of 62 K!
4. Critical (* or sonic) conditions do not actually occur anywhere in this flow, but critical conditions are still useful in solving problems like this.

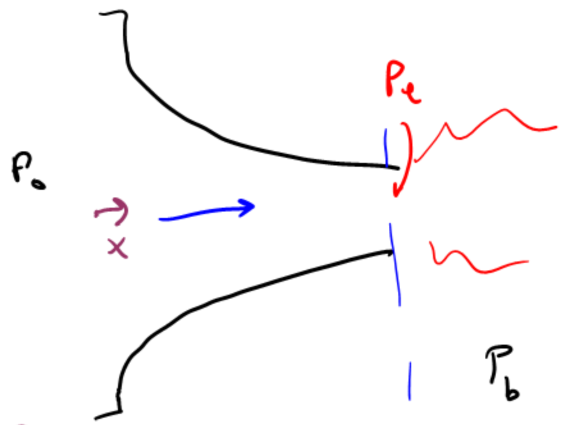
(c) Sonic when $P_b = P^* = \boxed{83.47 \text{ kPa}}$



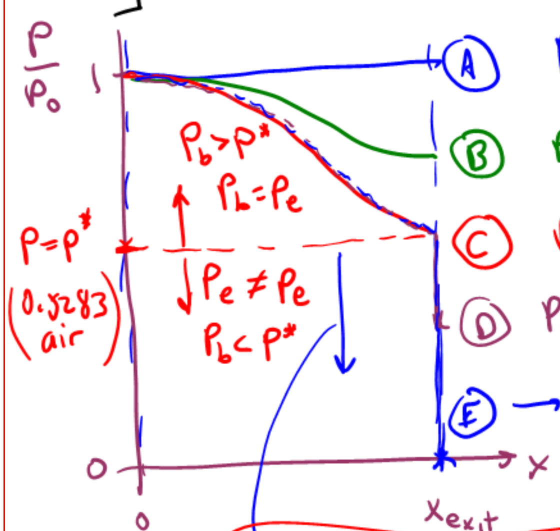
(d) If $P_b < p^*$ what happens?

The flow <u>stays</u> sonic @ exit plane (throat)

Plot $\frac{P}{P_0}$ as $P_b \downarrow$



we are lowering P_b for (A), (B), (C) ...



(A) $P_b = P_0$ → ∴ <u>no flow</u>

(B) $P_0 > P_b > p^*$ (our case here) Subsonic

(C) $P_b = p^*$ → sonic @ throat

(D) $P_b < p^*$ – still <u>sonic</u> @ <u>throat</u>

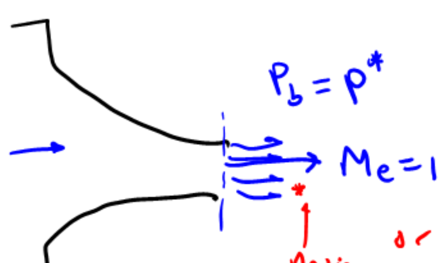
(E) → $P_b = 0$ → total vacuum!

Flow is <u>CHOKED</u>

✡ → if $P_b < P^*$

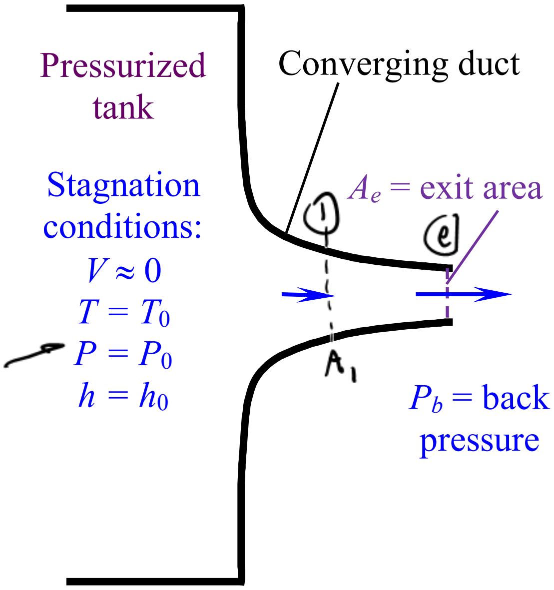
$M = 1$ @ exit plane

$P_e = P^* \neq P_b$



<u>noise</u> or <u>pressure change</u> cannot be felt inside the nozzle

Converging nozzle (continued)



Mass flow rate $\dot{m}$ is constant no matter what P_b is, if $P_b < P^*$ → choked

Let's calculate the **mass flow rate** through the nozzle as a function of the stagnation conditions, the back pressure, and the exit area.

* General case (may or may not be choked)

$\dot{m} = \rho V A$ anywhere

$a = \sqrt{\gamma R T}$

ideal gas → $\dot{m} = \frac{P}{RT} \cdot M \cdot a A = \frac{P}{RT} M \sqrt{\gamma R T}\, A = \boxed{PAM\sqrt{\frac{\gamma}{RT}} = \dot{m}}$

$V = M a$

Also → isentropic → use isentropic relations.

$$\dot{m} = PAM\sqrt{\frac{\gamma}{RT}} = \frac{P}{P_0} P_0 AM \sqrt{\frac{\gamma}{R}} \left[\frac{T}{T_0} T_0\right]^{-1/2}$$

using ratio "trick"

$$\dot{m} = \left(1 + \frac{\gamma-1}{2} M^2\right)^{\frac{-\gamma}{\gamma-1}} P_0 AM \sqrt{\frac{\gamma}{R}} \left[\left(1 + \frac{\gamma-1}{2} M^2\right)^{-1} T_0\right]^{-1/2}$$

combine exponents $(-)^a (-)^b = (-)^{a+b}$

$$\boxed{\dot{m} = P_0 A M \sqrt{\frac{\gamma}{RT_0}} \left(1 + \frac{\gamma-1}{2} M^2\right)^{-\frac{(\gamma+1)}{2(\gamma-1)}}} \qquad (1)$$

General case for $\dot{m}$ whether choked or not

- Applies @ any cross section of the duct
- Applies even if flow is not choked
- At the exit plane, use $A = A_e$ & $M = M_e$

· CHOKED CASE → $P_b < P^*$ $\quad A_e = A^*$; $M = 1$

Plug in $A = A_e$, $M = 1$ into eq. (1)

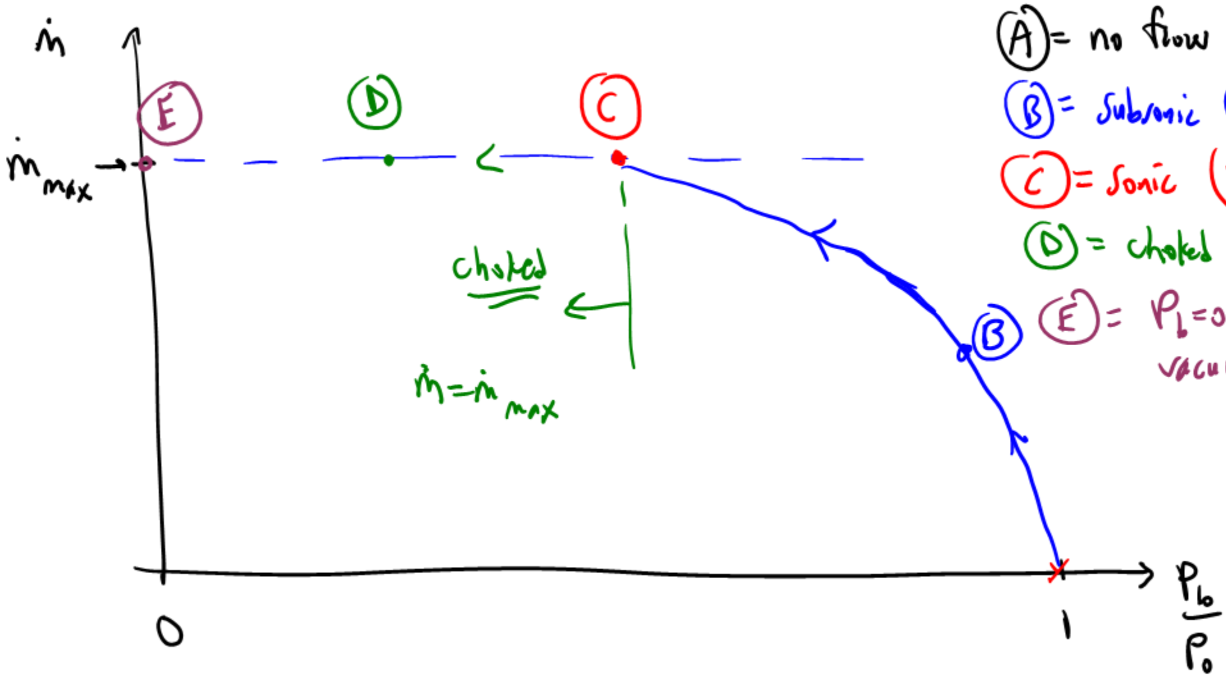
$$\dot{m}_{choked} = \dot{m}_{max} = P_0 A_e M \sqrt{\frac{\gamma}{RT_0}} \left(1 + \frac{\gamma-1}{2} M^2\right)$$

$$\boxed{\dot{m}_{max} = \dot{m}_{choked} = P_0 A^* \sqrt{\frac{\gamma}{RT_0}} \left(\frac{\gamma+1}{2}\right)^{\frac{-\gamma+1}{2(\gamma-1)}}}$$

$\dot{m}$ for choked flow only

A_e

Plot $\dot{m}$ vs $\frac{P_b}{P_0}$



(A) = no flow ($P_b = P_0$)

(B) = Subsonic ($P_b > P^*$)

(C) = Sonic ($P_b = P^*$)

(D) = choked ($P_b < P^*$)

(E) = $P_b = 0$ vacuum

(Q) How to increase $\dot{m}$?

Look @ eq. for $\dot{m}_{max}$

$\dot{m} \uparrow$ if

- $P_0 \uparrow$ higher stag P (P_0) in tank
- $A_e \uparrow$ bigger exit area
- $T_0 \downarrow$ lower T in tank (T_0)

Why $T_0 \downarrow$? $\rightarrow$ • $\boxed{\rho \uparrow \text{ as } T_0 \downarrow}$ $P_0 = \rho_0 R T_0$

THIS ONE WINS!

But • As $T_0 \downarrow$ $a = \sqrt{\gamma R T}$, $\therefore a \downarrow$

$M = \frac{V}{a}$ $\rightarrow$ we can achieve $M = 1$ @ a lower speed V