Today, we will:

- Review the equations across a normal shock
- Derive and discuss the Hugoniot equation and the Hugoniot curve
- Discuss some *qualitative* examples of the occurrence of normal shocks

Summary of Equations Across a Stationary Normal Shock for an Ideal Gas:

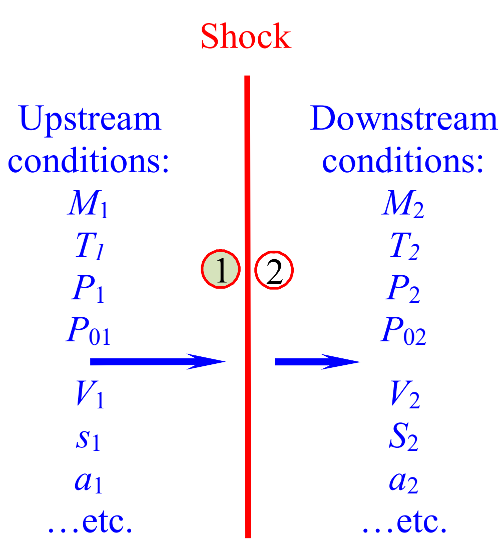
Consider a normal shock in an ideal gas. We combined conservation of mass, conservation of energy, and the linear momentum equation, along with some algebraic manipulation to get the following:

$$\boxed{T_{0,1} = T_{0,2}} \quad \boxed{M_2^{\,2} = \frac{1+\dfrac{\gamma-1}{2}M_1^{\,2}}{\gamma M_1^{\,2} - \dfrac{\gamma-1}{2}}} \quad \boxed{\frac{P_2}{P_1} = \frac{2\gamma M_1^{\,2} - \gamma + 1}{\gamma+1}} \quad \boxed{\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = \frac{(\gamma+1)M_1^{\,2}}{2+(\gamma-1)M_1^{\,2}}}$$

$$\boxed{\frac{T_2}{T_1} = \frac{1+\dfrac{\gamma-1}{2}M_1^{\,2}}{1+\dfrac{\gamma-1}{2}M_2^{\,2}}} \quad \boxed{\frac{P_{02}}{P_{01}} = \left[\frac{\dfrac{\gamma+1}{2}M_1^{\,2}}{1+\dfrac{\gamma-1}{2}M_1^{\,2}}\right]^{\frac{\gamma}{\gamma-1}} \left[\frac{1-\gamma+2\gamma M_1^{\,2}}{\gamma+1}\right]^{\frac{-1}{\gamma-1}}}$$

See other equations on the Equation Sheet for property changes across a normal shock.

Summary of changes across a *stationary* normal shock: Know these qualitatively.



Properties that ***increase*** across the shock:

- $P_2 > P_1$
- $T_2 > T_1$, thus:
  - $a_2 > a_1$
  - $h_2 > h_1$
- $\rho_2 > \rho_1$
- $s_2 > s_1$
- $A_2^* > A_1^*$

Properties that ***decrease*** across the shock:

- $M_2 < M_1$
- $P_{02} < P_{01}$
- $\rho_{02} < \rho_{01}$
- $V_2 < V_1$

Properties that ***stay the same*** across the shock:

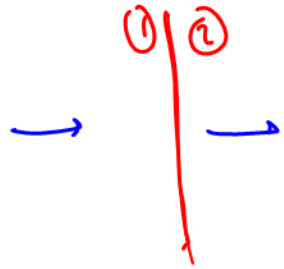
- $T_{02} = T_{01}$
- $h_{02} = h_{01}$
- $a_2^* = a_1^*$

THE HUGONIOT EQ   For normal shocks

Uses Thermodynamics rather than fluid dynamics

No MACH #

Eq. cons of mass $\rho_1 V_1 = \rho_2 V_2 \rightarrow \boxed{V_2 = V_1 \frac{\rho_1}{\rho_2}}$ (1)



+ mom eq. recall, $\boxed{P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2}$ (2)

Plug (1) into (2) solve for V_1^2

$$V_1^2 \left[\rho_1 - \rho_2 \left(\frac{\rho_1}{\rho_2} \right)^2 \right] = P_2 - P_1$$

$$\boxed{V_1^2 = \frac{P_2 - P_1}{\rho_2 - \rho_1} \frac{\rho_2}{\rho_1}} \qquad (4a)$$

Also, re-write (1) as $\boxed{V_1 = V_2 \frac{\rho_2}{\rho_1}}$ → Plug into (2), solve for V_2

$$\boxed{V_2^2 = \frac{P_2 - P_1}{\rho_2 - \rho_1} \frac{\rho_1}{\rho_2}} \qquad (4b)$$

· Energy Eq

$$h_{0_1} = h_{0_2} \rightarrow h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

(adiabatic)

$$\boxed{h = u + \frac{P}{\rho}}$$

u = specific internal energy

$$\boxed{u_1 + \frac{P_1}{\rho_1} + \frac{V_1^2}{2} = u_2 + \frac{P_2}{\rho_2} + \frac{V_2^2}{2}} \quad (5)$$

Plug (4a) & (4b) into (5) & eliminate V_1 & V_2

$$\boxed{u_2 - u_1 = \frac{P_1 + P_2}{2}\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)}$$

☆ HUGONIOT EQ FOR A NORMAL SHOCK (6f)

fluids version of H.E.

Thermo version:

$$\boxed{u_2 - u_1 = \frac{P_1 + P_2}{2}\left(v_1 - v_2\right)}$$

Thermo version

(6t)

Comments about Hugoniot Eq (H.E.)

- Holds for non-ideal gases! (we never made ideal gas approx)

- H.E. has no Mach #

- All the variables are state variables

$\boxed{\text{HE is a state eq across a shock}}$

In thermo, any state variable can be expressed as a fnc. of any 2 other independent state variables

Here let $u = \text{fnc}(P, v)$ (6t)

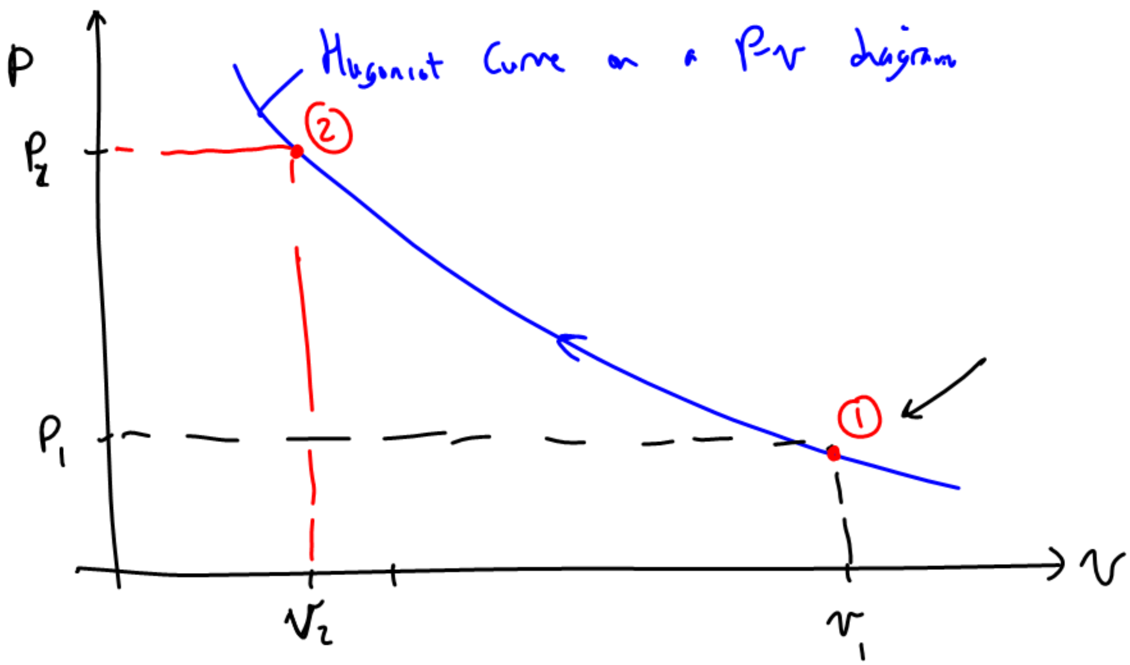
thermo version

$$u_2 - u_1 = \frac{P_1 + P_2}{2}(v_1 - v_2)$$

$$\text{fnc}(P_2, v_2) - \text{fnc}(P_1, v_1) = \text{fnc}(P_1, P_2, v_1, v_2)$$

get $\boxed{P_2 = \text{fnc}(P_1, v_1, v_2)}$

This can be plotted on a P-v diagram



$v_1 > v_2$
across a shock

Hugoniot curve : At some given state ①
The H. curve represents the locus of all
possible states downstream of the shock
(depends on strength of the shock)

★ IDEAL GAS

H.E.

SHOCK STRENGTH

$$\boxed{\frac{P_2}{P_1} = \frac{\left(\frac{\gamma+1}{\gamma-1}\right)\frac{\rho_2}{\rho_1} - 1}{\left(\frac{\gamma+1}{\gamma-1}\right) - \frac{\rho_2}{\rho_1}}} \quad (7)$$

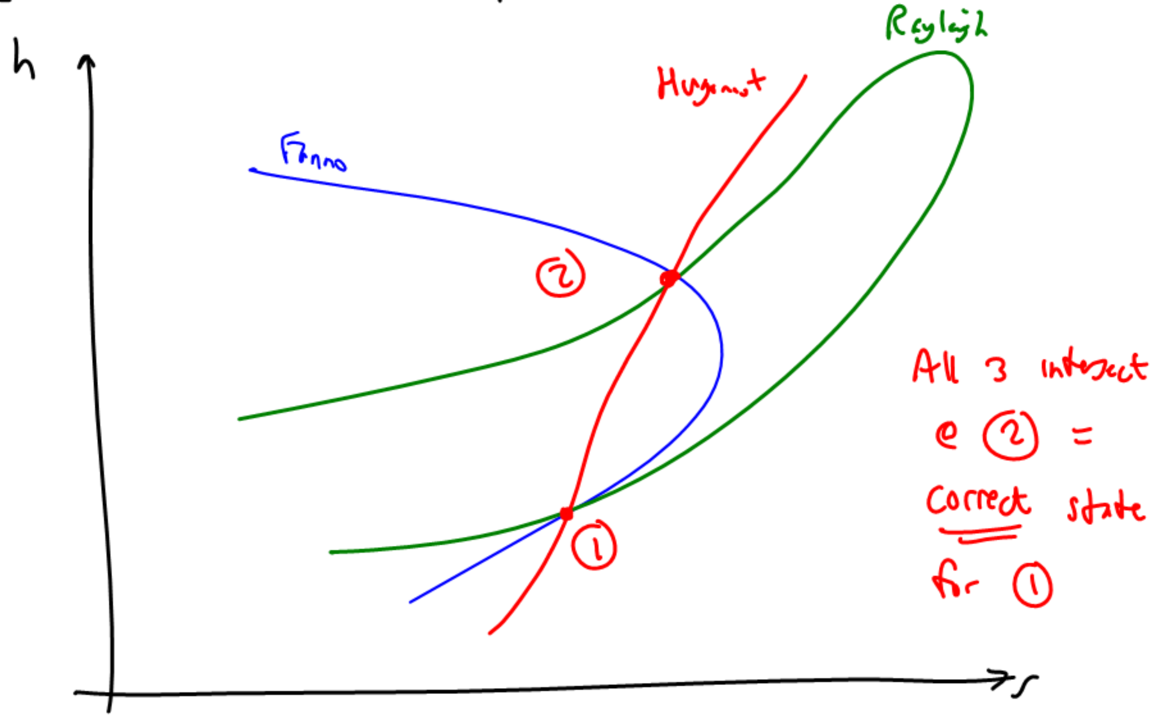
★ Another (a third) eq. for $\frac{P_2}{P_1}$ across a normal shock

Test case @ $M_1 = 2.60$ $\gamma = 1.40$

Calc $\frac{\rho_2}{\rho_1} = 3.44898$ } H.E. (7) → $\boxed{\frac{P_2}{P_1} = 7.7200}$

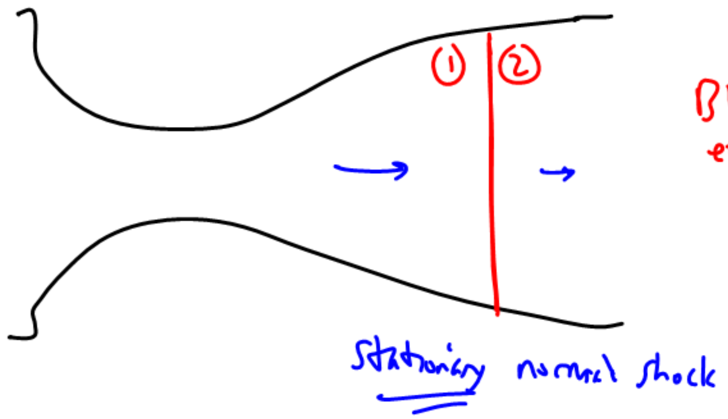
☺ Same as Fanno, Rayleigh!

HW 6: Add H.E. to our plot

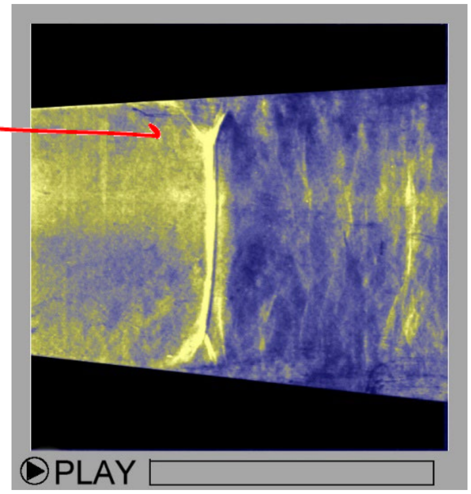


Examples of normal shocks: QUALITATIVE

1) In a converging-diverging nozzle:

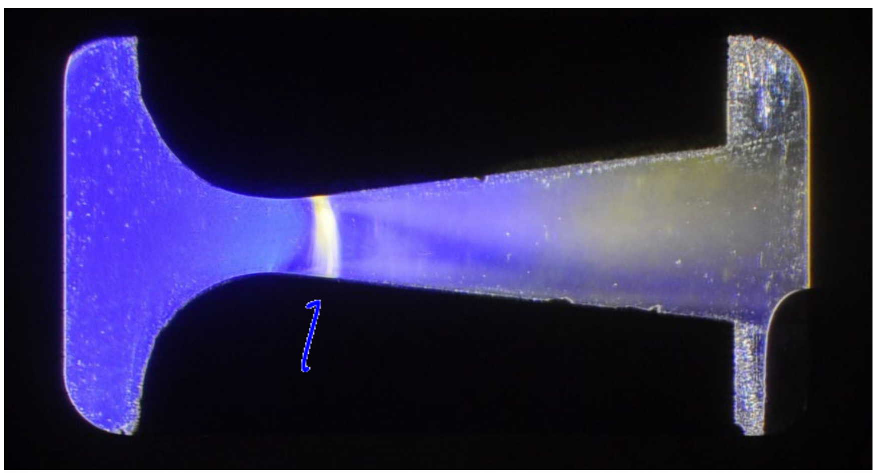


BL effects

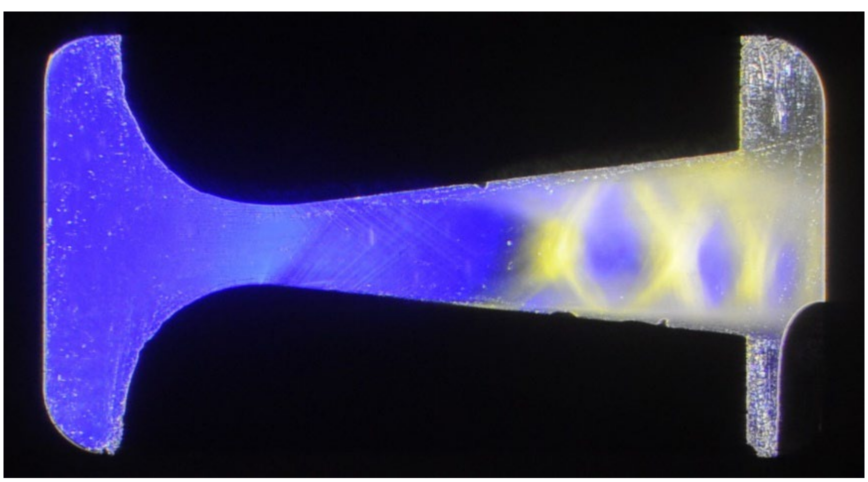


Schlieren image, shock in C-D nozzle. Photo by G. S. Settles.

(a)



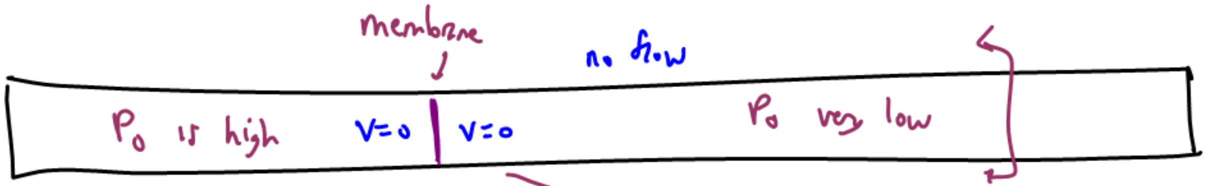
(b)



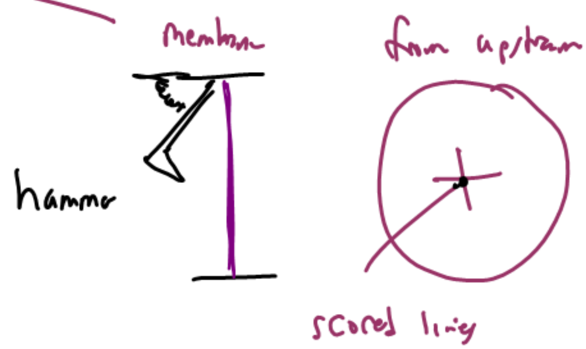
Color schlieren images in the ME 325 compressible flow rig: (a) back pressure such that a shock wave is observed just downstream of the throat, and (b) lower back pressure such that the shock wave is much farther downstream and Mach lines are observed in the isentropic flow region upstream of the shock structures.

2) In a shock tube: long tube with a diaphragm (membrane) that breaks

$t=0$

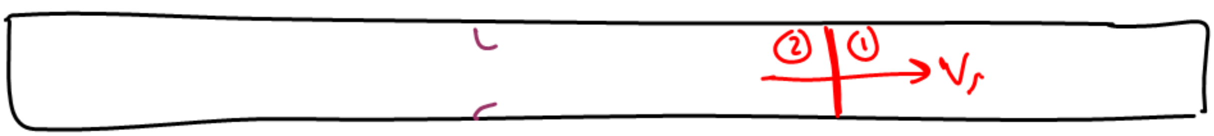


$@\ t=0^+ \rightarrow$ break the membrane



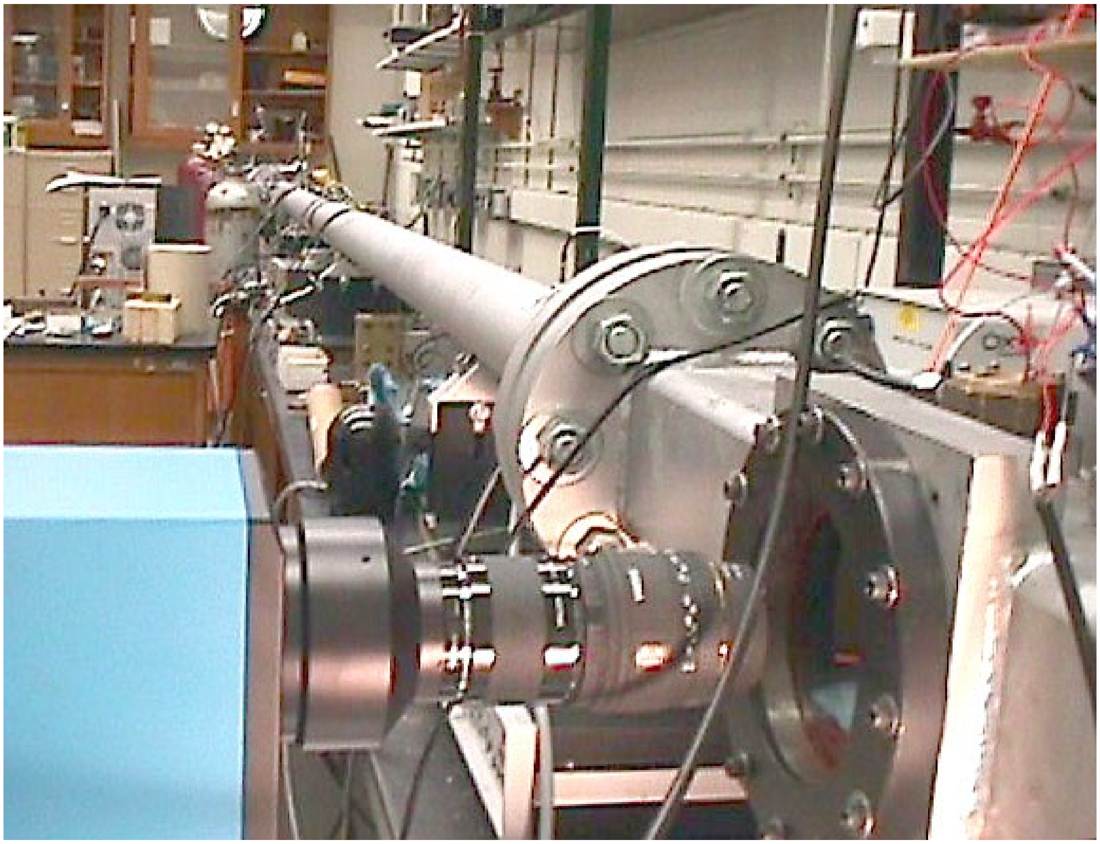
$t=t_1$

$t=t_2$



Moving shock

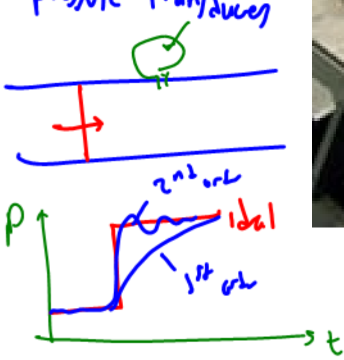
Example photographs of shock tubes:



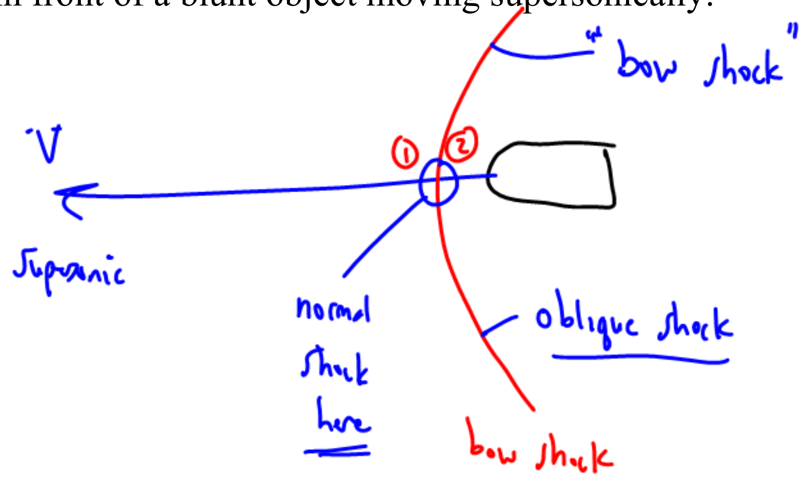
Shock tube.

Shock tube.

Can calibrate pressure transducer

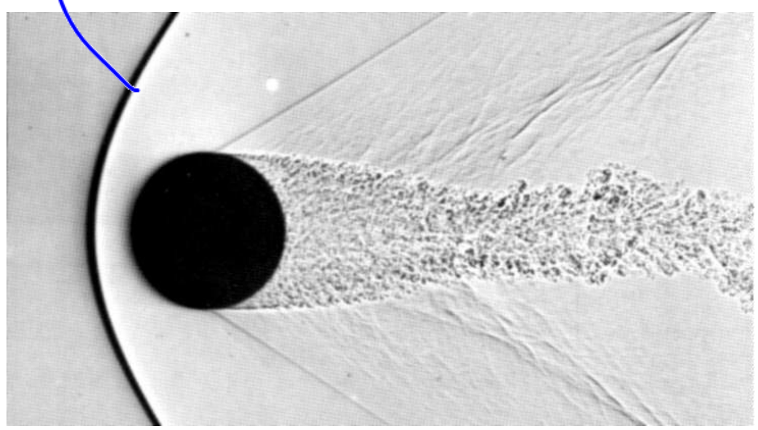


3) In front of a blunt object moving supersonically:



Moving F.O.R.

bow shock Stationary F.O.R.



oblique shocks