Today, we will:

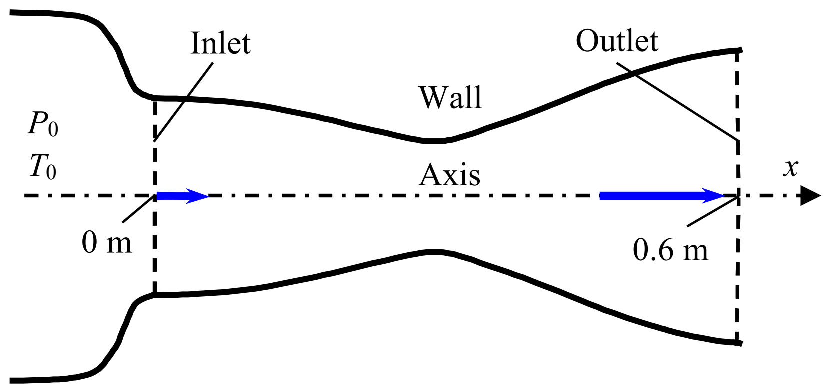
- Complete the example problem from last lecture – Part (b)
- Begin a ***qualitative*** discussion of normal shocks

Continuation of the example from last lecture:

Example: Converging-Diverging nozzle

Given: Air flows from a very large tank through a converging-diverging nozzle. The test section begins at $x = 0$. The outlet of the test section is at $x = 0.60$ m, where it is exposed to back pressure $P_b = 50.0$ kPa. In the tank,

- $P_{0,\text{inlet}} = 220$ kPa (absolute)
- $T_{0,\text{inlet}} = 300$ K



The cross-sectional area is known as a function of axial distance x. (see previous lecture)

(a) To do: For isentropic flow through the converging-diverging nozzle (no shocks), calculate and plot Mach number and nondimensional pressure $P/P_{0,\text{inlet}}$ as functions of x.

Solution: We assumed that the back pressure is low enough such that the flow in the C-D nozzle is isentropic everywhere (no shock). Numbers from last lecture:

We calculate $P_b/P_{0,\text{inlet}} = 50/220 = 0.2273$ – we *assume* this back pressure is low enough that the flow is supersonic through the entire diverging section of the nozzle, without any normal shocks in the nozzle. *Is this assumption true?*

For case E of our notes, a normal shock sits right at the exit plane of the CD nozzle. We calculated that downstream of this shock, $P_2 = 93.89$ kPa. So, as long as the actual P_b is less than this value of P_2, we are assured that our assumption was indeed correct.
Here, $P_b = 50.0$ kPa $< P_2 = 93.89$ kPa. ***Thus, we have verified our assumption.***

(b) To do: Calculate the average air speed at the throat and at the exit plane. Also calculate the mass flow rate through this CD nozzle.

Solution: To be completed in class.

At throat, $M = 1$ $\quad M = \frac{V}{a} \rightarrow \boxed{V = Ma}$

set $M = 1$

$a = \sqrt{\gamma R T}$ $\quad T = \frac{T}{T_0} T_0 = \left(1 + \frac{\gamma - 1}{2} M^2\right)(T_0)$ — 300 K

$T = 250.0\ K$ @ throat

T^*

$\therefore\ V = Ma \rightarrow V_{throat} = a^*$

$$V = \sqrt{\gamma R T^*} = \sqrt{(1.4)\left(287.0\ \frac{m^2}{s^2 \cancel{K}}\right)(250\ \cancel{K})}$$

$$\boxed{V_{throat} = 316.94\ \frac{m}{s}}$$

$\dot{m} \longrightarrow \dot{m} = \dot{m}_{max}$ = SAME EQ AS PREVIOUSLY FOR CONVERGING DUCT

$$\boxed{\dot{m}_{max} = P_0 A^* \sqrt{\frac{\gamma}{R T_0}} \left(\frac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}}$$

OR $\boxed{\dot{m} = \rho V A}$ — calc ρ @ throat

$\rho = \rho^*$

either way

$$\boxed{\dot{m} = 9.0722\ \frac{kg}{s}}$$

TRY ON YOUR OWN

Now $\boxed{\text{Calc } V_e}$ @ exit plane

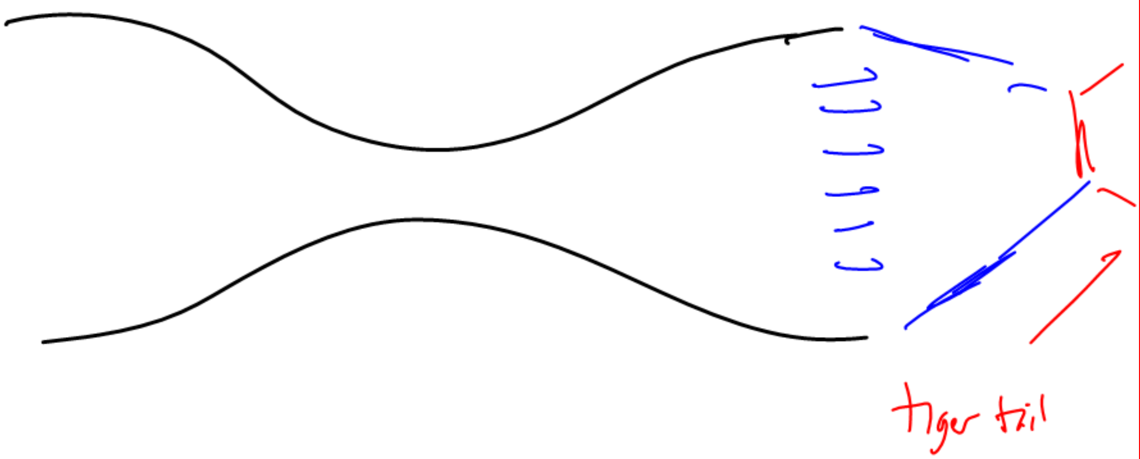
From table (last time) $\rightarrow$ $M_e = 2.4683$

$P_e = 13.5271$ for ideal expansion (P_F) condition (F)

Our $P_b = 50.0\ kPa > P_F$

$P_F < P_b < P_E$

THIS IS THUS $\boxed{\text{OVEREXPANDED}}$



$$T_e = \frac{T_e}{T_0} T_0 = \left(1 + \frac{\gamma - 1}{2} M_e^2\right)^{-1} (300\,K) = 135.226\ K$$

$$a_e = \sqrt{\gamma R T_e} = \sim 233.097\ \frac{m}{s}$$

$$\underline{\underline{V_e = M_e a_e}} = \boxed{575.35\ \frac{m}{s}} \qquad \boxed{V_e = 575.\ \frac{m}{s}}$$

<u>OR</u> We know $\dot m$ $\quad \underline{\dot m = \rho V A}$ anywhere in the C-D nozzle

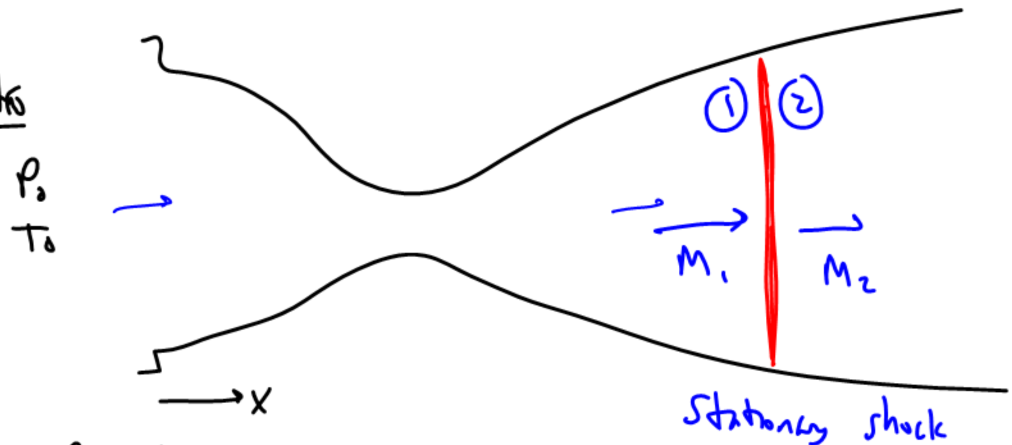
$$\rho_e = \frac{\rho_e}{\rho_0}\rho_0 = \left(1+\frac{\gamma-1}{2}M_e^2\right)^{-\frac{1}{\gamma-1}}\left(\frac{p_0}{RT_0}\right)$$

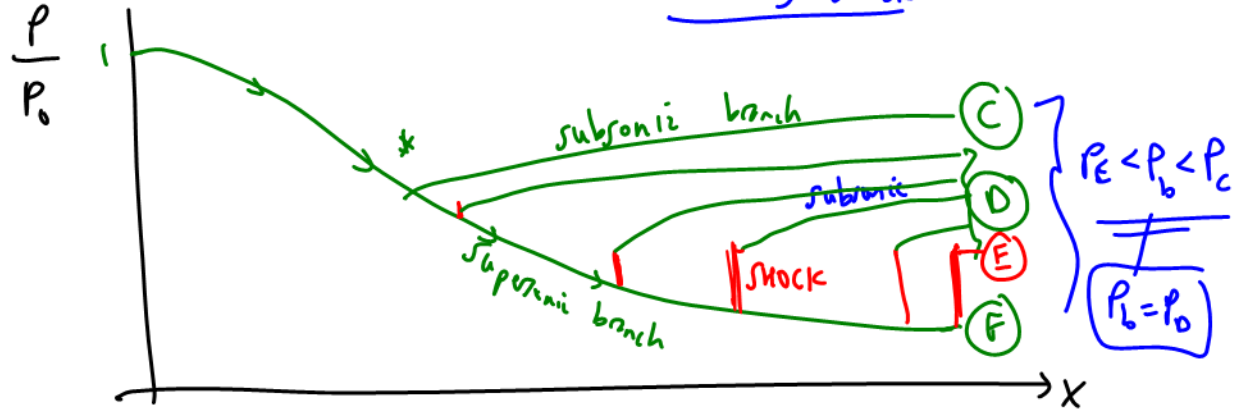
Isentropic

$$\boxed{V_e = \frac{\dot m}{\rho_e A_e}} = \sim = \boxed{575.35\ \frac{m}{s}}$$

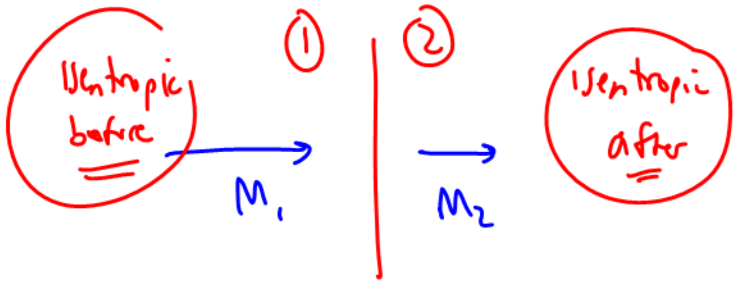
NORMAL SHOCKS

• <u>Intro</u>





$$\boxed{\text{WE ARE AT CONDITION D}}$$



- $\boxed{M_1 > 1}$ supersonic $\qquad \boxed{M_2 < 1}$ subsonic
- Normal shock is nonisentropic $\boxed{S_2 > S_1}$ ☆
- The location of the shock depends on P_b

$P_b \downarrow$
- if $P_b = P_c$ — no shock
- if $P_b < P_c < P_E$ → shock moves downstream & gets stronger as $M_1 \uparrow$

$P_b \downarrow$

$\boxed{\text{"shock strength"} \uparrow \text{ as } M_1 \uparrow}$

e.g. $\dfrac{P_2}{P_1}$

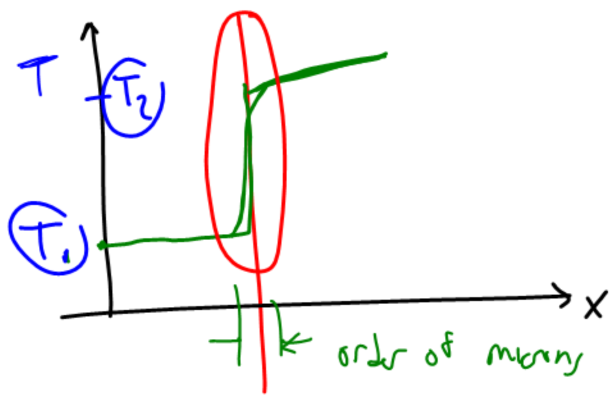
- If $P_b = P_E$ → shock @ exit plane

- $\boxed{P_2 > P_1}$ across a shock

• Shock is very thin (μm – microns)

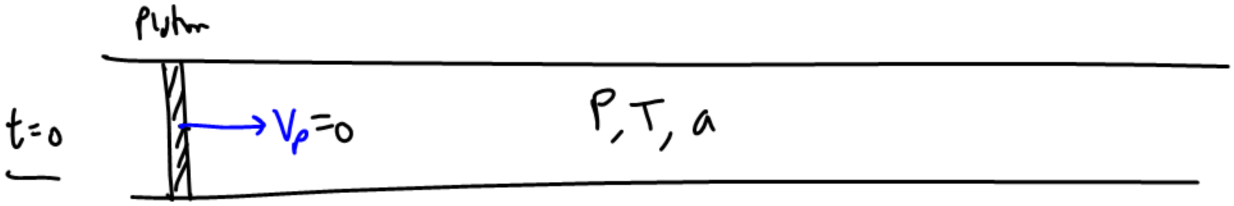
We approx. as sudden change

• $\boxed{T_2 > T_1}$ → similar behavior as p



"THOUGHT EXPERIMENT"

Long tube



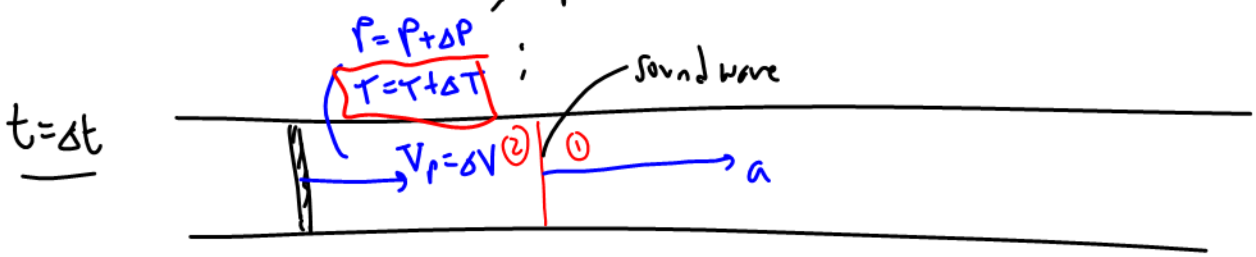
Imagine piston accelerates step-wise

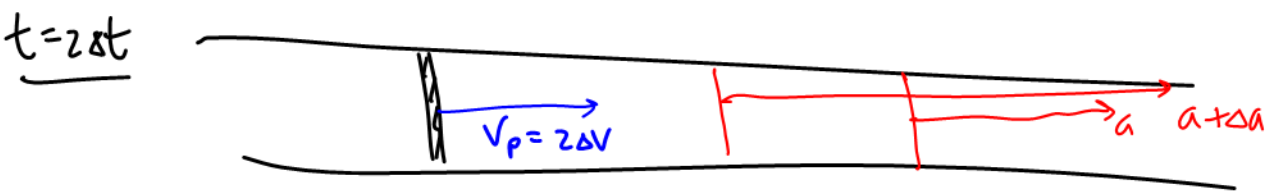
@ $t=0$, $V_p=0$

$t=\Delta t$, $V_p=\delta V$

$t=2\Delta t$, $V_p=2\delta V$

⋮

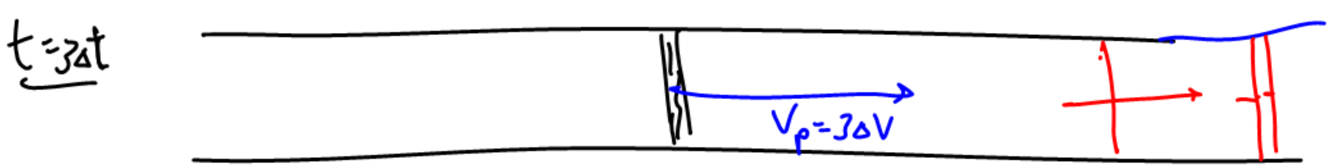




moves faster than 1st one since T ↑

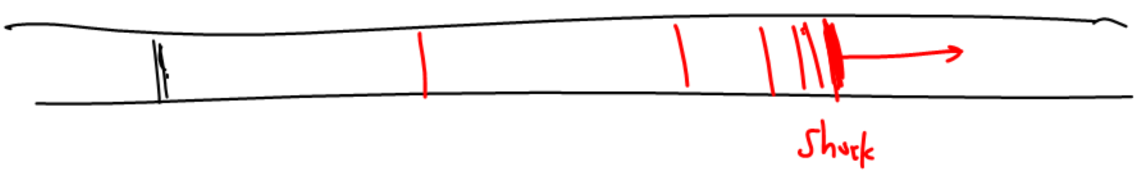
$\therefore a=\sqrt{\gamma RT}$ ↑

⋮

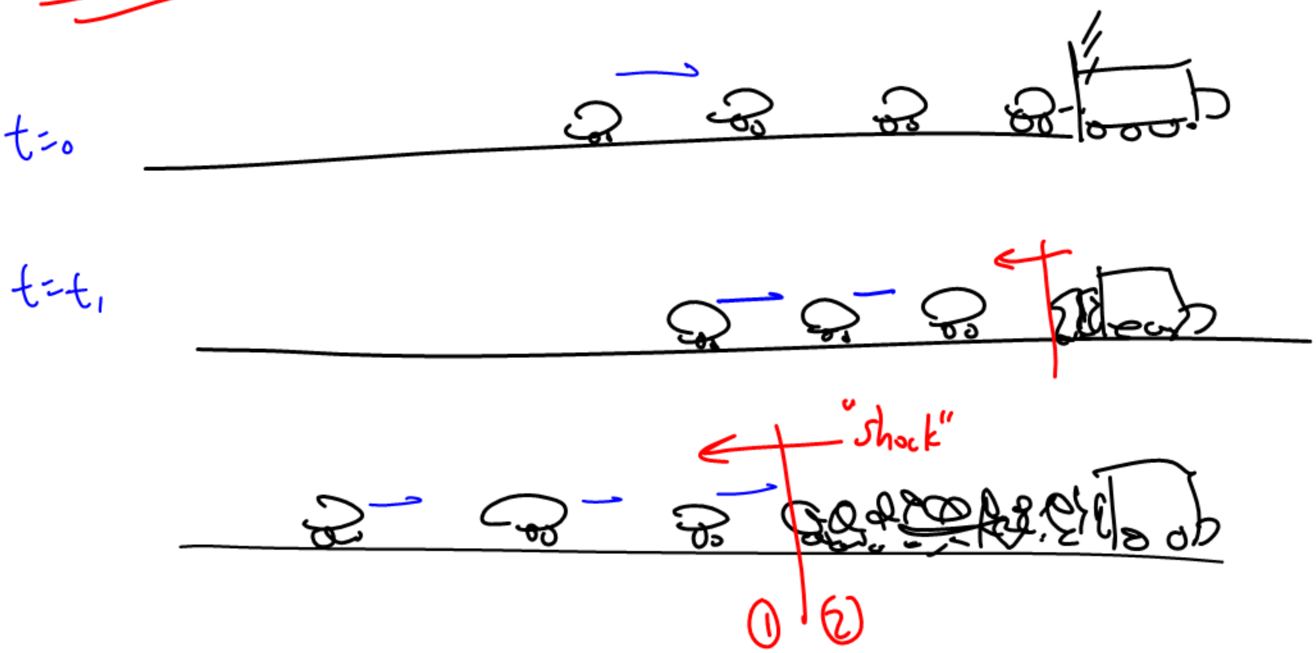


⋮

Farther away



ANALOGIES: "Traffic shock"

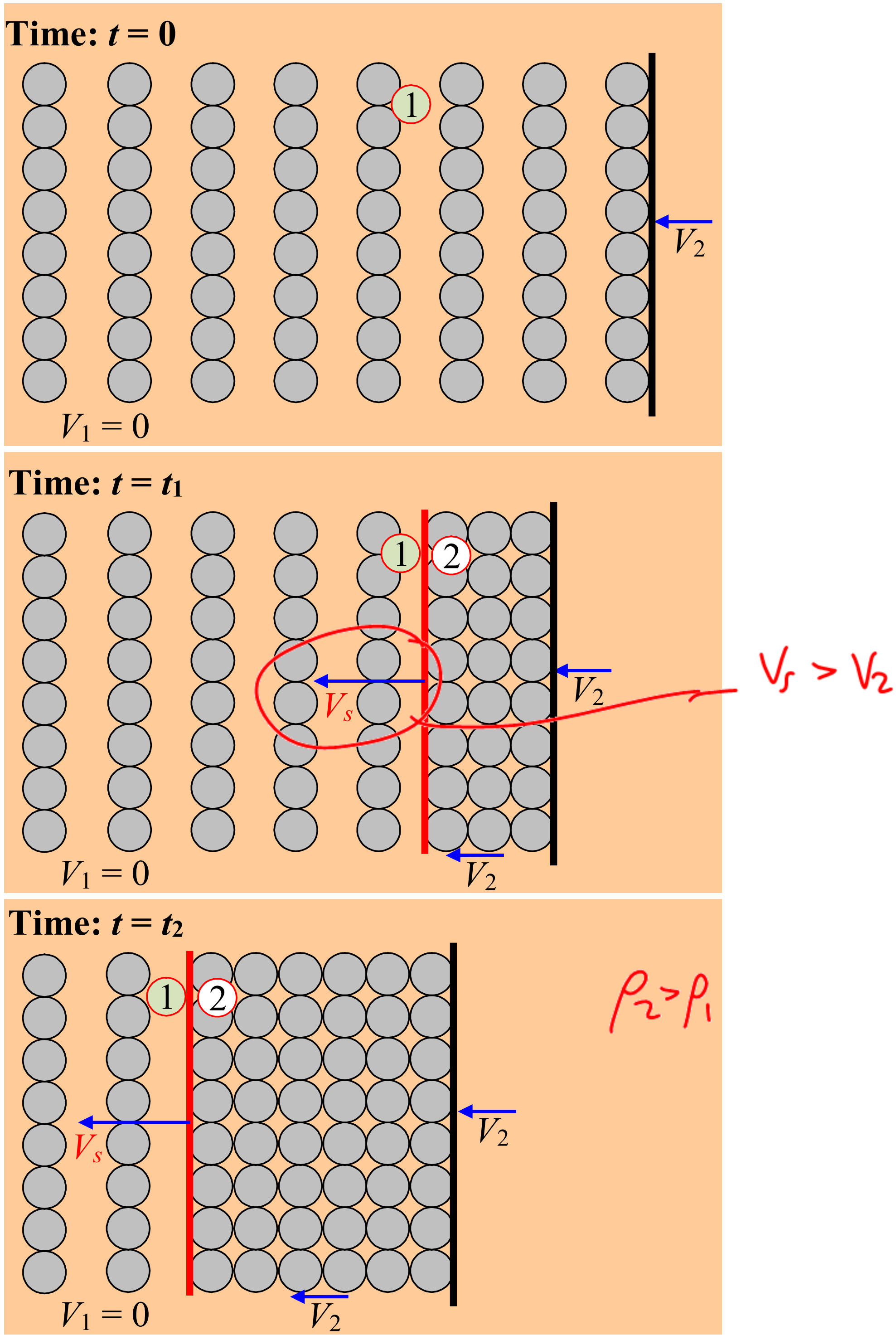


The **skier pile-up analogy** (similar to the traffic shock analogy):

with the local sound speed c ($c^2 = (\partial P/\partial \rho)_S$).

Photo from David Drewniak, from https://www.scribd.com/document/208788775/Shock-waves-vs-sounds-waves

The "*dime analogy*" (model a moving shock as rows of dimes that pile up when pushed by a rod or "piston" as sketched; three sequential times):



Comments:

- The vertical red line is analogous to a shock wave: $V_1 = 0$, $V_s > V_2$, $\rho_2 > \rho_1$ (there is sudden increase in density, and the "wave front" of dimes moves faster than the piston).
- The dimes in region 1 don't "know" anything is happening until the shock hits them.
- Similarly in a shock wave in air, the air in region 1 does not "know" anything is happening until the shock wave hits it.