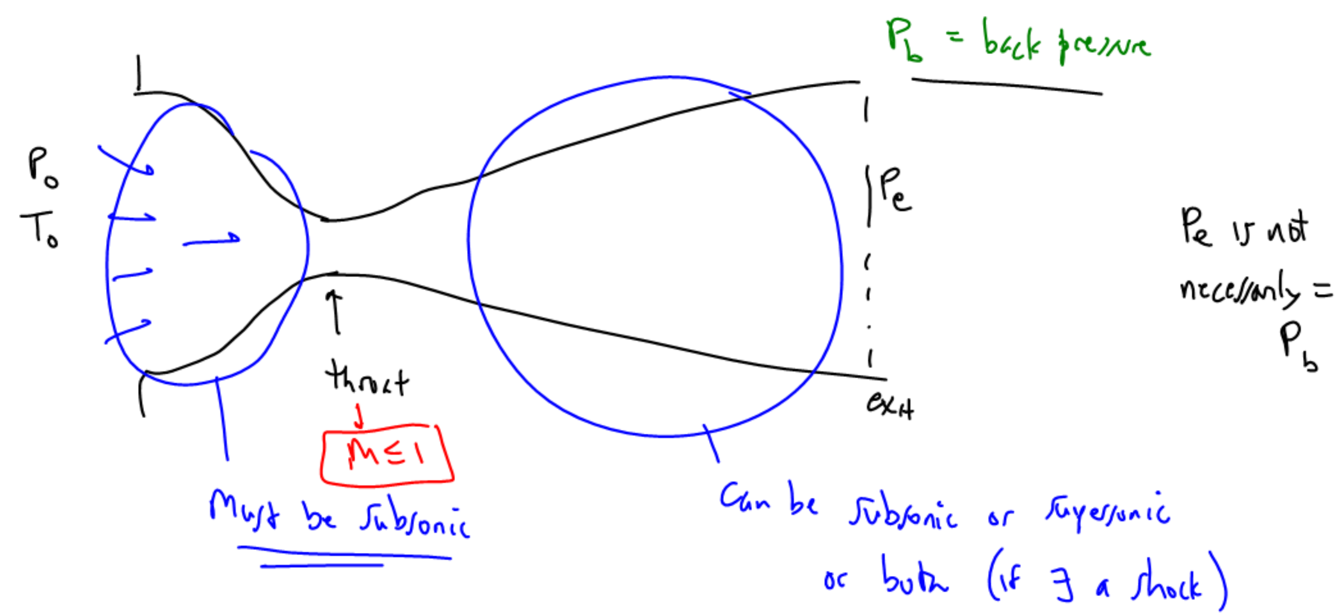
Today, we will:

- Begin a discussion of flow in ***C-D nozzles*** [converging-diverging nozzles (or converging-diverging ducts)]: We first do a *qualitative* analysis
- Discuss what happens downstream of the nozzle, depending on back pressure.
- Discuss a very practical application – ***rocket nozzle design***
- Do Candy Questions for Candy Friday

✯ CONVERGING-DIVERGING NOZZLES



✯ QUALITATIVE ANAL

"experiment" — Let $P_b = P_o$ & $P_b \downarrow$ all the way to 0

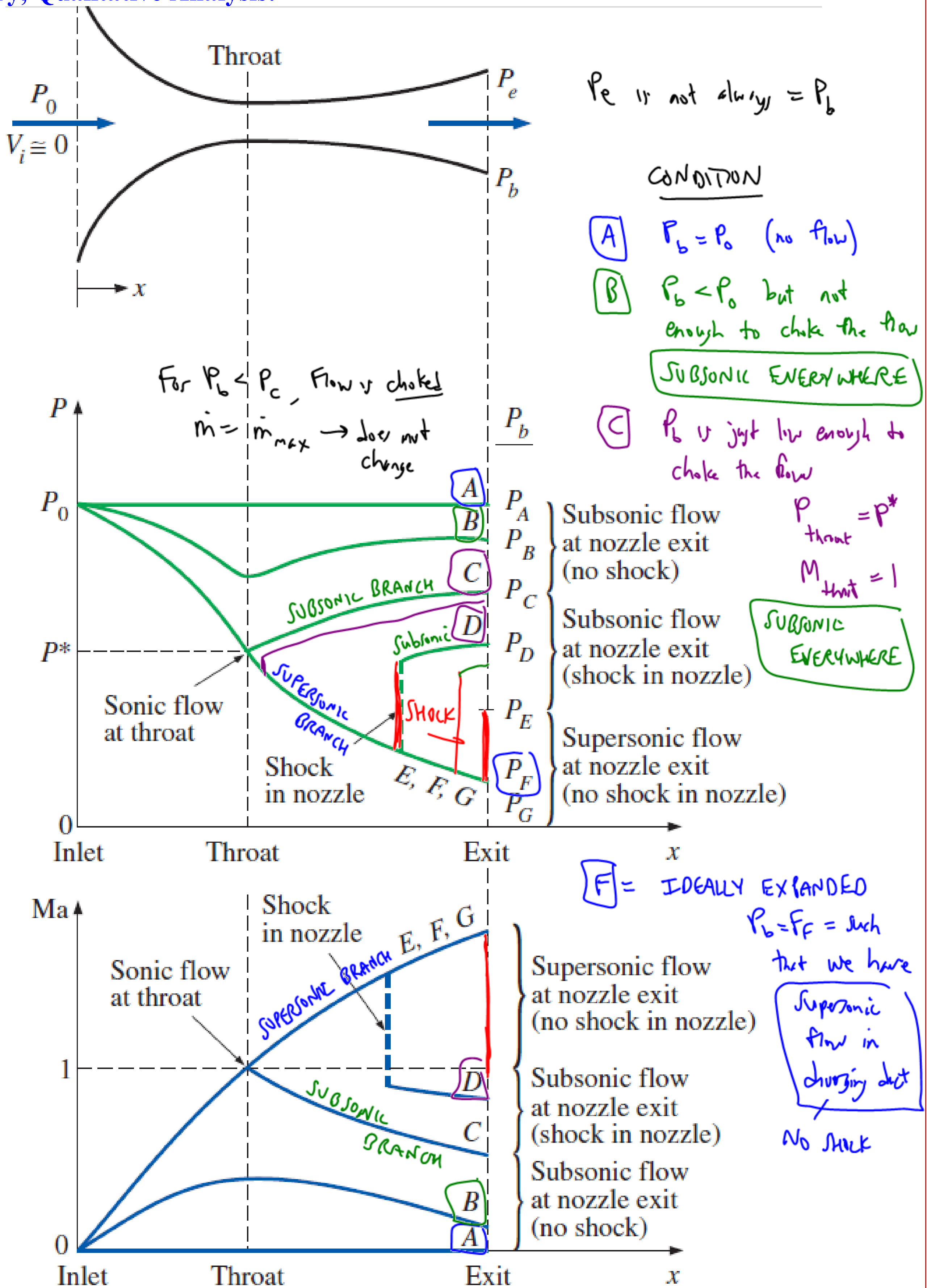
- Plot M & P vs x

A & A:

- Steady
- 1-D
- adiabatic
- ideal gas
- isentropic (except across a shock)

- We know A(x) & (throat area)

Summary, Qualitative Analysis:

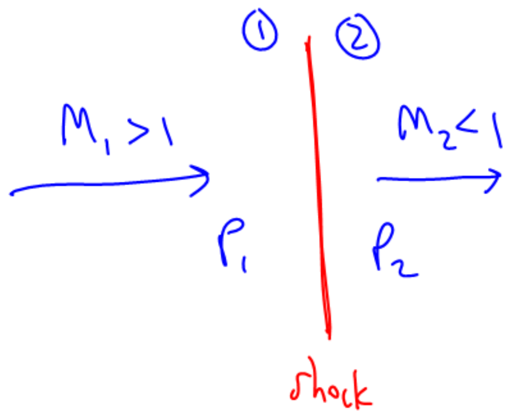


Effect of Back Pressure in a Converging-Diverging Nozzle: [Fig. 12-22 of C&C book]

D → $P_b < P_c$ $P_b > P_F$ → A SHOCK FORMS

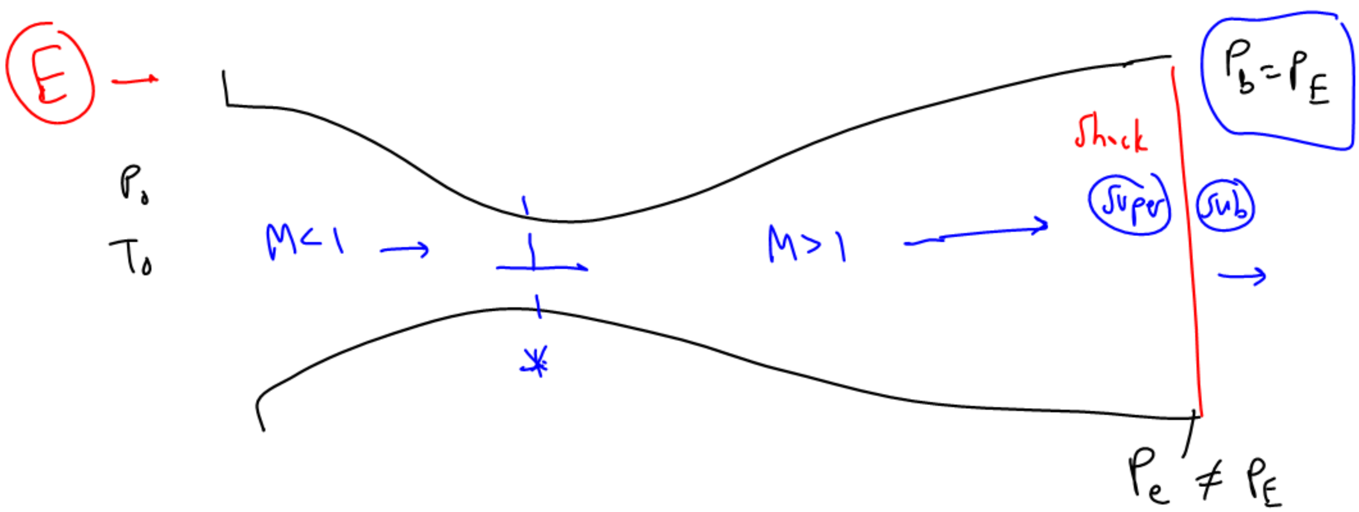
(NORMAL SHOCK)

ACROSS SHOCK, M goes from supersonic to subsonic



$P_2 \uparrow$ across a shock

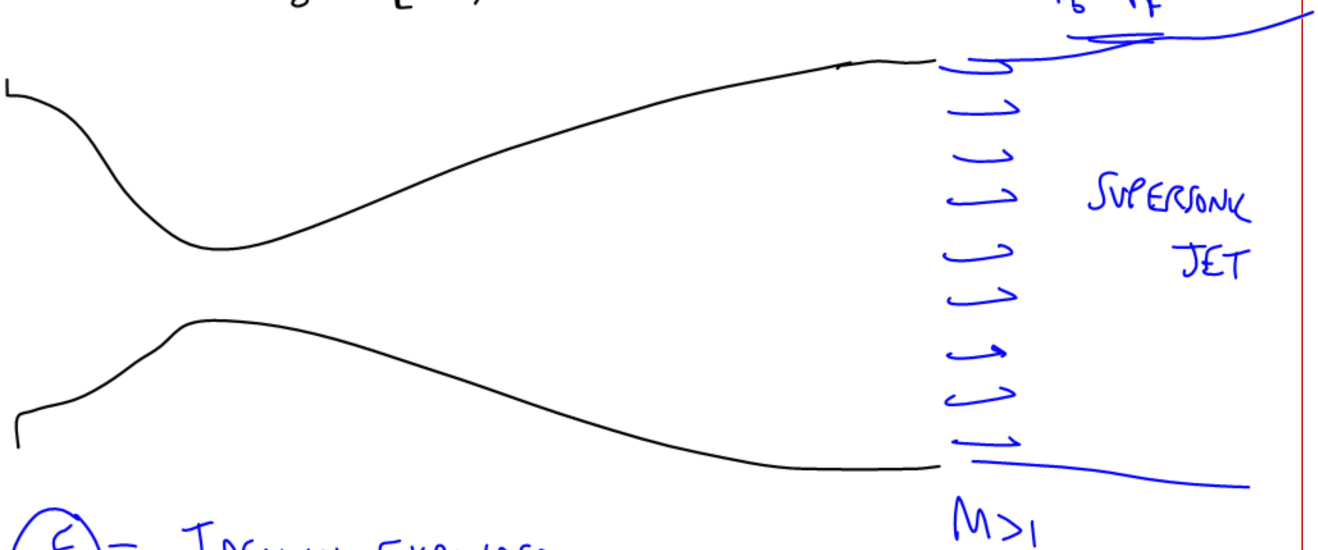
SHOCK MOVES DOWNSTREAM AS $P_b \downarrow$ Between D & E



FOR ANY $P_b < P_E$ the flow in entire C-D duct is choked & cannot change with further decrease in P_b

WHAT IF $P_b < P_E$?

$\boxed{F}$ $P_b = P_F$



(F) = IDEALLY EXPANDED

PERFECTLY "

$\boxed{G}$ → $P_b < P_E$ → We get complicated shocks downstream

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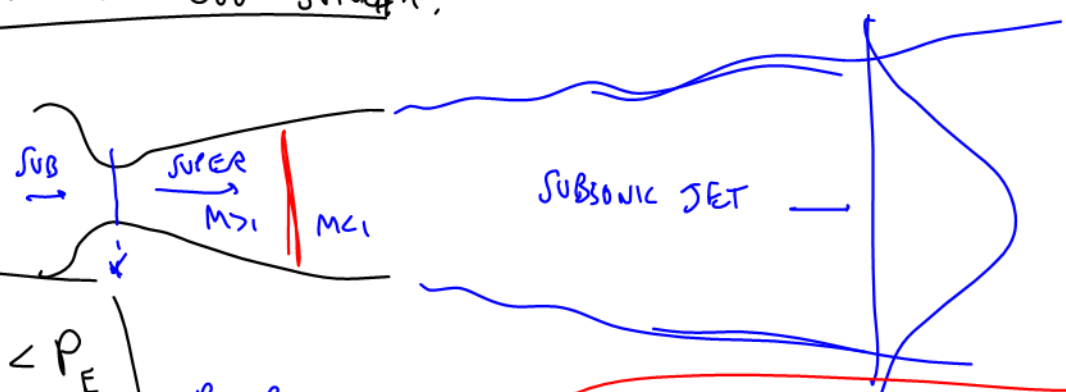
WHAT HAPPENS DOWNSTREAM?

$\boxed{\text{CASE D}}$



IF $\boxed{P_F < P_b < P_E}$

SHOCK DIAMONDS or "TIGER TAIL"

$\boxed{\text{OVEREXPANDED}}$ ☆

IF $P_b < P_F$ → $\boxed{G}$ not expanded enough

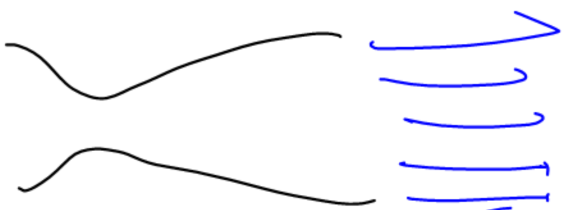
Flare out

$P_b < P_e$

AS $P_b \downarrow$

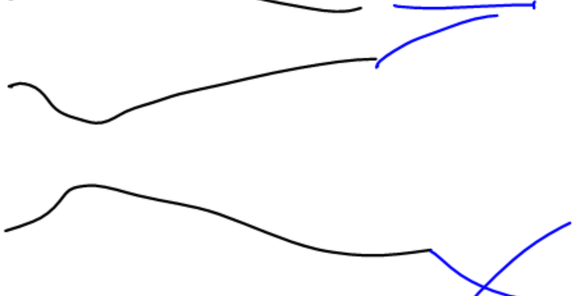
FLARE OUT INCREASES

$\boxed{F}$



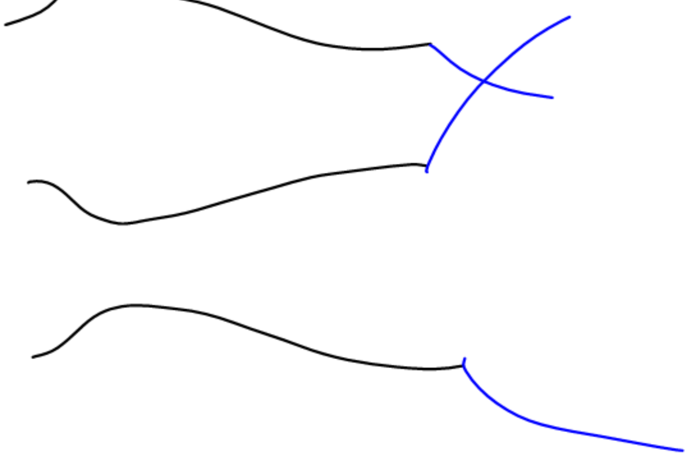
$P_b = P_F$

$\boxed{G_1}$



$P_b < P_F$

G_2



$P_b << P_F$

PRACTICAL APPLICATION – ROCKETS

@ ground

$P_b = P_{atm}$

@ TAKE-OFF

overexpanded

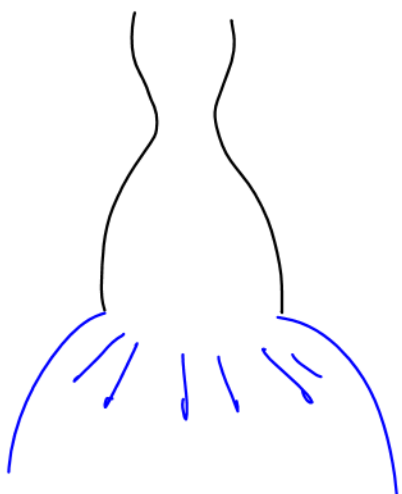
@ higher altitude

P_b is lower

max thrust

@ higher altitude

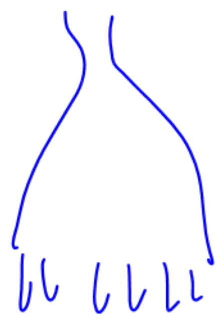
P_b is small



low thrust again

STAGE 2

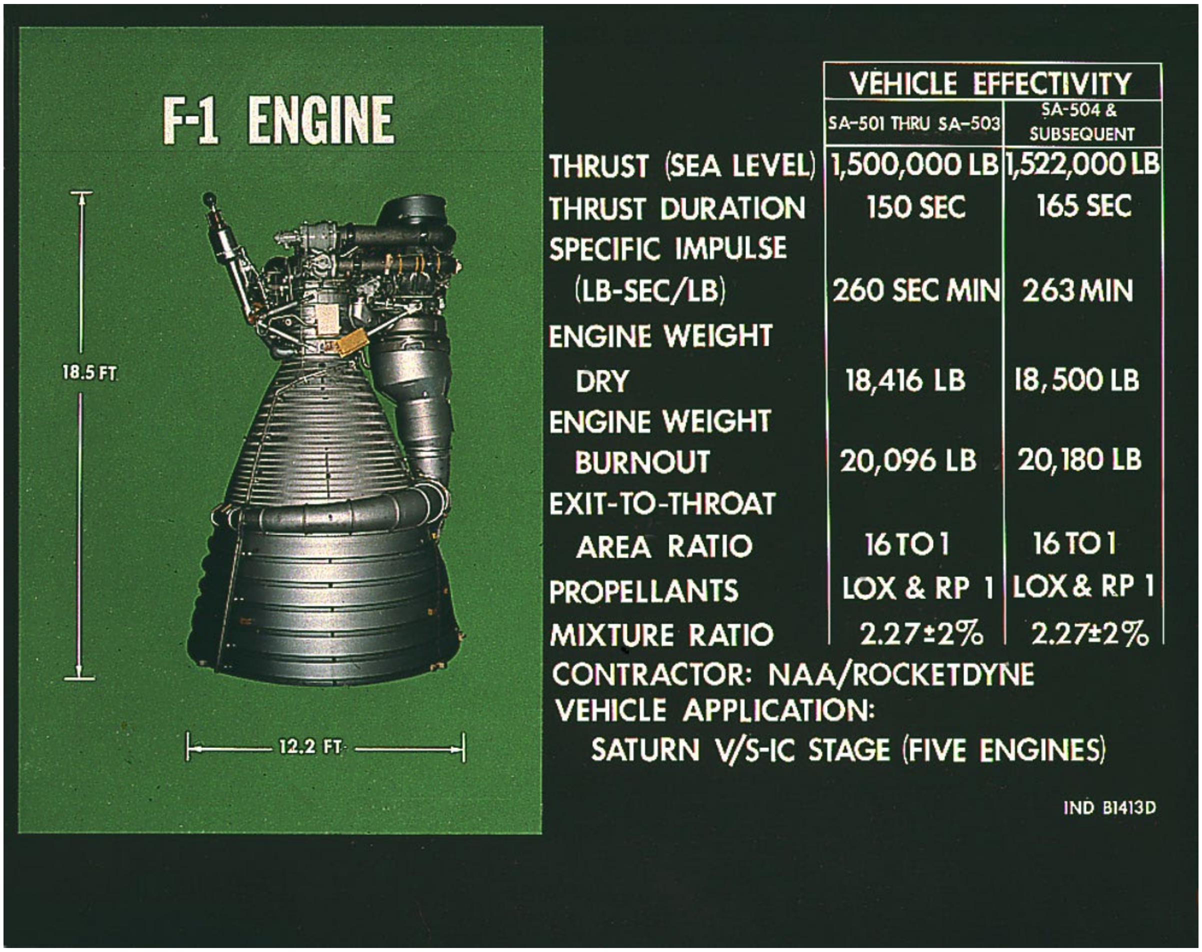
rocket →



ideally expanded @ a low P

Example rocket engines (comparison of first stage, second stage, and satellite engines):

First stage rocket engine: (Rocketdyne F-1)



Example: Rocket nozzle exit Mach number

Given: The rocket engine shown above, with an **exit-to-throat area ratio of 16 to 1**.

To do: Calculate the Mach number at the exit plane. Assume that $\gamma = 1.33$, typical of rocket exhaust.

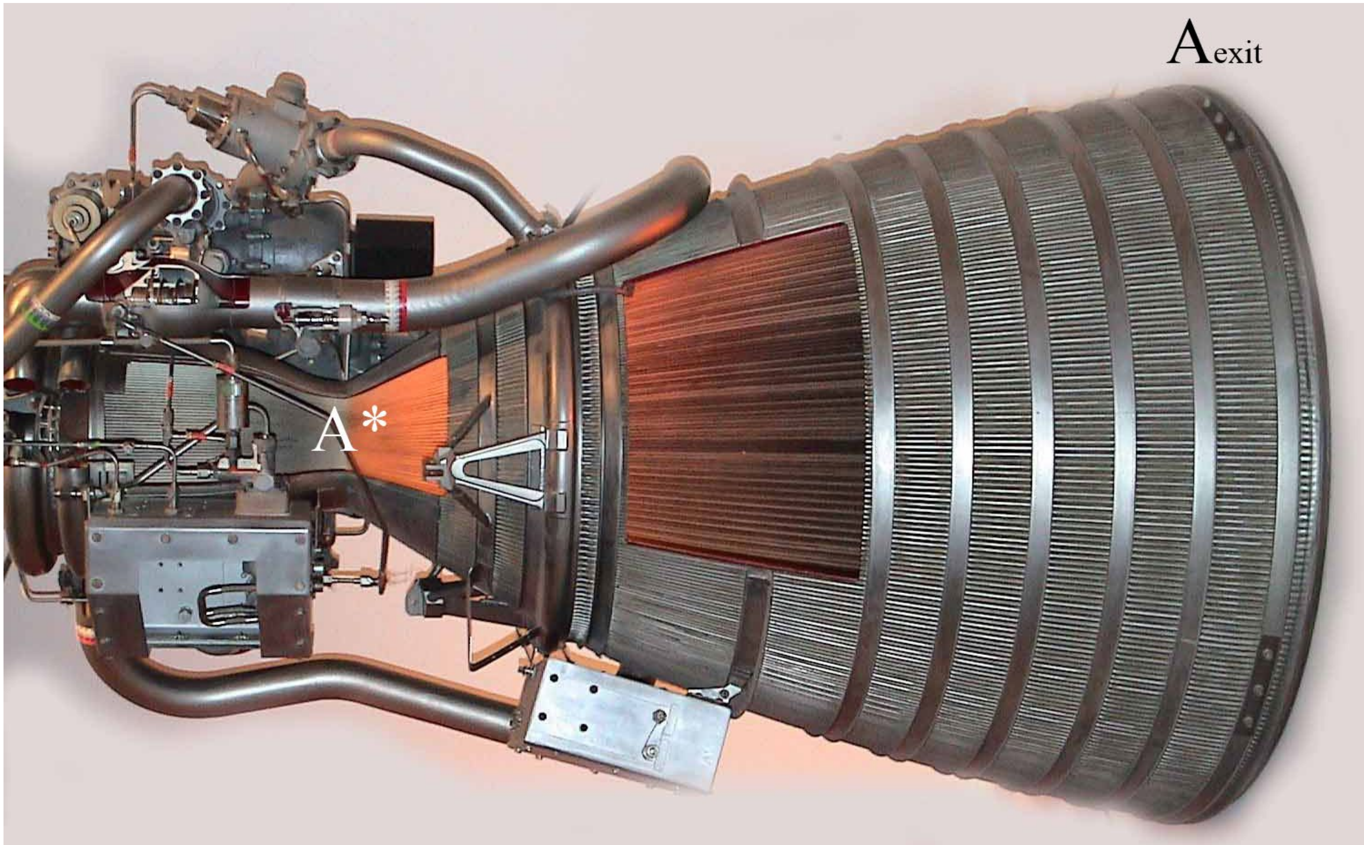
Assumptions and Approximations: The exhaust gas is ideal with $\gamma = 1.33$. The flow is steady and can be approximated as isentropic, adiabatic, and one-D.

Solution:

Use $\boxed{\dfrac{A}{A^*} = \dfrac{1}{M}\left[\left(\dfrac{2}{\gamma+1}\right)\left(1+\dfrac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$ at the exit plane to calculate M_e.

$$\boxed{M_e = 4.13}$$

Second stage rocket engine:



Pratt &Whitney RL-10 rocket motor designed for a specific M_{exit} (photographed at the National Air & Space Museum). 1960-vintage, $\boldsymbol{M_{exit}}$ **= 5**, γ = 1.33, thrust = 15,000 lbf, $D_e \sim 1$ m, used in the Saturn IV **2nd stage**.

Example: Rocket nozzle exit area ratio

Given: The rocket engine shown above, with an exit Mach number of 5.

To do: Calculate the exit-to-throat area ratio.

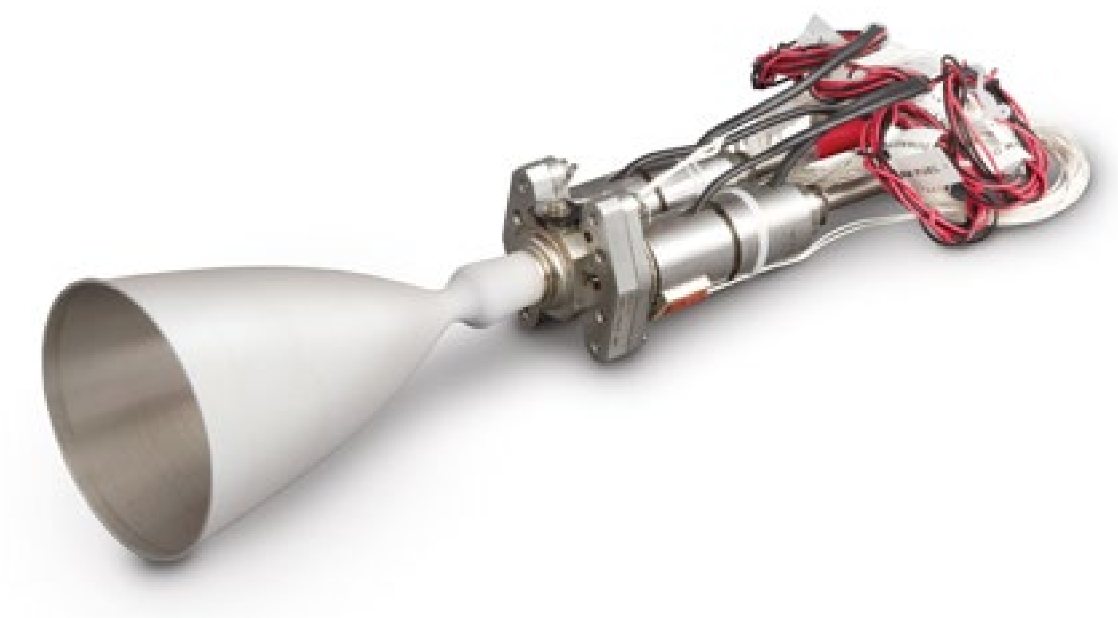
Assumptions and Approximations: The exhaust gas is ideal with γ = 1.33. The flow is steady and can be approximated as isentropic, adiabatic, and one-D.

Solution:

Use $\boxed{\dfrac{A}{A^*} = \dfrac{1}{M}\left[\left(\dfrac{2}{\gamma+1}\right)\left(1+\dfrac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}}$ at the exit plane to calculate A_e/A^*.

let $M_e = 5$ → solve $\boxed{\dfrac{A}{A^*} = 37.4}$

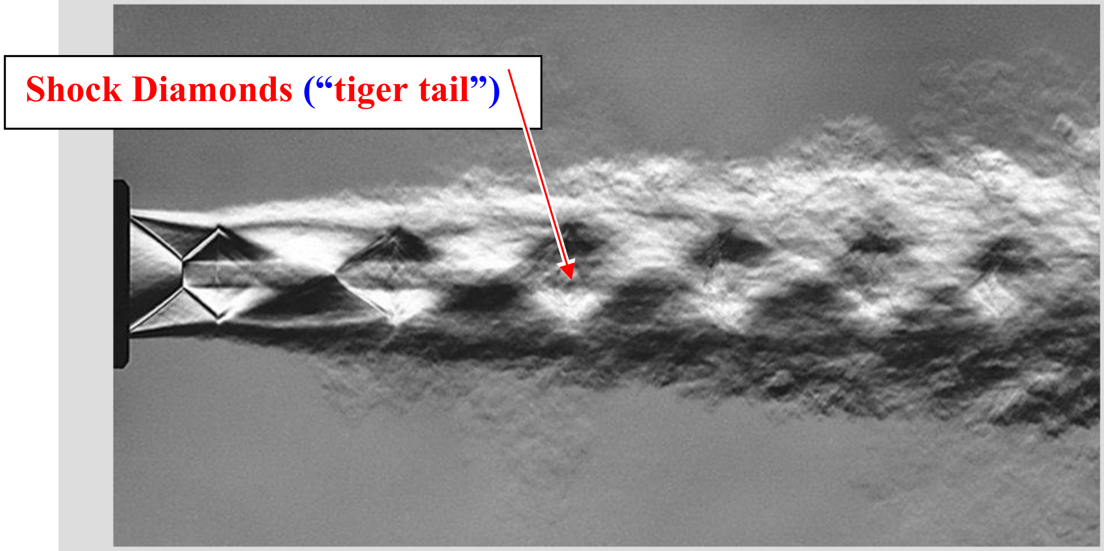
Satellite thruster engines:



Notice the huge area ratio – can be as high as 150! These engines are designed to operate in near vacuum (nearly zero) back pressure in space.

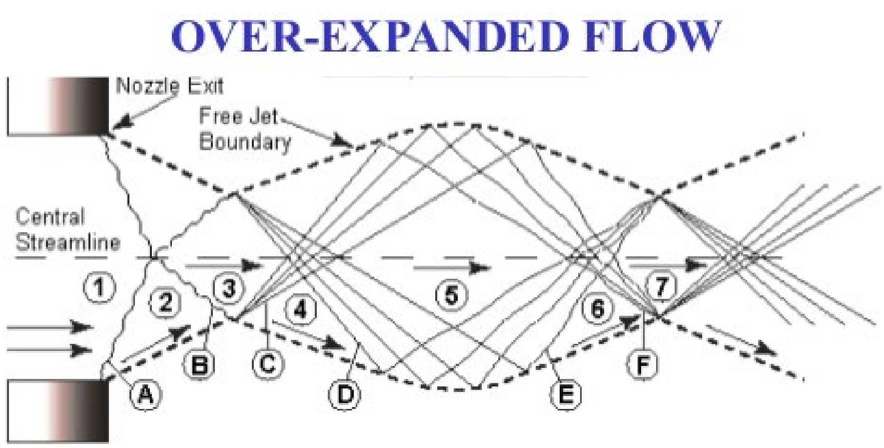
$$\frac{A_e}{A^*} = 150 \; !$$

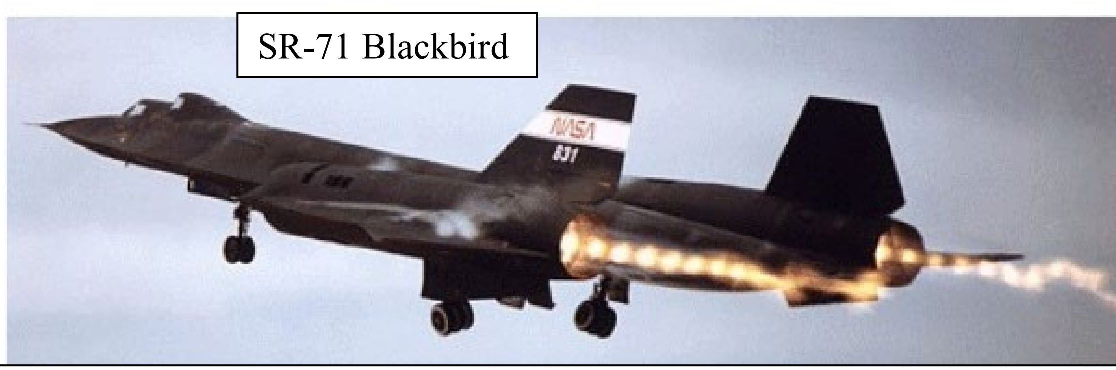
Over-expanded nozzles:



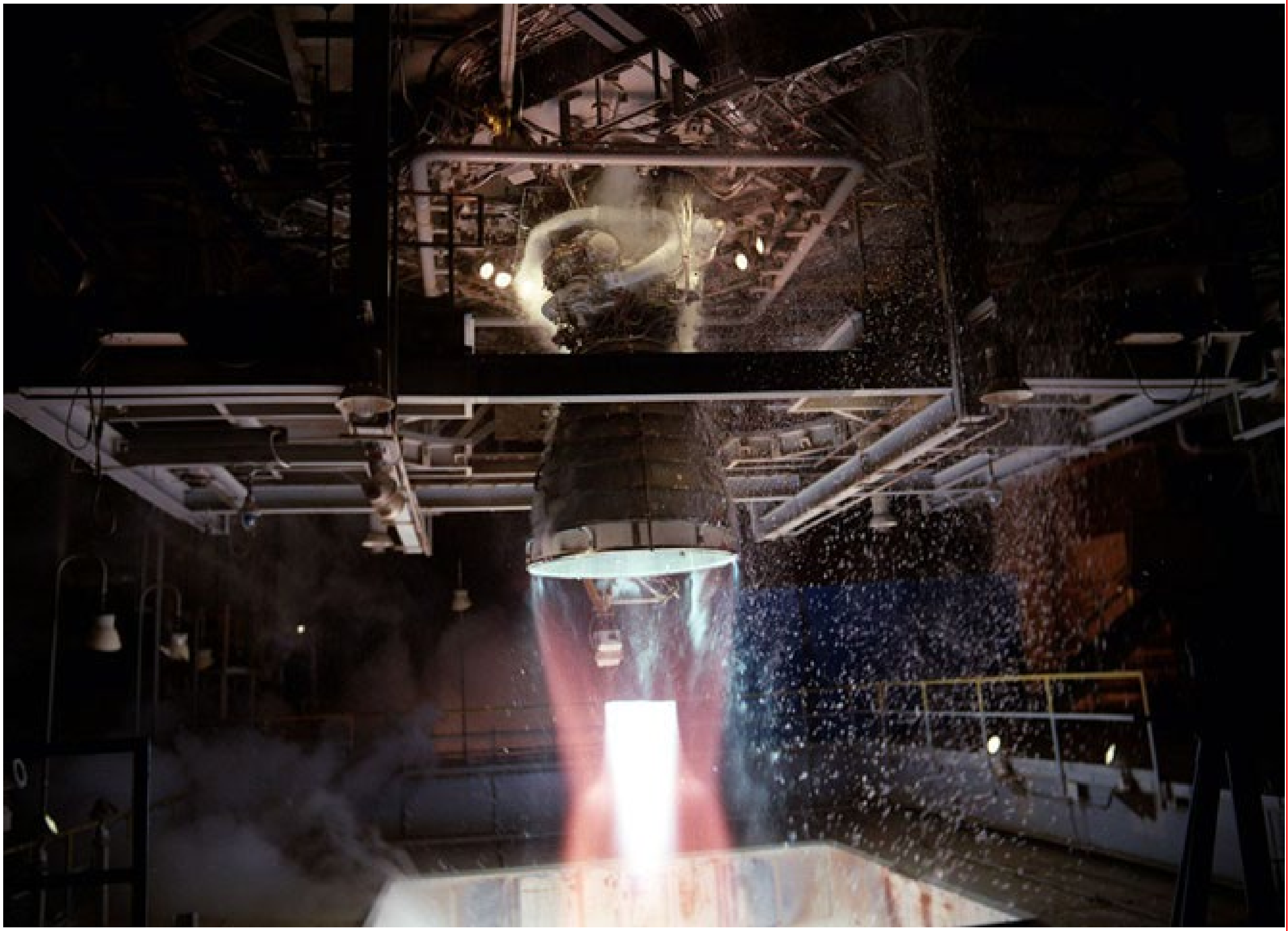
The complex interactions between shock waves and expansion waves in an "overexpanded" supersonic jet. The flow is visualized by a schlierenlike differential interferogram.

From https://slideplayer.com/slide/4179827/ .





From https://www.slideshare.net/sabirahmed796/nozzles-45206272 .



Rocket engine on test stand. Notice the very over-expanded conditions.

More shock diamonds:

Example – Speed of a high speed jet aircraft

Given: The SR-71 travels at $M = 3.2$ at 24 km altitude (80,000 ft)!

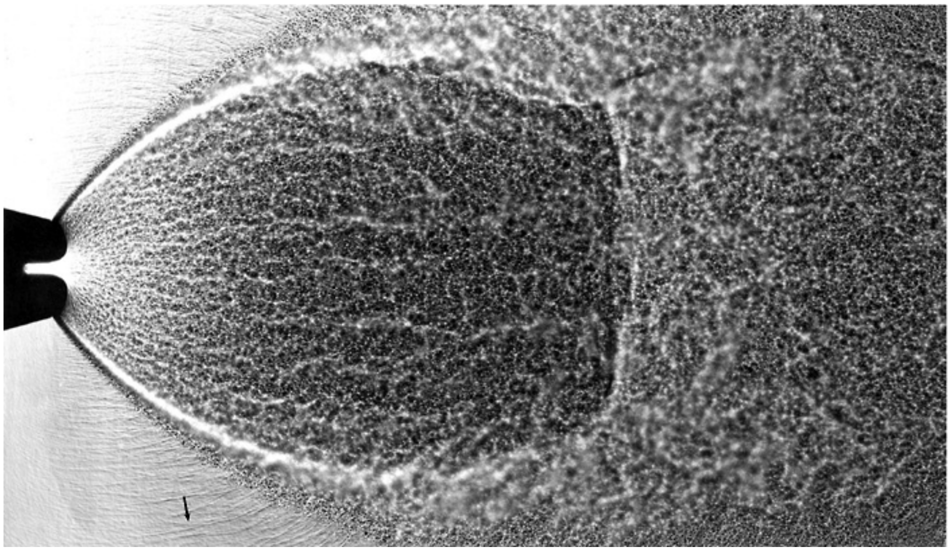
To do: Calculate its air speed.

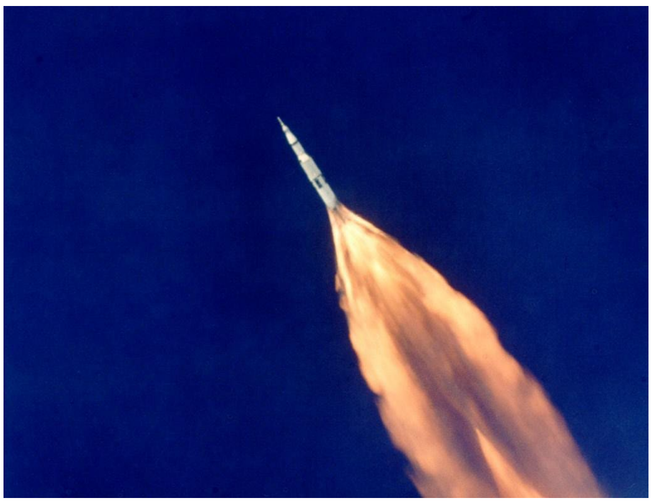
Solution:

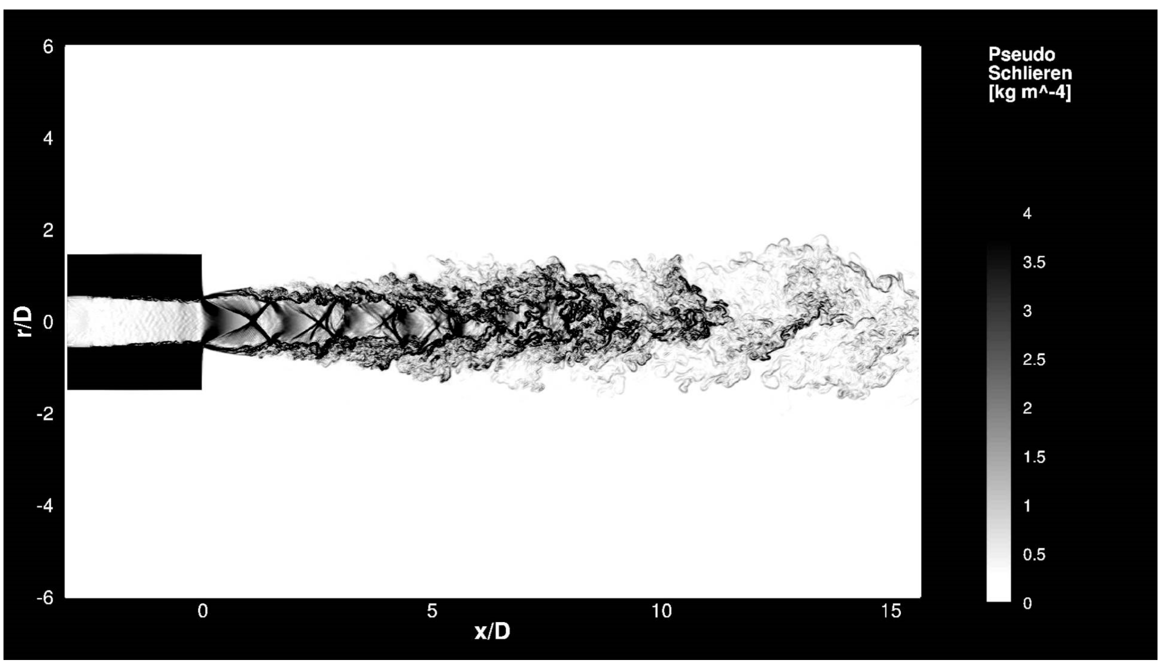
- From standard atmosphere table, $T_{24\text{ km}} = -69.7°\text{F} = 217\text{ K}$.
- Air: $\gamma = 1.4$ and $R_{air} = 287\text{ m}^2/(\text{s}^2\cdot\text{K})$, $a = (\gamma RT)^{1/2} = 295\text{ m/s}$.
- Thus, $V = M \cdot a = 3.2(295\text{ m/s})$ $\boxed{= 944\text{ m/s } (= 2110\text{ mph})!}$

Beautiful Shock Diamonds!

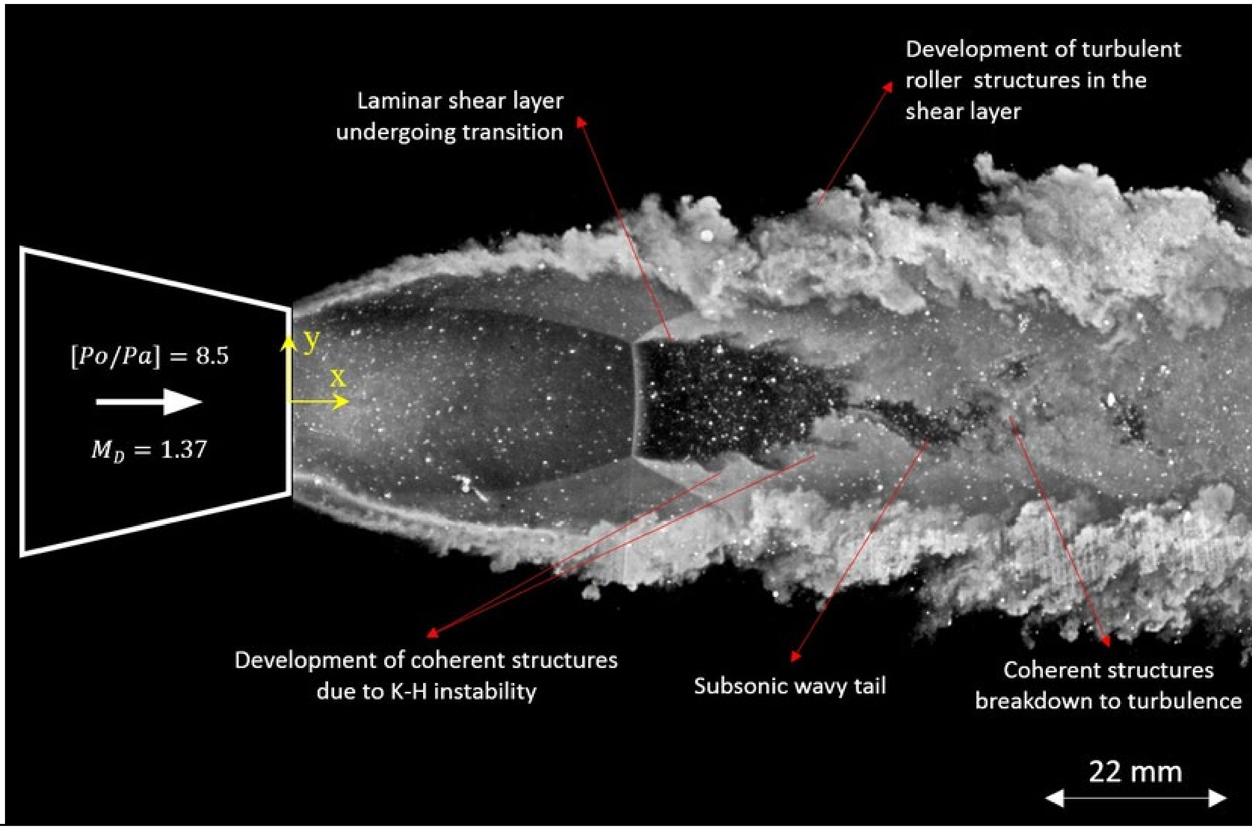
Under-expanded Nozzles:







Supersonic underexpanded jet at M_e = 1.55 (pseudo Schlieren image from DNS calculations). From https://www.youtube.com/watch?v=OTfQ-CRnUlk .



From https://www.researchgate.net/publication/299637500_Visualization_of_supersonic_free_and_confined_jet_using_planar_laser_mie_scattering_technique/figures?lo=1 .