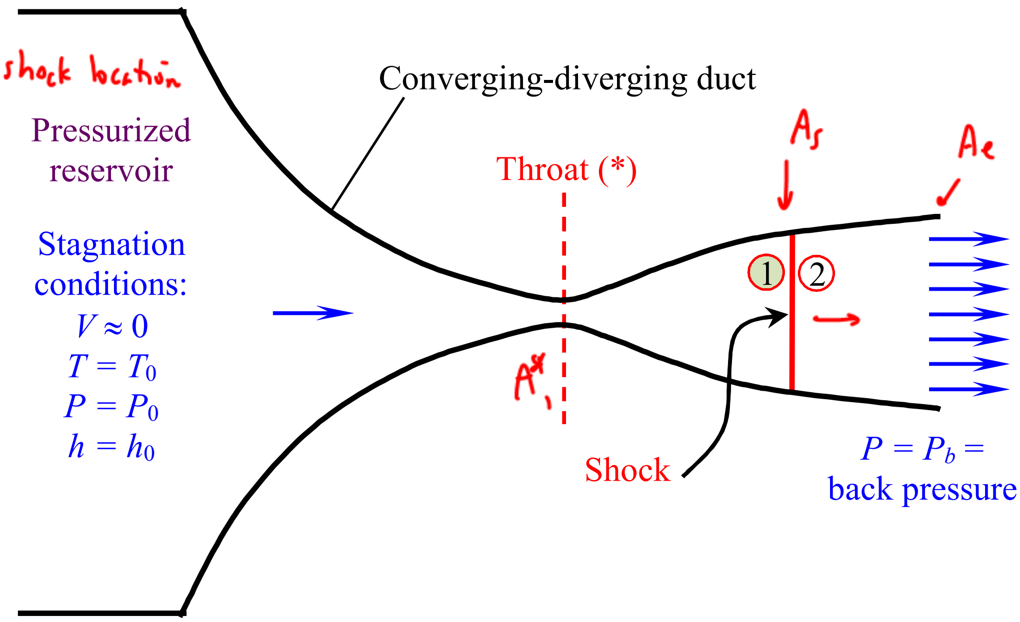
Today, we will:

- Finish the example: normal shock in a C-D nozzle; ***known*** shock location
- Do another example: normal shock in a C-D nozzle; ***unknown*** shock location
- If time, begin to discuss ***moving normal shocks*** and how they differ from stationary normal shocks

[Example continued from previous lecture]

Example – Normal shock at a *known* location in a converging-diverging nozzle

Given: A large tank has upstream stagnation properties T_0 = 800 K and P_0 = 1.00 MPa. Air flows through a well-insulated converging-diverging nozzle. The back pressure is adjusted such that a normal shock sits at a location in the diverging portion of the nozzle where the area is twice the throat area. The nozzle exit area is three times the throat area.



To do: Calculate the pressure and Mach number at the exit plane.

Solution:

Assumptions and Approximations:

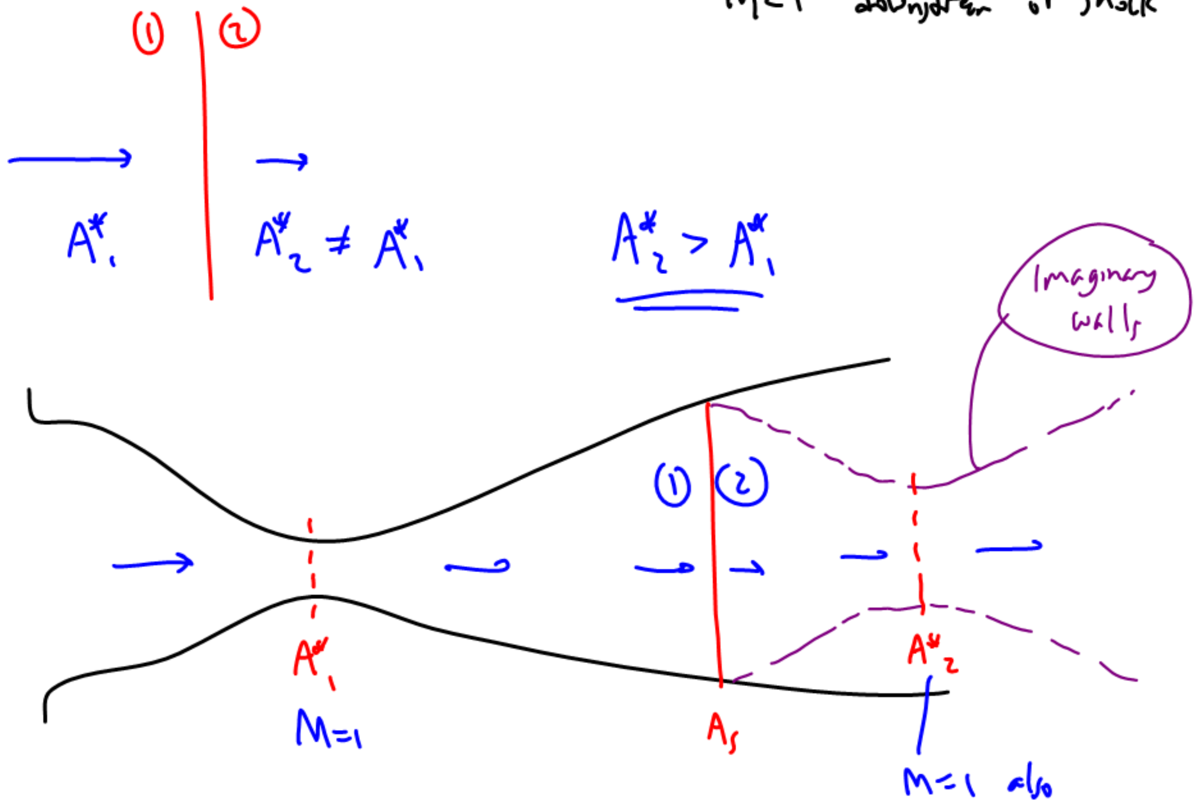
The air is an ideal gas. The flow is steady. The flow is approximated as adiabatic, one-D, and isentropic up to the shock and after the shock.

So far, from last time,

- We used isentropic relations from the reservoir to the shock. Got M_1 = 2.1972.
- Used normal shock relations *across* the shock. Got M_2 = 0.54743, P_2/P_1 = 5.4656, and P_{02}/P_{01} = 0.62941.
- Now we want to use isentropic relations again from the shock to the exit plane. But we do not know A^*_2.

We need $\underline{A_2^*}$ to use $\frac{A}{A_1^*} = fnc(M, \gamma)$ to get M_2 downstream of shock

M<1 downstream of shock



HOW TO CALCULATE A_2^*? ← KEY Cons. of mass

$\dot{m}_1 = \rho_1^* V_1^* A_1^*$ @ real throat

$\dot{m}_2 = \rho_2^* V_2^* A_2^*$ @ our imaginary throat

$\dot{m} = \dot{m}_{max}$ since choked

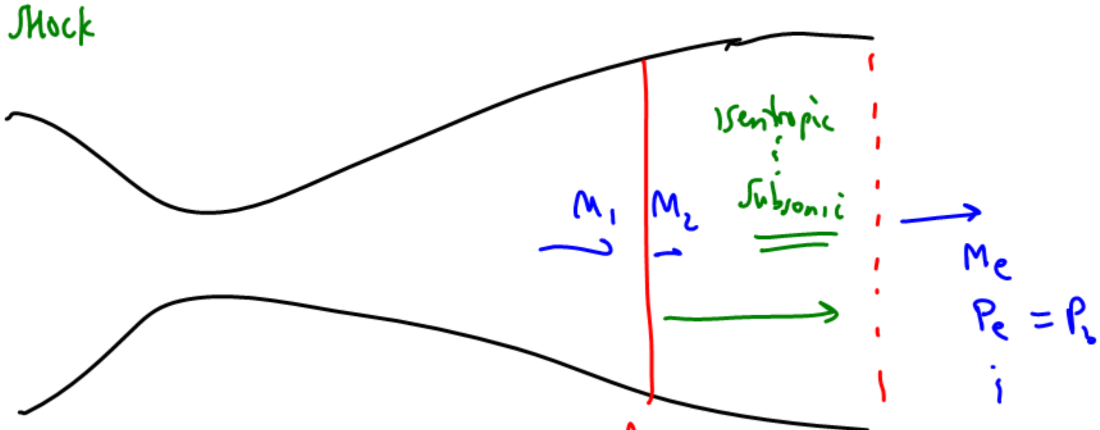
$$\therefore \quad P_{0_1} A_1^* \sqrt{\frac{\gamma}{R T_{0_1}} \left(\frac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}} = P_{0_2} A_2^* \sqrt{\frac{\gamma}{R T_{0_2}} \left(\frac{\gamma+1}{2}\right)^{-\frac{\gamma+1}{2(\gamma-1)}}}$$

$$\therefore \quad P_{0_1} A_1^* = P_{0_2} A_2^*$$

$$\therefore \quad \boxed{\frac{A_2^*}{A_1^*} = \frac{P_{0_1}}{P_{0_2}}} \; ☆ = \frac{M_2}{M_1}\left[\frac{1+\frac{\gamma-1}{2}M_1^2}{1+\frac{\gamma-1}{2}M_2^2}\right]^{\frac{\gamma+1}{2(\gamma-1)}} \quad (2) \; ☆$$

KEY TO SOLVING FLOW DOWNSTREAM OF SHOCK

$P_{0_2} < P_{0_1} \rightarrow \therefore A_2^* > A_1^*$



$\frac{A_s}{A_1^*} = 2$ $\qquad$ $\frac{A_e}{A_1^*} = 3$

E.g. @ exit plane, calc M_e & P_e

Need RATIOS to get

$$\text{need} \quad \frac{A_e}{A_2^*} = \frac{A_e}{A_1^*} \cdot \frac{A_1^*}{A_2^*}$$

$\frac{A_e}{A_1^*}$ → given – 3.0

$\frac{A_1^*}{A_2^*}$ → get from Eq (2) — Eq (2) gives 0.62941 for air ($\gamma = 1.40$)

[@ $M_1 = 2.1972$, $M_2 = 0.54734$]

From this, get M_e (implicitly any way you want)

Then $\dfrac{A_e}{A_2^*} = \dfrac{A_e}{A_1^*}\dfrac{A_1^*}{A_2^*} = 3.0\cdot(0.62941) = 1.8882$

↓ USE THIS TO GET M_e

$$\boxed{\frac{A}{A^*} = fnc(M, \gamma)}$$

Solve implicitly for M_e @ exit

Pick subsonic root. I get

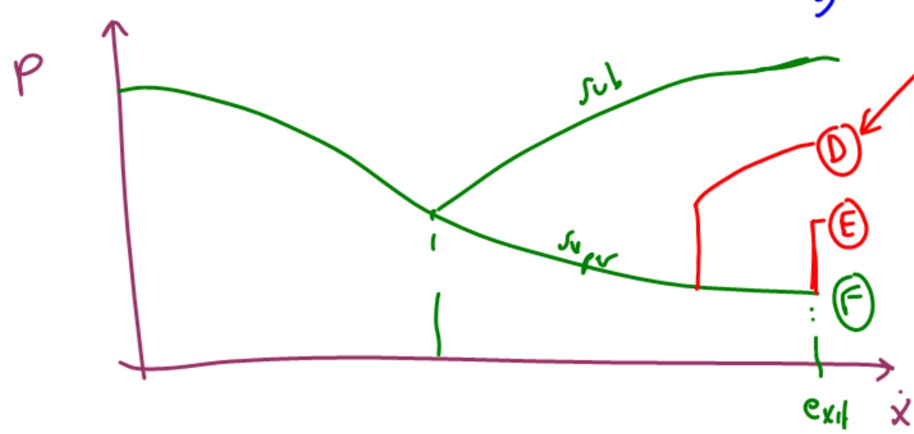
$$\boxed{M_e = 0.32650}$$

← ANS

Plus, need P_e →

$$P_e = \frac{P_e}{P_{o_2}} \frac{P_{o_2}}{P_{o_1}} P_{o_1} = \boxed{584.6 \text{ kPa}}$$

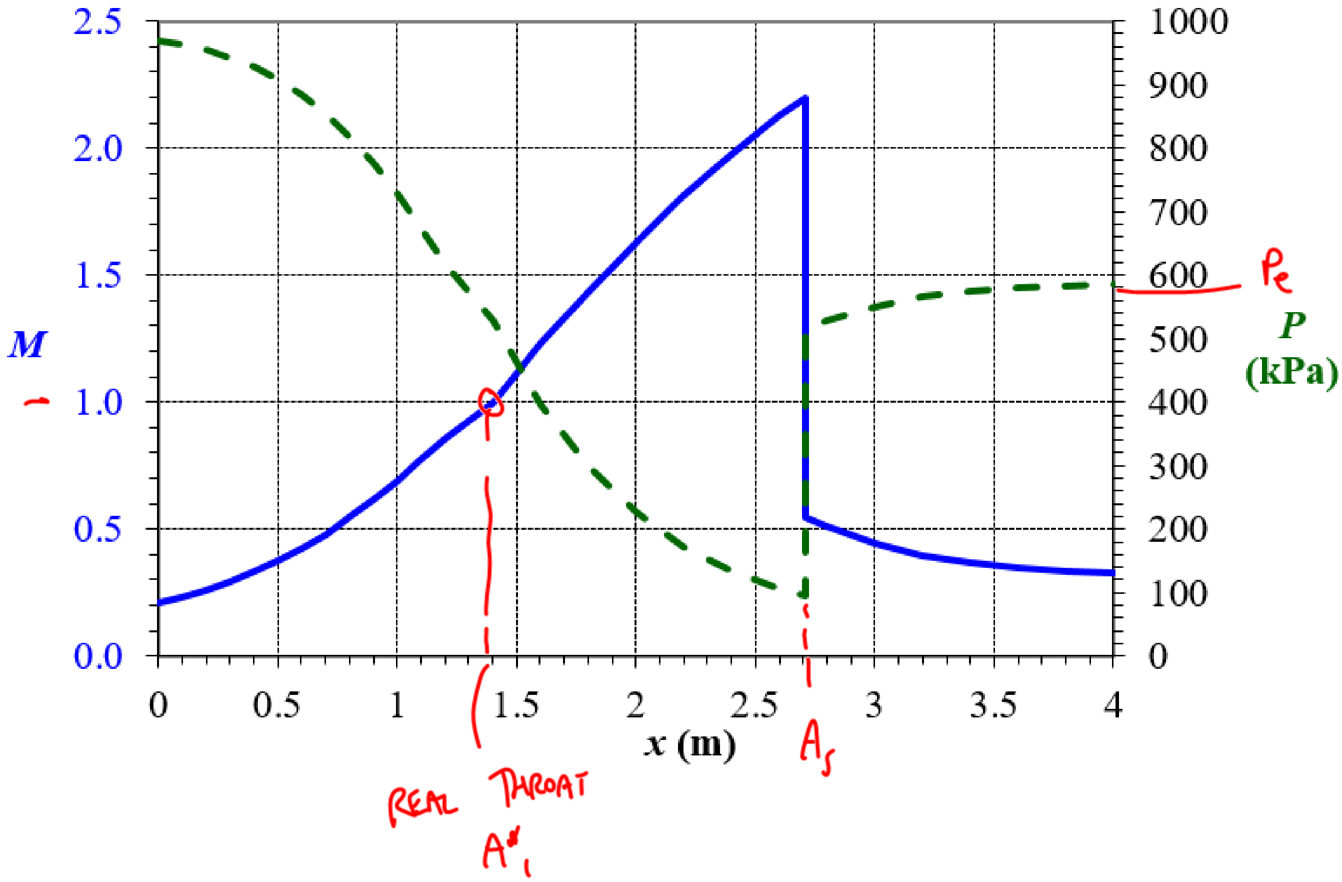
- $\frac{P_e}{P_{o_2}}$: Isentropic relations
- $\frac{P_{o_2}}{P_{o_1}}$: we calculated already
- P_{o_1}: known



WE ARE CASE D HERE

Check $P_b > P_E$

I verified all calculations in Excel, using a given C-D nozzle geometry:

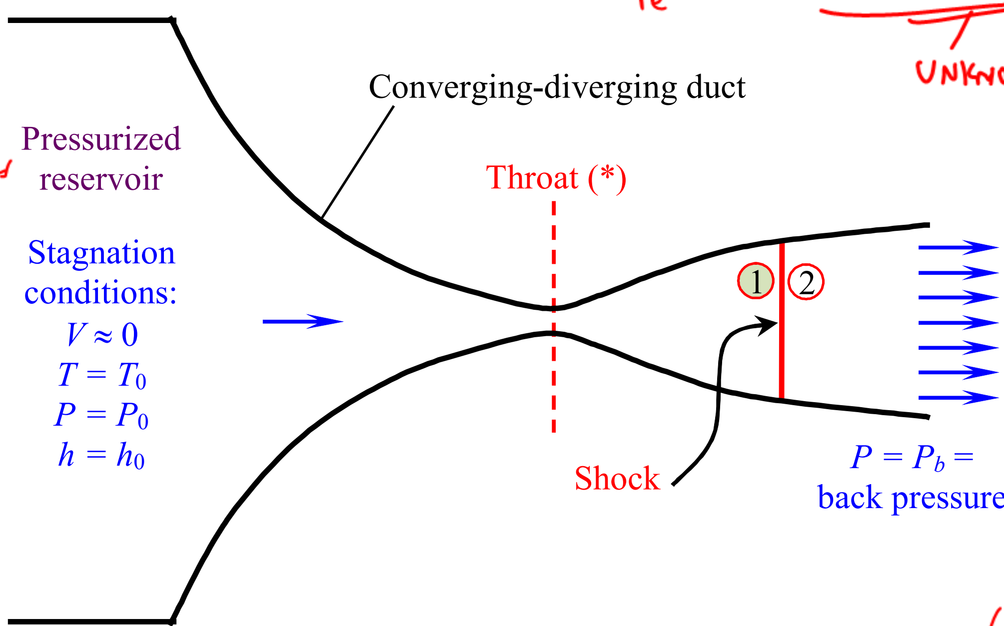


Example – Normal shock at an *unknown* location in a converging-diverging nozzle

Given: A large tank has upstream stagnation properties T_0 = 800 K and P_0 = 1.00 MPa. Air flows through a well-insulated converging-diverging nozzle. The nozzle exit area is three times the throat area. The back pressure is adjusted to 500 kPa. A normal shock sits somewhere in the diverging portion of the nozzle.

(P_e) HARDEST CASE — UNKNOWN SHOCK LOCATION

SAME AS PREVIOUS 2 EXAMPLES



To do: Calculate the location of the shock in terms of its area ratio A/A^*. — (A^*_1)

Solution:

Assumptions and Approximations:

1. The air is an ideal gas.
2. The flow is steady.
3. The flow is approximated as adiabatic, one-D, and isentropic up to the shock. (i. after the shock)

To be completed in class.

SOLUTION → WILL NEED TO ITERATE!

ITERATION PROCEDURE

1) Guess M_1 (given P_{0_1}, A^*_1, $x_s \rightarrow A_s$)

2) across shock → calc $\frac{P_{0_2}}{P_{0_1}} = fnc(M_1, \gamma)$

3) Calc $\frac{A_e}{A^*_2} = \frac{A_e}{A^*_1}\frac{A^*_1}{A^*_2}$ — $\left[\frac{A^*_1}{A^*_2} = \frac{P_{0_2}}{P_{0_1}}\right]$

4) Use $\frac{A}{A^*} = fnc(M, \gamma)$ → get $\underline{M_e}$

↑ USE A^*_2

5) Calc $P_e = \frac{P_e}{P_{o_2}} \frac{P_{o_2}}{P_{o_1}} P_{o_1}$

6) Compare P_e calculated (step 5) to P_b (given)

P_e must $= P_b$ since subsonic

• FIRST GUESS → I use previous problem,

Guess $M_1 = 2.1972$

Steps 1-5

$P_e = 584.6$ kPa

actual P_e is 500 kPa

Guess again Pick M_1 > guess 1 since shock is stronger (moves downstream)

• I picked $M_1 = 2.3$

Redo steps (1)-(5)

get $P_e = 534.2$ kPa Still ≠ 500 kPa

I used False Position Method from here on

	M_1	P_e (kPa)
guess 1 →	2.1972	584.6
guess 3 →		500 ← I WANT
guess 2 →	2.3	534.2

linear interp to get Guess 3

I get $M_1 = 2.370$

steps 1-5

$P_e = 500.4$ kPa

See Excel for rest

I put my calculations into Excel. The linear interpolation starts on the third iteration.

Iteration procedure to calculate shock location:								
	Guess M_1	A_s / A^*_1	M_2	P_{02}/P_{01}	A_e / A^*_2	Final M_e	P_e/P_{02}	P_e (kPa)
	2.1972	2.0000033	0.547431	0.62941204	1.88823612	0.3265025	0.9288219	584.61166
	2.3	2.1931308	0.534411	0.58329451	1.74988352	0.3565877	0.9158567	534.21416
linear interp →	2.3697895	2.337261	0.526399	0.55297951	1.65893852	0.3799332	0.9051873	500.55003
	2.3709298	2.3397074	0.526273	0.55249177	1.6574753	0.3803367	0.9049984	500.00418
	2.3709385	2.3397262	0.526272	0.55248803	1.65746409	0.3803398	0.904997	500
	2.3709385	2.3397262	0.526272	0.55248803	1.65746409	0.3803398	0.904997	500
	2.3709385	2.3397262	0.526272	0.55248803	1.65746409	0.3803398	0.904997	500
Final answers →	2.3709385	2.3397262	0.526272	0.55248803	1.65746409	0.3803398	0.904997	500

↗ FINAL ANSWERS !