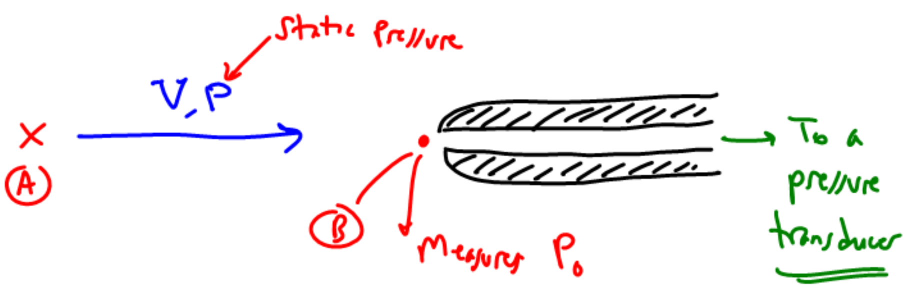
Today, we will:

- Discuss equations for Pitot tubes: incompressible flow, subsonic compressible flow, and supersonic compressible flow
- Do example problems – normal shocks in C-D nozzles

5)

Pitot Probes: WE CONSIDER 3 CASES

1) INCOMPRESSIBLE FLOW



We also measure P (static pressure)

We apply Beloved Bernoulli !

$$P_A + \frac{1}{2}\rho V_A^2 = P_B + \frac{1}{2}\rho \cancel{V_B}^2 \quad \text{(ignore gravity)}$$

0 (stag. pt)

$$\boxed{V = \sqrt{\frac{2(P_0 - P)}{\rho}}}$$

INCOMP. PITOT FORMULA

$P = P_A$ measured somewhere else

$P_0 = P_B =$ measured by transducer

2) Compressible, but Subsonic

$\rho \neq$ const B.B. does not apply

$M_1, \rho_1, P_1, T_1,$ etc

Use isentropic relations

$$\boxed{\frac{P_{0_1}}{P_1} = \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}}}$$

Know: $a_1 = \sqrt{\gamma R T_1}$ $\quad M_1 = \frac{V_1}{a_1} \rightarrow \boxed{V_1 = M_1 \sqrt{\gamma R T_1}}$

Solve for M @ measured P_{0_1} (from the Pitot probe)

" " " P_1 (from somewhere else)

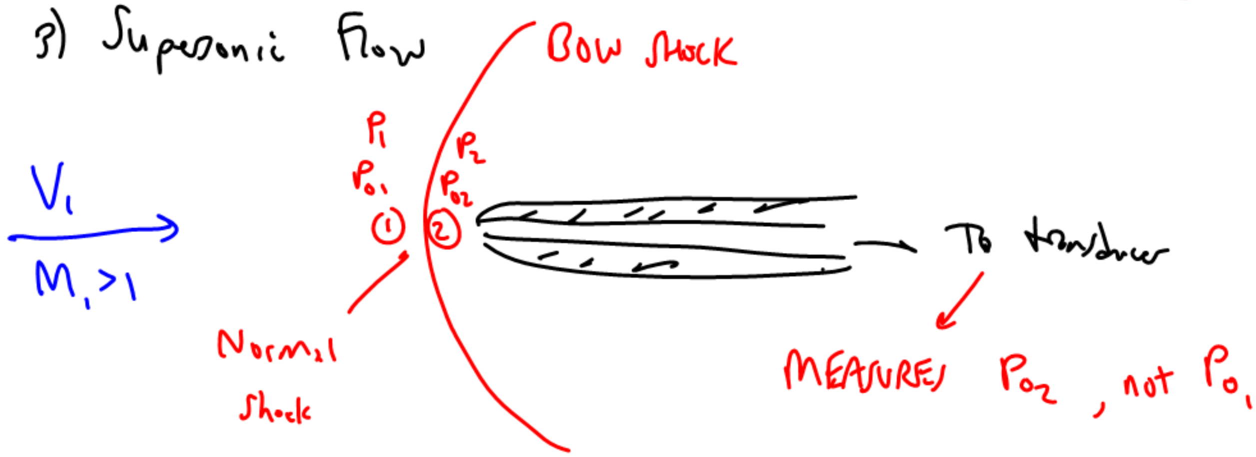
$$M_1^2 = \frac{2}{\gamma-1}\left[\left(\frac{P_{0_1}}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]$$

Need T_1 also

$$\boxed{V_1 = \sqrt{\frac{2\gamma R T_1}{\gamma-1}\left[\left(\frac{P_{0_1}}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}}$$

Compressible Subsonic Pitot Formula

3) Supersonic Flow



ratios:

$$\boxed{\frac{P_1}{P_{o_2}} = \frac{P_1}{P_{o_1}}\,\frac{P_{o_1}}{P_{o_2}}}$$

use isentropic relation (upstream of shock)

use shock relationships for $\frac{P_{o_1}}{P_{o_2}}$

$$\frac{P_1}{P_{o_2}} = \cancel{\left(1+\frac{\gamma-1}{2}M_1^2\right)^{\frac{-\gamma}{\gamma-1}}}\;\cancel{\left(1+\frac{\gamma-1}{2}M_1^2\right)^{\frac{\gamma}{\gamma-1}}}\left(\frac{\gamma+1}{2}M_1^2\right)^{\frac{-\gamma}{\gamma-1}}\left(\frac{1-\gamma+2\gamma M_1^2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}$$

$$\therefore\; \boxed{\frac{P_1}{P_{o_2}} = \left(\frac{1-\gamma+2\gamma M_1^2}{\gamma+1}\right)^{\frac{1}{\gamma-1}}\left(\frac{\gamma+1}{2}M_1^2\right)^{\frac{-\gamma}{\gamma-1}}}$$

RAYLEIGH SUPERSONIC PITOT FORMULA

WE MEASURE P_1 & P_{o_2} (Pitot probe)

(known)

IMPLICIT → IF WE WANT M_1

NEED TO SOLVE FOR M_1 implicitly

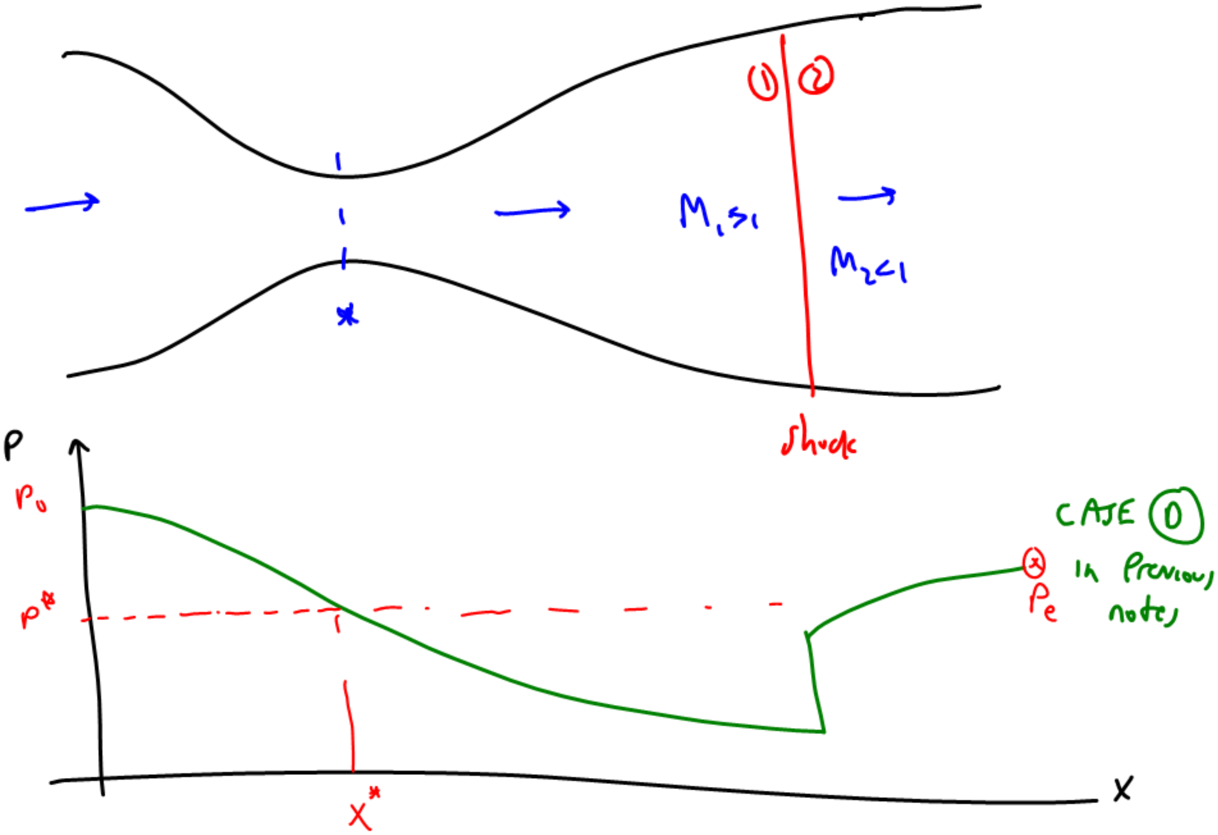
However you want!

E.g., Comp. Aero. calculator

@ our example $M_1 = 2.60$ → $\frac{P_1}{P_{o_2}} = 0.10892$

Then, $\boxed{V_1 = M_1\sqrt{\gamma R T_1}}$ → get V_1

☆ NORMAL SHOCKS IN CONVERGING-DIVERGING DUCTS



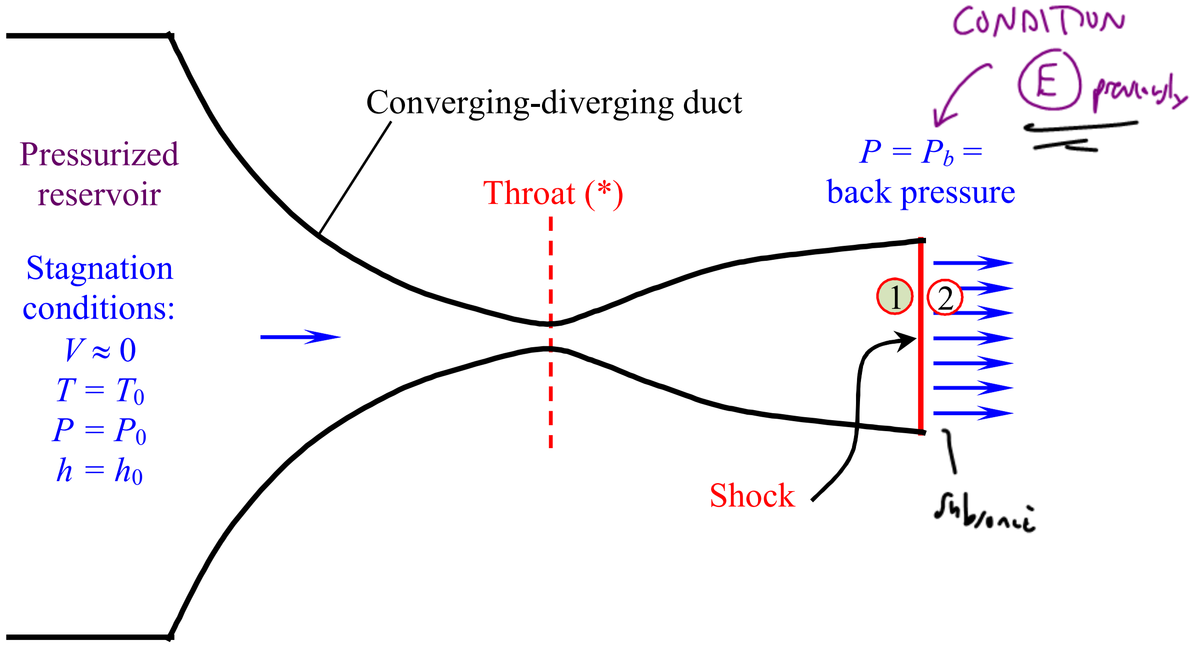
We will do 2 analyses:

(easy) 1) Known shock location @ some x (calc P_e)

(harder) 2) Known P_b ($P_b = P_e$ since subsonic after the shock) → calc shock location

Example – Normal shock at the exit plane of a converging-diverging nozzle

Given: A large tank has upstream stagnation properties T_0 = 800 K and P_0 = 1.00 MPa. Air flows through a well-insulated converging-diverging nozzle. The back pressure is adjusted such that a normal shock sits right at the exit of the nozzle where the area is three times the throat area.



To do: Calculate the pressure and Mach number at the exit plane. (downstream of shock)

Solution:

Assumptions and Approximations:

The air is an ideal gas. The flow is steady. The flow is approximated as adiabatic, one-D, and isentropic up to the shock and after the shock. (BUT NOT THROUGH THE SHOCK)

To be completed in class.

Here, $P_e = P_b = P_2 = P_E$

- Calc M_1 @ exit plane before the shock → use $\frac{A}{A^*} = fnc(M_1, \gamma)$

Here $\frac{A}{A^*} = 3.00 \rightarrow \boxed{M_1 = 2.6374}$ (upstream of shock)

- Across shock: $\frac{P_2}{P_1} = 7.94862$, $\frac{P_{0_2}}{P_{0_1}} = 0.446174$, $M_2 = 0.500692$

etc

$$\cdot \text{Calc } P_b = P_2 = P_{0_1} \underbrace{\frac{P_1}{P_{0_1}}}_{\text{isentropic}} \underbrace{\frac{P_2}{P_1}}_{\text{across shock}} = (1000\text{ kPa})\left(1+\frac{\gamma-1}{2}M_1^2\right)^{\frac{-\gamma}{\gamma-1}}(7.94862)$$

$$\boxed{P_b = 375.97 \text{ kPa}} \ \star$$

Back pressure that gives a shock right @ the exit plane

Ⓠ Is P_b (< = >) P_1^* [P^* change across a shock]

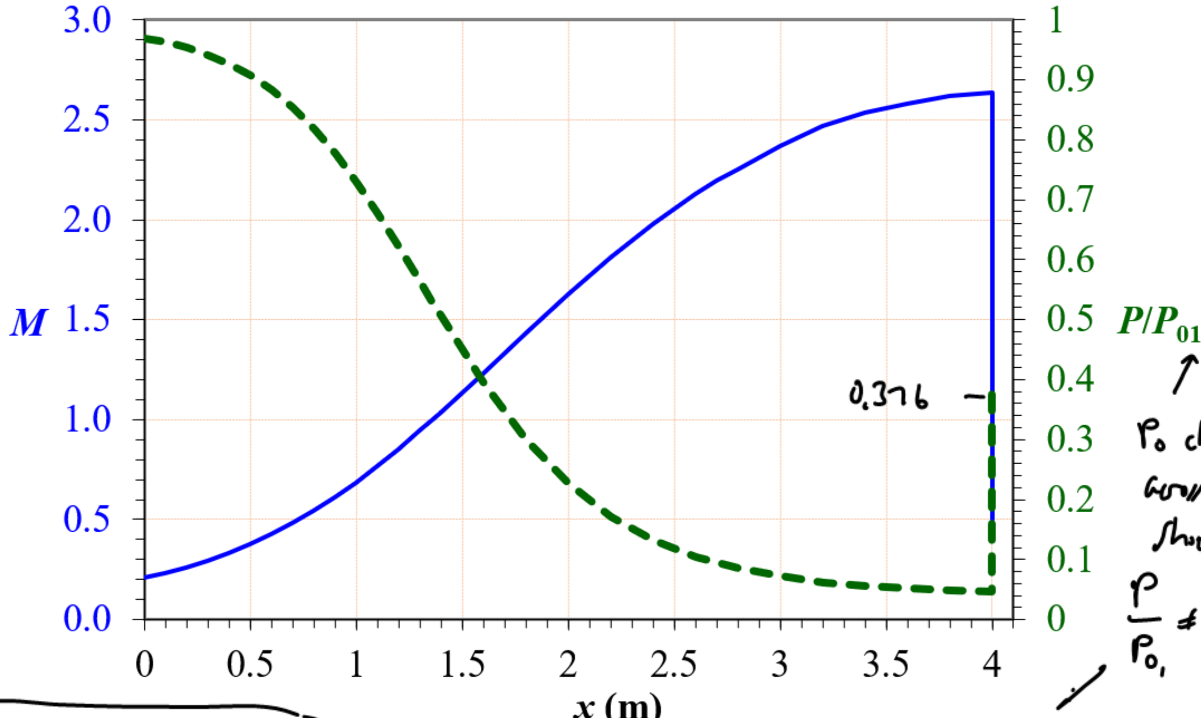
Ⓐ $$P_1^* = \frac{P_1^*}{P_{0_1}} P_{0_1} = \left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{-\gamma}{\gamma-1}}(1000\text{ kPa}) = \underline{528.28\text{ kPa}} = P_1^*$$

$M = 1$ @ *

Ans $\underline{P_b < P_1^*}$

I verified these results using Excel:

Result: pressure at exit plane:
P_e = 375.96 kPa



P_0 changes across a shock

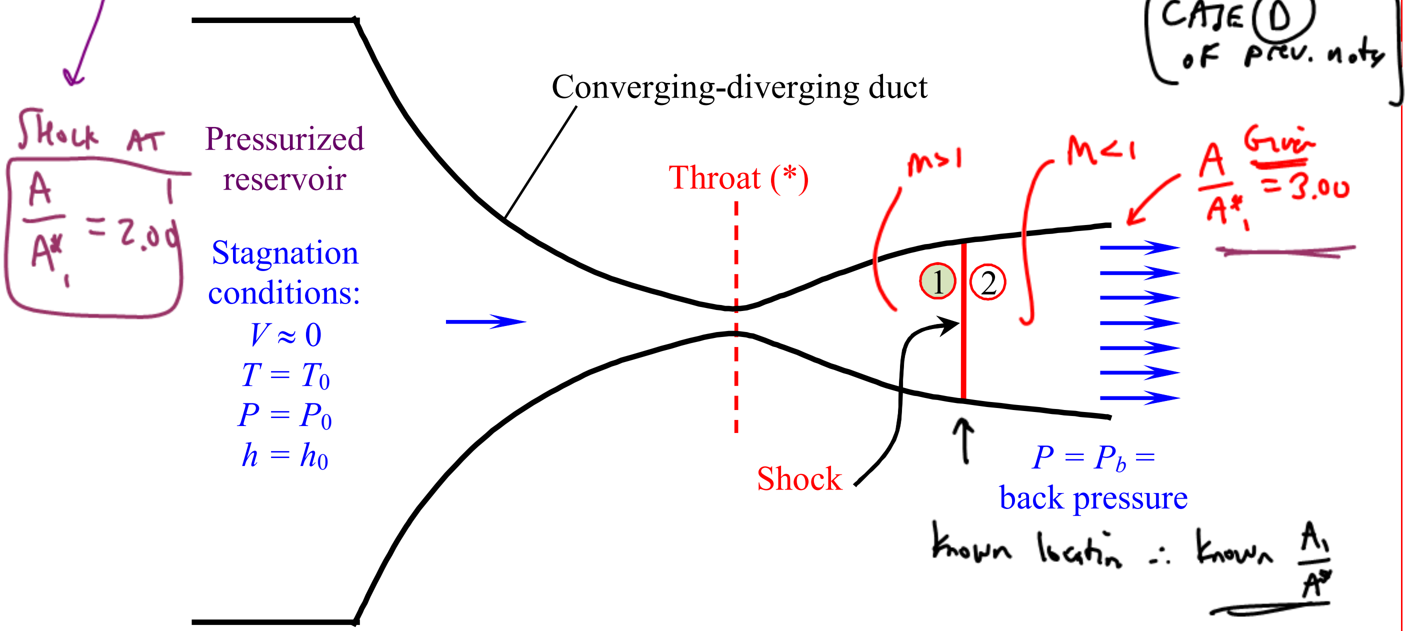
$$\frac{P}{P_{0_1}} \neq \frac{P}{P_{0_2}}$$

$$\boxed{P_e / P_{0_2} = 0.843}$$

Example – Normal shock at a *known* location in a converging-diverging nozzle

Given: A large tank has upstream stagnation properties T_0 = 800 K and P_0 = 1.00 MPa. Air flows through a well-insulated converging-diverging nozzle. The back pressure is adjusted such that a normal shock sits at a location in the diverging portion of the nozzle where the area is twice the throat area. The nozzle exit area is three times the throat area.

CASE (D) of prev. notes



To do: Calculate the pressure and Mach number at the exit plane.

Solution:

Assumptions and Approximations:

The air is an ideal gas. The flow is steady. The flow is approximated as adiabatic, one-D, and isentropic up to the shock and after the shock.

To be completed in class.

Procedure

- Use isentropic eqs upstream of shock up to ①

$$\frac{A_1}{A^*_1} = \frac{1}{M_1}\frac{(1+0.2\,M_1^2)^3}{1.728} \quad \text{— Solve for } M_1 \text{ implicitly}$$

$\frac{A_1}{A^*_1} = 2$ (location of shock)

Supersonic

Get $\boxed{M_1 = 2.1972}$

- Go across shock. Use shock eqs @ $M_1 = 2.1972$

$$\boxed{\frac{P_2}{P_1} = 5.4656, \quad \frac{P_{0_2}}{P_{0_1}} = 0.62941 \quad M_2 = 0.54743}$$

$$\frac{T_2}{T_1} = 1.8544 \quad , \text{etc.} \ \ldots \quad \left[\frac{T_{0_2}}{T_{0_1}} = 1\right]$$

- Now use isentropic relations again <u>downstream of shock</u>

<u>SUBSONIC</u>

cannot use $\frac{A}{A^*}$ eq to get M

Because <u>A^* changes across shock</u>