Today, we will:

- Continue our ***quantitative*** analysis of normal shocks – derivation of equations for stationary normal shocks in an ideal gas
- Discus and compare **Fanno curve** and **Rayleigh curve** and their significance to normal shocks
- Do some example problems – normal shocks

Previous lecture: we derived the **Fanno equation** across a stationary normal shock, 1-D flow, ideal gas:

$$\boxed{\frac{P_2}{P_1} = \frac{M_1}{M_2}\sqrt{\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}}} \quad (13)$$

we don't know M_2

Note that the Fanno equation was obtained by combining

- conservation of mass $\quad \rho_1 V_1 = \rho_2 V_2 \quad$ (1)
- conservation of energy $\quad h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \quad$ (2)

We did not yet use the linear momentum equation $\quad P_1 - P_2 = \rho_2 V_2^2 - \rho_1 V_1^2 \quad$ (5)

Across a shock

HW 5 → Plot Fanno curve

Give M_1, T_1, s_1

In Excel

M_2	P_2/P_1 (Eq 13)	T_2/T_1	T_2	$h_2 = c_p T_2$	s_2
0.1					
0.15					
0.2					
correct					

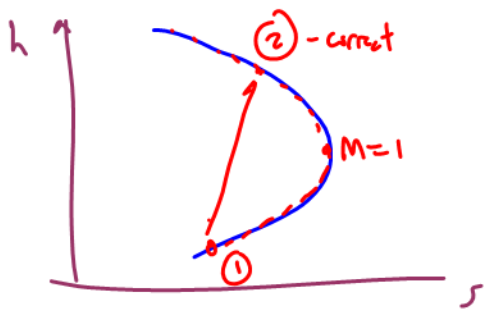
① | ②

Not $\frac{T_2}{T_1}$ across a shock

T-ds eq. Not isentropic

Use fundamentals (ideal gas law, $a = \sqrt{\gamma R T}$, etc)

Get $\frac{T_2}{T_1} = \text{fnc}\left(\frac{P_2}{P_1}, \frac{M_2}{M_1}, \gamma\right)$



Mom Eq (5)

$$\boxed{P_1 - P_2 = \rho_2 V_2^2 - \rho_1 V_1^2} \qquad (5)$$

← we used cons of mass to get Eq (5)

"Manipulate" to get $\frac{P_2}{P_1}$

$$= \frac{P_2}{RT_2}(M_2 a_2)^2 - \frac{P_1}{RT_1}(M_1 a_1)^2$$

$a = \sqrt{\gamma R T}$ →

$$P_1 - P_2 = P_2 \gamma M_2^2 - P_1 \gamma M_1^2$$

$$P_1 (1 + \gamma M_1^2) = P_2 (1 + \gamma M_2)^2$$

$$\boxed{\frac{P_2}{P_1} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}}$$

☆ Rayleigh Eq. (14)

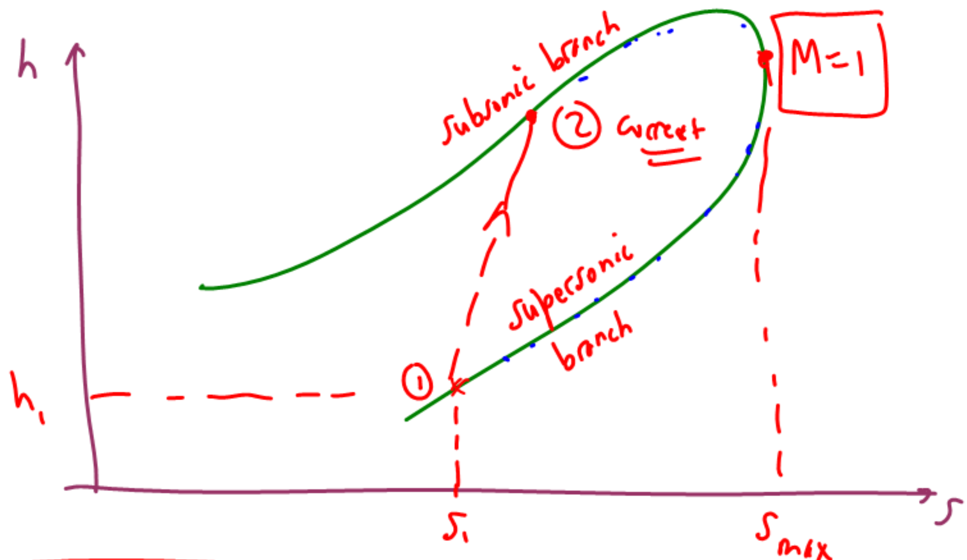
plot it → "Rayleigh Curve"

Alternate eq for $\frac{P_2}{P_1}$ across a shock

RAYLEIGH → we used mass & momentum

FANNO → we used mass & energy

The Rayleigh Curve



Rayleigh curve is the locus of all possible states that have the same mass flow (per unit area) — conserve mass & satisfy linear momentum eq. across a shock

(But do not necessarily satisfy the energy eq.)

Any point is a potential ② state, but only one that is correct — it's the one that also satisfies Fanno eq.

EXAMPLE Given: $M_1 = 2.60$ $\gamma = 1.40$ (air)

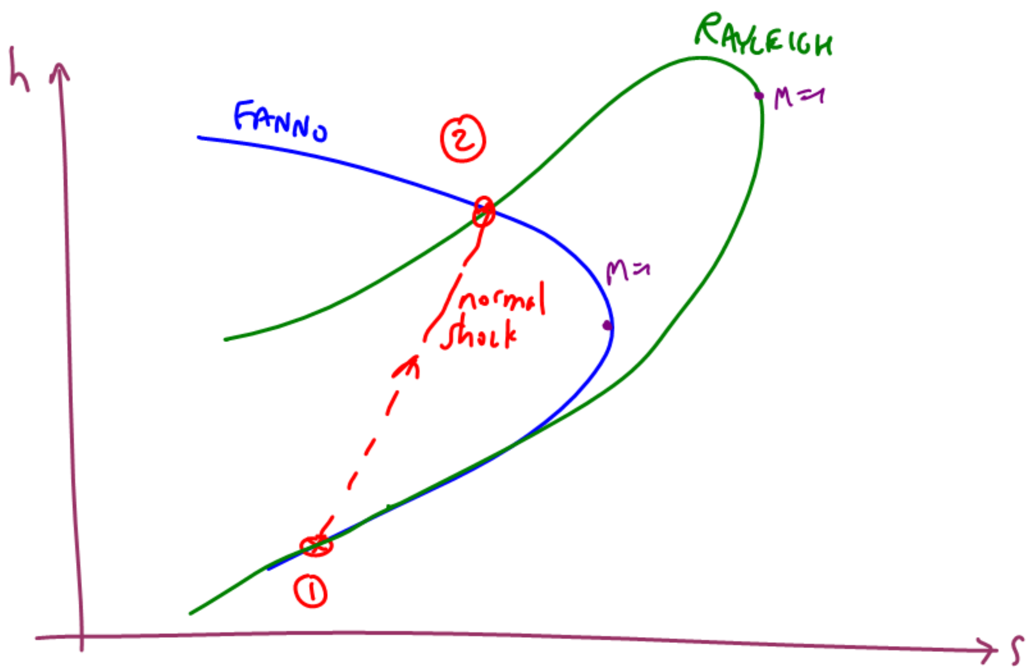
use calculator to get M_2 → $M_2 = 0.503871$, $\dfrac{P_2}{P_1} = 7.72000$

• Fanno → $\dfrac{P_2}{P_1} = \dfrac{M_1}{M_2}\sqrt{\dfrac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}} = 7.720000$ ✓

· Rayleigh $\quad \frac{P_2}{P_1} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2} = \underline{\underline{7.720000}}$ ✓ ☺

☆ <u>HOW TO GET M_2?</u> → $\boxed{\text{want } M_2 = \text{fnc}(M_1, \gamma)}$ ☆ goal

One way → set Fanno & Rayleigh together & solve for M_2



<u>Method 1</u> → set $\left.\frac{P_2}{P_1}\right)_{Fanno} = \left.\frac{P_2}{P_1}\right)_{Rayleigh}$ & solve for M_2

(Fanno) (Rayleigh)

$$\frac{P_2}{P_1} = \boxed{\frac{M_1}{M_2}\sqrt{\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2}} \qquad (15)$$

one eq, one unknown, M_2 → <u>solve for M_2</u>

Solve for M_2

$\Big\downarrow$ algebra, quadratic rule

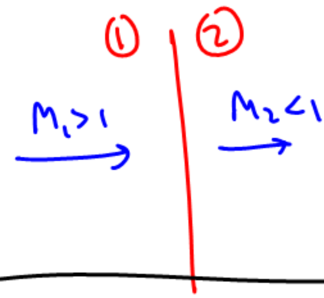
$$a M_2^4 + b M_2^2 + c = 0$$

(variable is M_2^2)

$$M_2^2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

2 roots → $\boxed{\text{one subsonic}}$ ← correct one

one supersonic ✗



Sol'n:

$$\boxed{M_2^2 = \frac{-B - \sqrt{-(B+2A)}}{1+\gamma B}}$$

where

$$\boxed{A = \frac{M_1^2\left(1 + \frac{\gamma-1}{2} M_1^2\right)}{\left(1+\gamma M_1^2\right)^2}} \qquad \boxed{B = 2\gamma A - 1}$$

(on eq sheet)

Test @ $M = 2.60$, $\gamma = 1.40$ (as we used for Fanno & Rayleigh)

Get → $\boxed{M_2 = 0.50387}$ ✓ ☺ → get $\frac{P_2}{P_1}$, $\frac{T_2}{T_1}$, ...