Today, we will:

- Discuss the physical meaning and significance of choked flow and the speed of sound
- Introduce ***Mach waves***
- Do more example problems – converging nozzle and choked vs. unchoked flow
- Do Candy Questions for Candy Friday

Choked Flow and the Speed of Sound

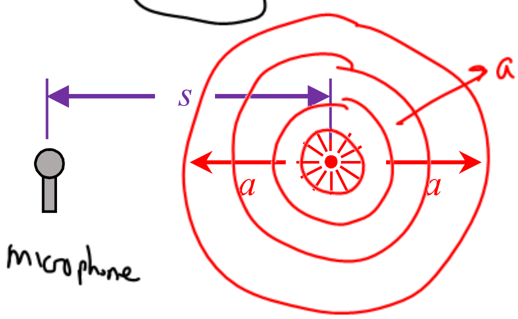
Let's do a "thought experiment" in which there is a small pressure disturbance (noise) at a point in a uniform flow of air

Travel @ speed a

Pressure waves spread in all directions

We examine 4 cases, as V is increasing $V \uparrow$

Case a) $\boldsymbol{V = 0}$ $\boxed{M=0}$

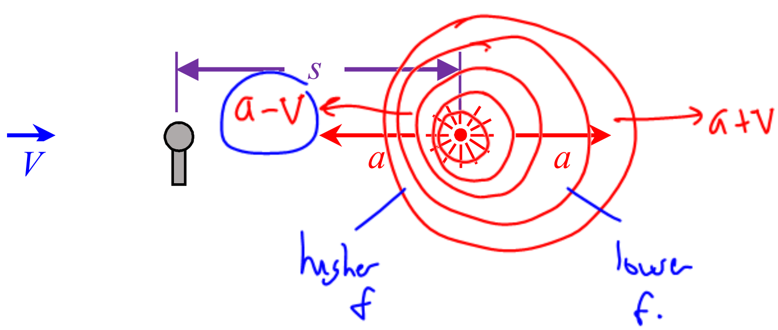


Microphone hears the sound @ $\boxed{\Delta t = \frac{s}{a}}$

frequency is unchanged

Case b) $\boxed{\boldsymbol{0 < V < a}}$ $\boxed{M<1}$ (Subsonic)

Not symmetric

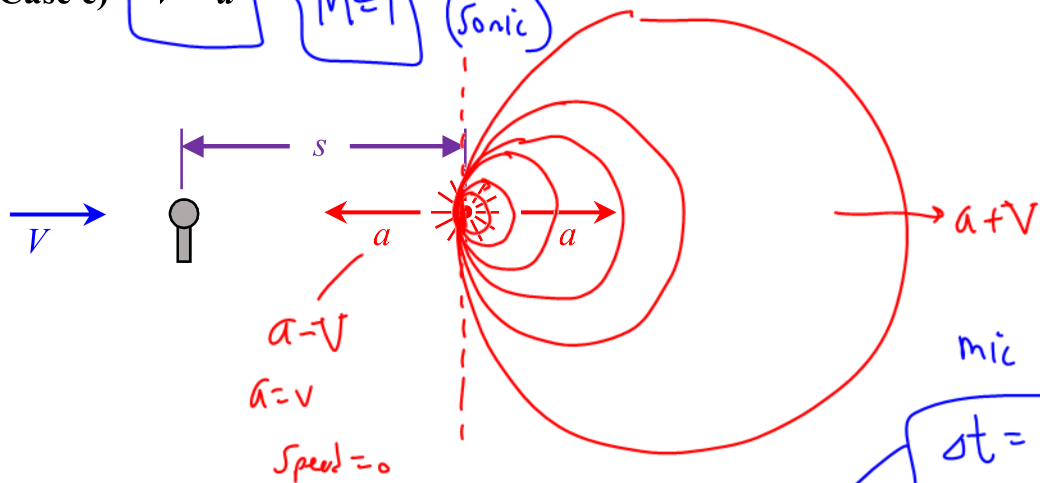


mic. hears @ $\boxed{\Delta t = \frac{s}{a-V}}$

frequency is changed

"Doppler effect"

Case c) $\boxed{V = a}$ $\boxed{M=1}$ (sonic)

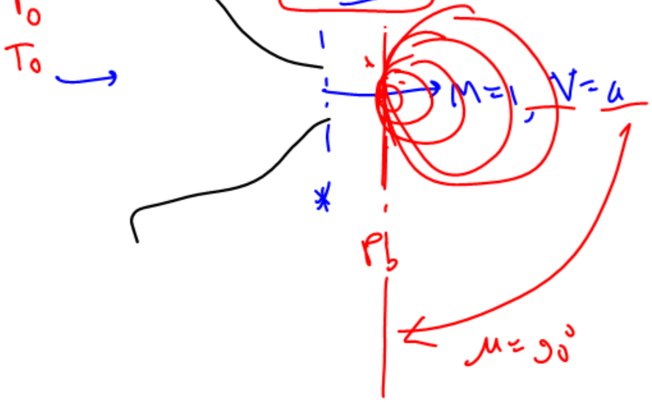


mic hears @

$$\boxed{\Delta t = \frac{s}{a-V} = \frac{s}{0} = \infty}$$

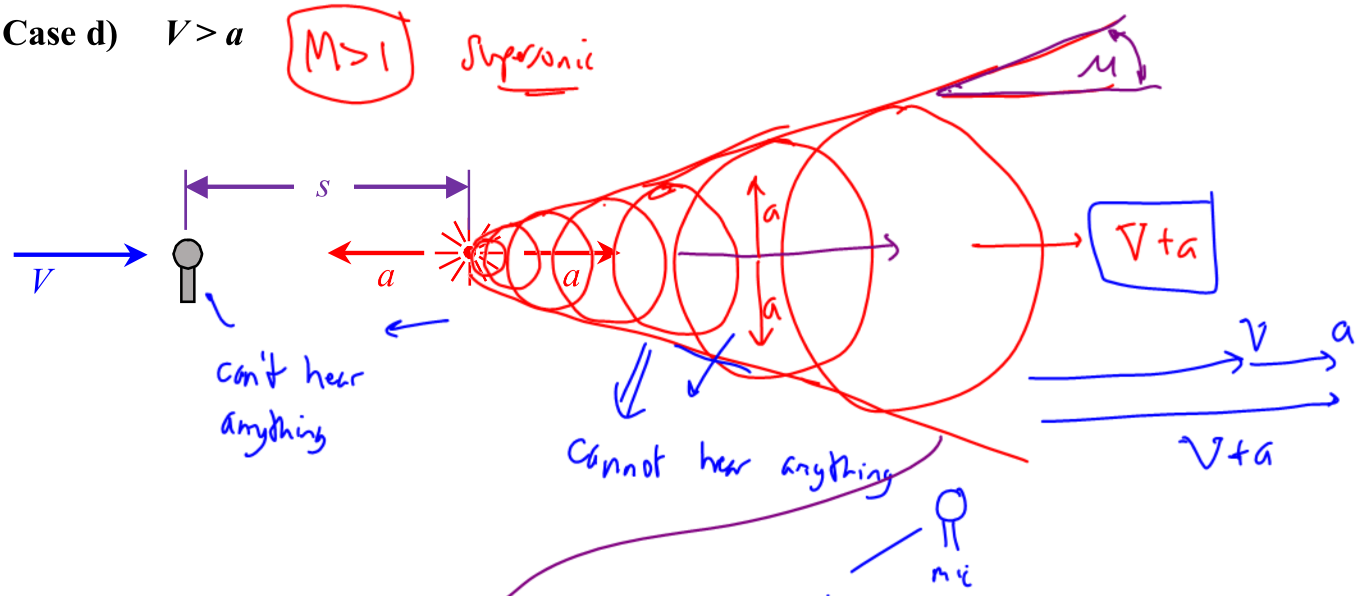
never hears the sound!

$\boxed{\text{CHOKED}}$



$\boxed{\text{No disturbances can travel upstream into the nozzle if it is choked}}$ *

Case d) $V > a$ $\boxed{M>1}$ Supersonic



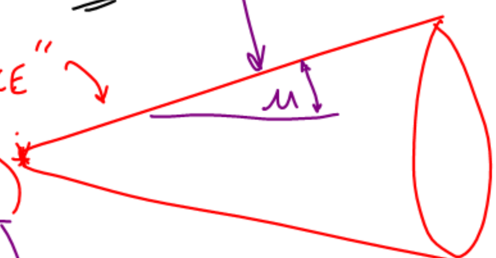
Called a Mach wave

never hear the sound

$\boxed{\mu = \text{Mach angle}}$

In 3-D it is a cone

"ZONE OF SILENCE" (outside the Mach cone)

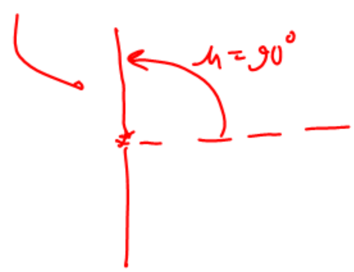


MACH CONE

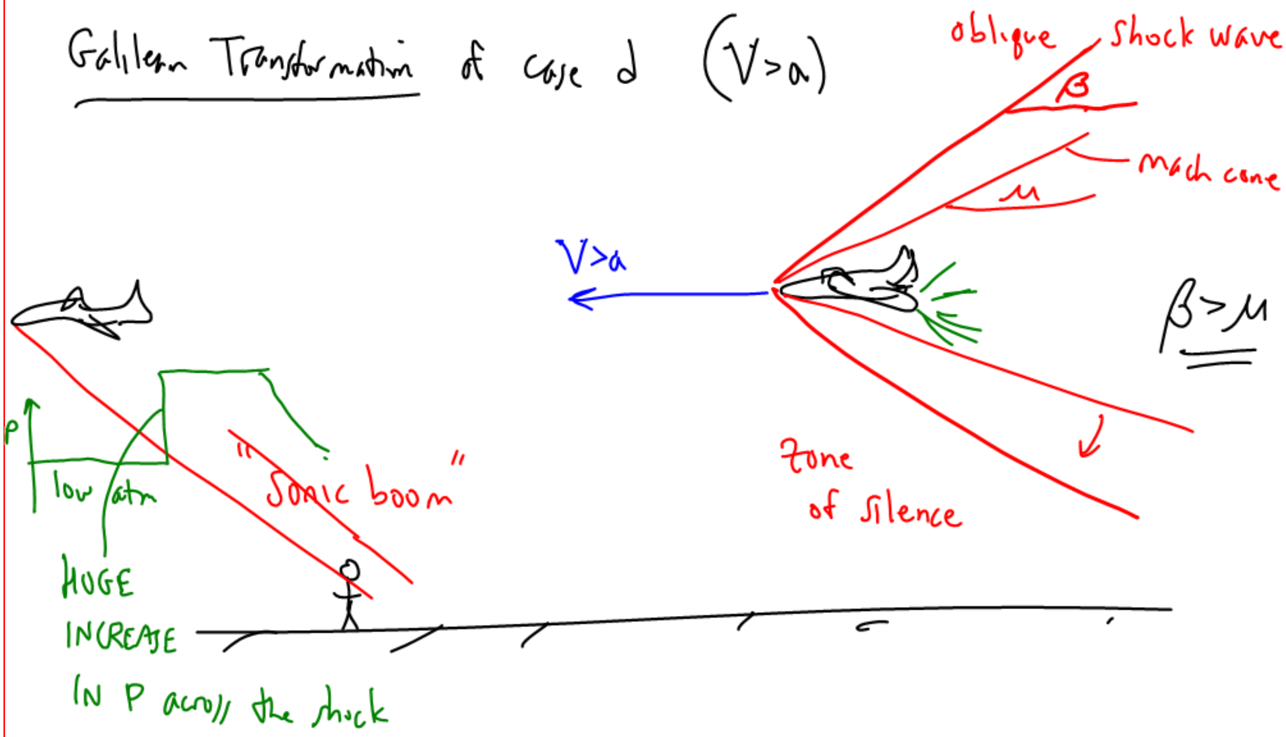
$\boxed{\mu = \sin^{-1}\frac{1}{M}}$ ☆ = Mach angle

Eg $M=2$ $\mu = \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ$

$M=1$ $\mu = \sin^{-1}\left(\frac{1}{1}\right) = \frac{\pi}{2}$ or 90°



Galilean Transformation of case d $(V>a)$

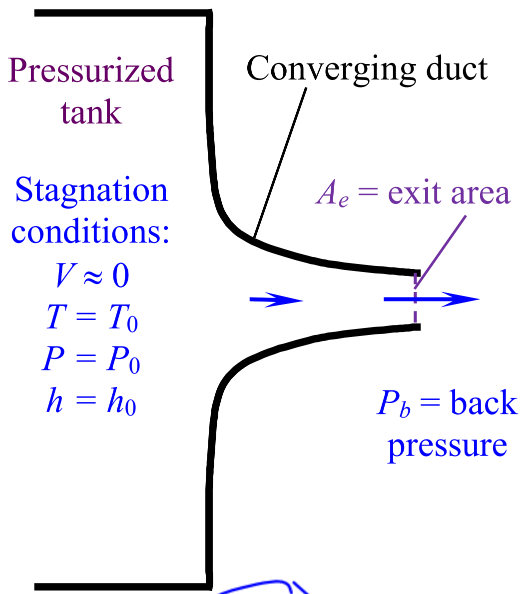


Example: Choked converging nozzle flow

Given: Air flows steadily from a large pressurized tank at pressure P_0 into a converging duct that is open to another large tank at pressure P_b. The duct is insulated and we ignore friction. We measure:

- $P_0 = 158.0$ kPa
- $T_0 = 520.0$ K
- P_b = back pressure < P^* (flow is choked)
- A_e = exit area = 0.0130 m^2

(Same as previously)



To do:

$\dot{m}_{max} = 3.64\ \frac{kg}{s}$

(a) Calculate the mass flow rate.

(b) Calculate the air speed V_e at the exit plane in m/s.

Solution: To be completed in class.

Assumptions and Approximations: steady, ideal gas, adiabatic, isentropic, 1-D approx.

(a) CHOKED $\rightarrow \therefore$

$$\dot{m} = \dot{m}_{max} = P_0 A^* \sqrt{\frac{\gamma}{RT_0}} \left(\frac{\gamma+1}{2}\right)^{\frac{-(\gamma+1)}{2(\gamma-1)}}$$

$\frac{-(\gamma+1)}{2(\gamma-1)} \rightarrow -3$ for air

$A^* = A_e$ (given) $\qquad \frac{1.4+1}{2}$

$$\dot{m} = \dot{m}_{max} = (158.0\ kPa)(0.0130\ m^2)\sqrt{\frac{1.40}{\left(287\ \frac{m^2}{s^2 K}\right)(520\ K)}}\ (1.2)^{-3}\left(\frac{1000\ N}{m^2 \cdot kPa}\right)\left(\frac{kg\ m}{s^2\ N}\right)$$

$$\boxed{\dot{m}_{max} = 3.64\ \frac{kg}{s}}$$

(b) $V_e = a_e = a^*$ since $M_e = 1$ @ exit

$$V_e = a^* = \sqrt{\gamma R T^*} \qquad \frac{T^*}{T_0} = \frac{2}{\gamma - 1}$$

METHOD A

$$V_e = \sqrt{\gamma R T_0 \left(\frac{T^*}{T_0}\right)} = \sqrt{\gamma R T_0 \left(\frac{2}{\gamma-1}\right)} = 417.\ \frac{m}{s}$$

METHOD B

$$\dot{m} = \dot{m}_{max} = \rho_e V_e A_e$$

$$V_e = \frac{\dot{m}_{max}}{\rho_e A_e}$$

$$\rho_e = \rho^* = \rho_0 \left(\frac{\rho^*}{\rho_0}\right)$$

$$\boxed{\rho_0 = \frac{P_0}{R T_0}}$$

$$\frac{\rho^*}{\rho_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}$$

$$\therefore \quad \boxed{V_e = \frac{\dot{m}_{max} R T_0}{P_0 A_e \left(\frac{2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}}} = \boxed{417.\ \frac{m}{s}} \checkmark$$