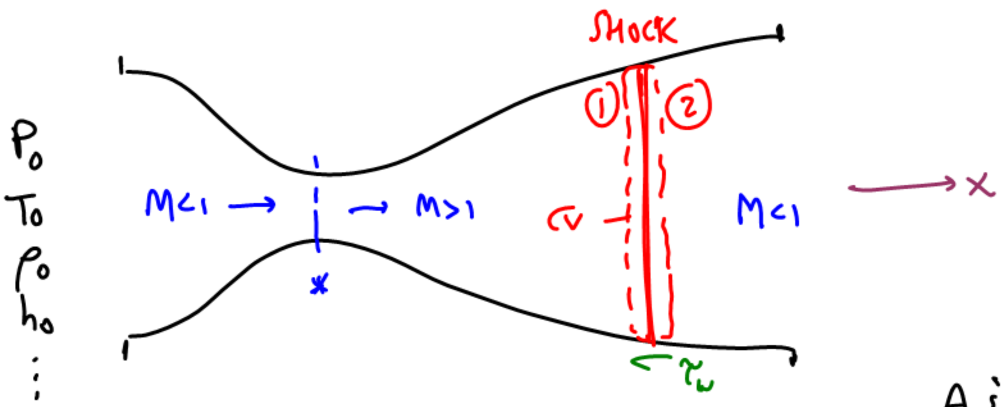
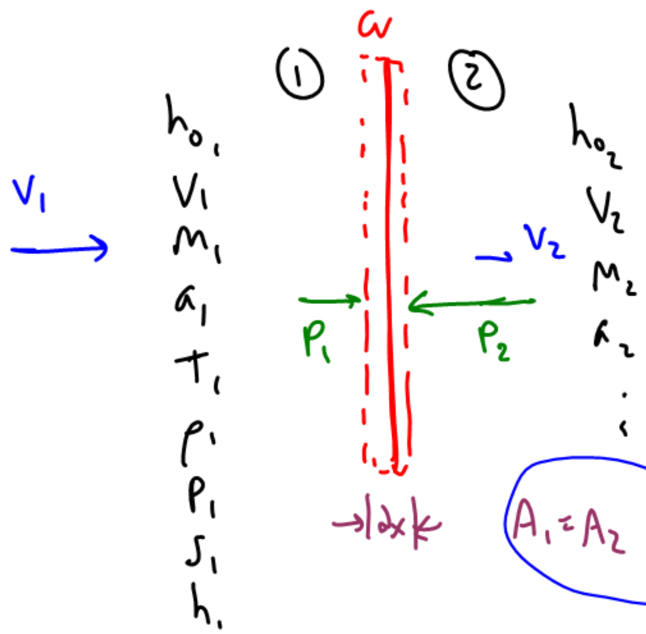
Today, we will:

- Begin our ***quantitative*** analysis (equations) of normal shocks in an ideal gas
- Discuss **Fanno curve** its significance to normal shocks
- If time, discuss what to expect for **Exam 1** (this Friday in class)

Quantitative analysis of normal shocks (for ideal gases):





$A_1 = A_2$ as $dx \to 0$

A & A:

- One-D
- ideal gas
- Steady (in this F.O.R.) stationary shock
- adiabatic
- isentropic BEFORE & AFTER the shock [NOT ISENTROPIC ACROSS shock]

CONS. LAWS FOR THIS CV

- Cons of mass:

$$\dot{m}_1 = \dot{m}_2 = \rho_1 V_1 \cancel{A_1} = \rho_2 V_2 \cancel{A_2}$$

$$\boxed{\rho_1 V_1 = \rho_2 V_2} \quad (1)$$

· cons of energy: adiabatic & steady & no pump or turbine

$$h_{o_1} = h_{o_2} \Rightarrow \boxed{h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}} \quad (2)$$

STILL HOLDS ACROSS A SHOCK!

· Linear Momentum (in x direction)

ME 320:

$$\boxed{\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}} \quad (3)$$

mom. flux correction factor

In x-direction

Struts, cable

$$\sum F_x = \sum F_{x\,pressure} + \cancel{F_{x\,gravity}} + \cancel{F_{x\,viscous}} + \cancel{F_{x\,other}}$$

$$(P_1 - P_2)A \qquad -\tau_w\, dx\,(\text{Perimeter}) \qquad dx \to 0$$

$$= \beta_2 \dot{m} V_2 - \beta_1 \dot{m} V_1$$

$\beta_2 = 1$, $\beta_1 = 1$

$\beta = 1$ for uniform one-D flow

ignore BLs $\therefore$ $\beta = 1$

(3) becomes $\boxed{(P_1 - P_2)A = \dot{m}(V_2 - V_1)}$ (4)

$\dot{m} = \rho V A$

$$(P_1-P_2)\cancel{A} = \underbrace{\rho_2 V_2 \cancel{A_2} V_2 - \rho_1 V_1 \cancel{A_1} V_1}$$

$$P_1 - P_2 = \rho V_2^2 - \rho V_1^2 \rightarrow \boxed{P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2} \quad (5) \quad *$$

looks like Beloved Bernoulli, but is a very different eq. !!

IDEAL GAS:

$$\boxed{P = \rho R T} \quad (6)$$

$$\boxed{h = C_p T} \quad (7)$$

$$\boxed{S_2 - S_1 = C_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}} \quad (8)$$

$$\boxed{S_2 - S_1 = C_v \ln \frac{T_2}{T_1} - R \ln \frac{\rho_1}{\rho_2}} \quad (9)$$

$$a = \sqrt{\gamma R T} \qquad M = \frac{V}{a} \quad (10)$$

Lets "manipulate" & combine eqs (1)-(10) to get our results

Start w/ $\boxed{h_{0_1} = h_{0_2}}$

$$\underbrace{C_p T_{0_1} = C_p T_{0_2}} \rightarrow \boxed{T_{0_1} = T_{0_2}} \qquad \boxed{\frac{T_{0_2}}{T_{0_1}} = 1} \quad (11)$$

* STAGNATION TEMP DOES NOT CHANGE

① | ②

T_{0_1} | T_{0_2} $\qquad T_{0_1} = T_{0_2}$

isentropic approx $\quad S_1$ | $S_2 \quad$ isentropic approx $\qquad (S_2 > S_1)$

$$\frac{T_{0_1}}{T_1} = 1 + \frac{\gamma-1}{2} M_1^2 \qquad \text{(a)}$$

$$\frac{T_{0_2}}{T_2} = 1 + \frac{\gamma-1}{2} M_2^2 \qquad \text{(b)}$$

$$\frac{\cancel{T_{0_1}}}{T_1}\frac{T_2}{\cancel{T_{0_2}}} = \boxed{\frac{T_2}{T_1} = \frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}} \qquad (12)$$

☆ Temp ratio across a normal shock

We don't know M_2

Always use abs. T

$(P = \rho R T)$

· IDEAL gas & isentropic

$$\frac{P_2}{P_1} = \frac{\rho_2}{\rho_1}\frac{T_2}{T_1} \qquad \left[= \left(\frac{T_2}{T_1}\right)^{-1/2}\right]$$

$$\text{Eq.(1)} \rightarrow \frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{a_1 M_1}{a_2 M_2} = \frac{\sqrt{\gamma R T_1}}{\sqrt{\gamma R T_2}}\frac{M_1}{M_2} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{2}}\frac{M_1}{M_2}$$

$$\boxed{\frac{P_2}{P_1} = \frac{M_1}{M_2}\sqrt{\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}}} \qquad (13)$$

$$\sqrt{T_2/T_1}$$

☆ FANNO EQ. → ☆

Eq (13) is called the FANNO CURVE (FANNO LINE)

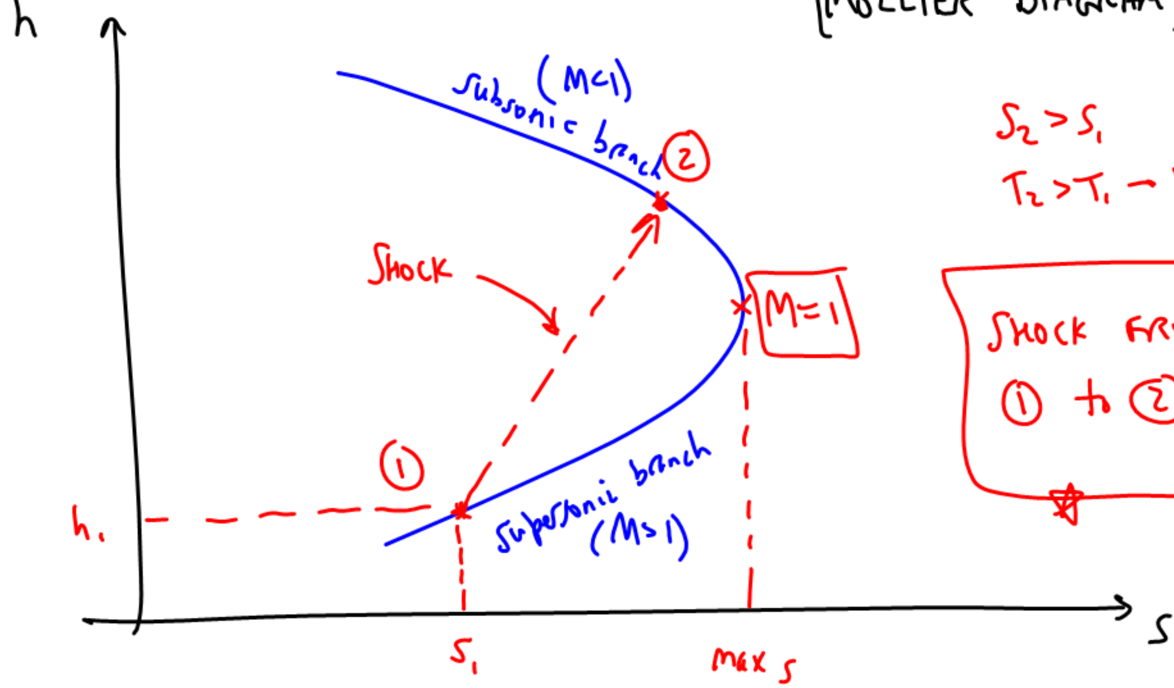
Comments:

- Fanno eq comes from cons. of mass & cons. of energy (ONLY)

WE DID NOT USE CONS. OF MOMEN.

- Still not useful since we don't know M_2

(MOLLIER DIAGRAM)



$S_2 > S_1$

$T_2 > T_1 \to h_2 > h_1$

SHOCK FROM ① to ② ☆

For some known ① $\longrightarrow$ know h_1 & s_1

($M_1 > 0$)

FANNO CURVE = locus of all possible states that have the same mass flow $\rho_1 V_1 = \rho_2 V_2$ (per or unit depth)

& same specific total enthalpy (energy eq.)

☆ WE NEED THE MOMEN. EQ TO FIGURE OUT WHERE ② IS ON THIS CURVE