ME 420 Professor John M. Cimbala Lecture 22

Today, we will:

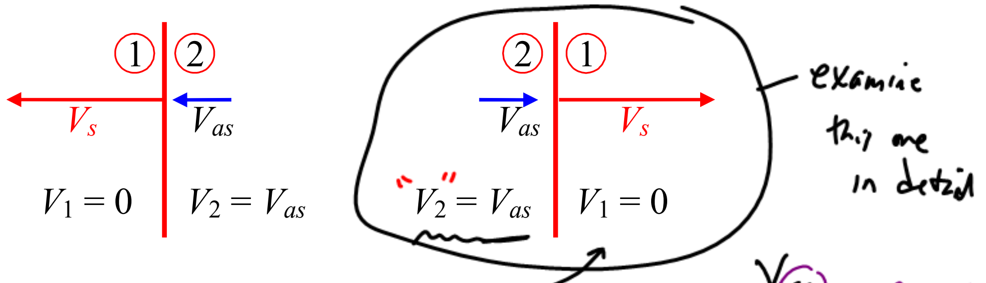
- Discuss ***moving normal shocks*** and how they differ from stationary normal shocks
- Do an example problem – moving shocks
- Do Candy Questions for Candy Friday

Moving Shocks:

Consider a *moving* normal shock wave (as in a blast wave from an explosion, or a normal shock moving in a shock tube)

Stationary F.O.R. – Moving Shock FOR

Absolute


Shock moving to left Shock moving to right

V_{as} – after shock

- The shock is moving into quiescent air (region 1)
- In this frame of reference we define $M_s = M_1 = V_s / a_1$
- The shock wave travels into region 1 at supersonic speed ($M_1 > 1$)
- The air behind the shock (region 2) follows the shock, but at a slower speed = V_{as}

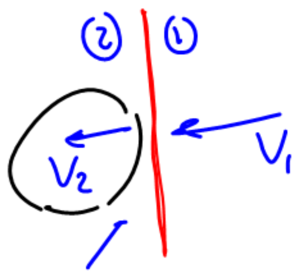
Eqs for a shock in stationary shock FOR

mass $$\rho_1 V_1 = \rho_2 V_2 \qquad (1)$$

mom $$P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2 \qquad (2)$$

en. $$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2} \qquad (3)$$

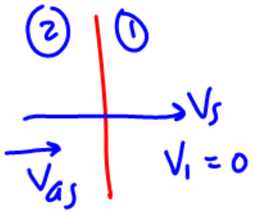
These are defined for a Stationary Shock FOR



These 3 eqs still hold for a moving shock

BUT, V_1 & V_2 must be defined relative to the shock

ABSOLUTE FOR (moving shock FOR)



To transform FOR's, subtract V_s from V_1 & V_2

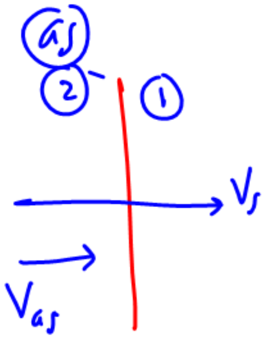
So, $V_1 = |V_1 - V_s| = |0 - V_s| = V_s$ (>0) $\boxed{V_1 = V_s}$ ★

$V_2 = |V_{as} - V_s| = V_s - V_{as}$ >0 $\boxed{V_2 = V_s - V_{as}}$ ★

Correct speeds relative to the shock (shock stationary FOR)

Summary so far

ABSOLUTE FOR



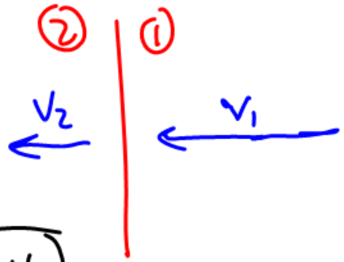
$\boxed{V_1 = 0}$

("V_2" $= V_{as}$)

$\boxed{M_{as} = \frac{V_{as}}{a_2}}$

$\boxed{M_s = \frac{V_s}{a_1}}$

STATIONARY SHOCK FOR



$\boxed{V_1 = V_s}$

$\boxed{V_2 = V_s - V_{as}}$

$\boxed{M_1 = \frac{V_1}{a_1} = \frac{V_s}{a_1} = M_s}$

$\boxed{M_2 = \frac{V_2}{a_s}}$

$\boxed{M_1 = M_s}$ BUT $\boxed{M_2 \neq M_{as}}$

PLUG RELATIVE VEL's INTO EQs (1)-(3)

$$\rho_1 V_s = \rho_2 (V_s - V_{as})$$

$$P_1 + \rho_1 V_s^2 = P_2 + \rho_2 (V_s - V_{as})^2$$

$$h_1 + \frac{1}{2} V_s^2 = h_s + \frac{1}{2} (V_s - V_{as})^2$$

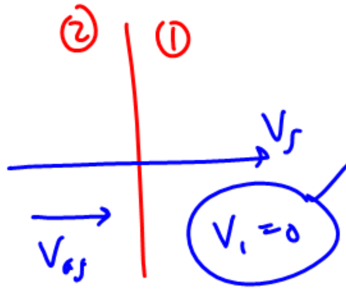
★ ← CONS EQs FOR A MOVING SHOCK IN ABSOLUTE FOR

Recall, for a stationary shock, $\boxed{h_{0_1} = h_{0_2} \rightarrow T_{0_1} = T_{0_2} \text{ for ideal gas}}$

But for a moving shock $h_{0_1} \neq h_{0_{as}}$!

$T_{0_1} \neq T_{0_2}$!

moving shock FOR



$$h_{0_1} = h_1 + \cancel{\frac{V_1^2}{2}}^{\,0}$$

$$\boxed{h_{0_1} = h_1}$$

@②, $\boxed{h_{0_{as}} = h_2 + \frac{V_{as}^2}{2}}$ always >0

$h_2 > h_1$ across a shock since $T_2 > T_1$

$\boxed{h_{0_{as}} > h_{0_1}}$ ★ $\rightarrow$ $\boxed{T_{0_{as}} > T_{0_1}}$

HOW TO ANALYZE MOVING SHOCKS?

- Static properties are defined as the property a sensor would measure if moving with the fluid

★★ Static properties are independent of F.O.R.

∴ $P_1, P_2, T_1, T_2, \rho_1, \rho_2, h_1, h_2 \rightarrow$ do not change with FOR

∴ Our old shock eqns still hold for static properties

$$\therefore \quad \frac{\overset{P_{as}}{P_2}}{P_1} = \text{fnc}\left(\overset{M_s}{M_1}, \gamma\right) \qquad \frac{\overset{T_{as}}{T_2}}{T_1} = \text{fnc}\left(\overset{M_s}{M_1}, \gamma\right) \quad \text{. etc.}$$

BUT Stagnation properties are defined by bringing the flow to rest

∴ Stag. properties DO depend on F.O.R. ✯

eg. $P_{0_1}, P_{0_2}, T_{0_1}, T_{0_2}, h_{0_1}, h_{0_2}, \rho_{0_1}, \rho_{0_2}$

change with FOR

FOR MOVING SHOCKS, USE SAME EQS FOR $\frac{P_2}{P_1}, \frac{T_2}{T_1} \ldots$ (STATIC PROPS)

BUT DO NOT USE ANY OLD EQS FOR STAGNATION PROPS

Also be careful since $M_1 = M_s$

BUT $M_2 \neq M_{as}$!

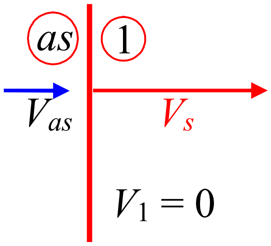
You can use M_2 as a "convenience" to calculate $\frac{T_2}{T_1}$

BUT keep in mind that $\underline{\underline{M_{as} \neq M_2}}$

Example: Moving normal shock

Given: A normal shock from a blast wave moves at speed 880 m/s into air at SATP. (25°C, 101.325 kPa)

To do: Calculate properties before and after the moving shock and compare to values obtained from a stationary shock frame of reference (FOR).



Solution:

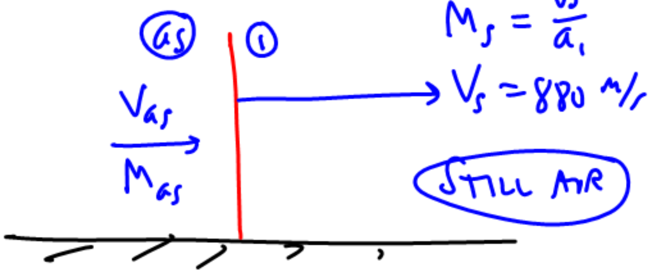
Assumptions and Approximations:

1. The air is an ideal gas.
2. Ground effects, friction, etc. are ignored.

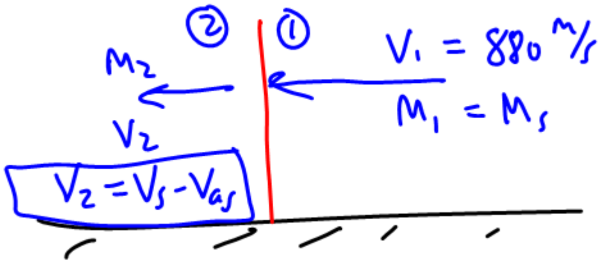
$T_1 = 298.15\ K$

$M_s = \frac{V_s}{a_1}$

MOVING SHOCK FOR
ABSOLUTE



STATIONARY SHOCK FOR



eqs:

$$a_1 = a_{\text{still air}} = \sqrt{\gamma R T_1} = \sqrt{(1.40)\left(287.0 \frac{m^2}{s^2 K}\right)(298.15 K)}$$

$$= \boxed{346.12\ \text{m/s} = a_1}$$

$$M_s = \frac{V_s}{a_1} = \frac{880\ \text{m/s}}{346.12\ \text{m/s}} = \boxed{2.5425 = M_s = M_1}$$

Use old eqs across a shock for static props:

$$\frac{T_2}{T_1} = 2.17990 \longrightarrow T_2 = T_{as} = \frac{T_2}{T_1}(T_1) = \underline{649.94\ K}$$

HOT!

$$M_2 = 0.50900$$

Similarly, $\frac{P_2}{P_1} = 7.3750 \rightarrow P_2 = P_{as} = \underline{747.273 \text{ kPa}}$

High P !

″ $\frac{\rho_2}{\rho_1} = \frac{V_1}{V_2} = 3.38318 \rightarrow V_2 = \underline{\underline{260.110 \text{ m/s}}}$

Stationary Shock FOR

$$\boxed{\frac{V_1}{V_2} = \frac{V_s}{V_s - V_{as}}} \rightarrow \text{solve for } V_{as}$$

$$\boxed{V_{as} = \left(1 - \frac{V_2}{V_1}\right) V_s}$$

$$\underline{\underline{V_{as} = 619.89 \text{ m/s}}}$$

FAST !

AFTER SHOCK

$\underline{T_{as} = 649.94 \text{ K}}$ $\underline{P_{as} = 747.273 \text{ kPa}}$ $\underline{V_{as} = 619.89 \text{ m/s}}$

CALC. M_{as} compare to $M_2 = 0.50900$

$$\boxed{M_{as} = \frac{V_{as}}{a_{as}} = 1.213}$$

$$\boxed{M_{as} \neq M_2}$$

M_{as} can be supersonic !!