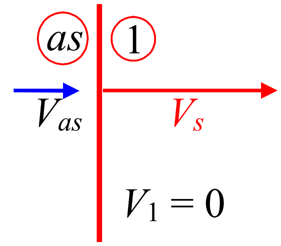
Today, we will:

- Finish the example problem from last time, and discuss more moving shock equations
- Discuss piston-driven shock tubes and expansion fans in long tubes
- Introduce *x-t* and *x-P* diagrams for moving shocks and moving expansion waves

Example: Moving normal shock (continued from previous lecture)

Given: A normal shock from a blast wave moves at speed 880 m/s into air at SATP.

To do: Calculate properties before and after the moving shock and compare to values obtained from a stationary shock frame of reference (FOR).



Moving shock FOR

Solution:

Assumptions and Approximations:

1. The air is an ideal gas.
2. Ground effects, friction, etc. are ignored.

We calculated: $M_1 = M_s = 2.5425$.

Static properties are the same in either FOR. Thus,

- $T_1 = 298.15$ K, $T_2 = T_{as} = 649.94$ K
- $P_1 = 101.325$ kPa, $P_2 = P_{as} = 747.27$ kPa
- $V_1 = 880$ m/s in stationary shock FOR, $V_2 = 260.11$ m/s in stationary shock FOR
- $V_1 = 0$ m/s in moving shock FOR, $V_{as} = 619.89$ m/s in moving shock FOR
- $M_2 = 0.50900$ in stationary shock FOR, and can be used as an intermediate property to calculate other static properties, **but M_{as} in moving shock FOR is *not* equal to M_2 in the stationary FOR.** We calculated M_{as} **= 1.213** in the moving shock FOR.

Comments: $M_{as} \neq M_2$ – can use M_2 as an intermediate step

M_{as} can be subsonic or supersonic

$V_{as} \neq V_2$

Static props are same $T_2 = T_{as}$, $P_2 = P_{as}$ etc

BUT STAGNATION PROPS are not same!!

Do depend on FOR.

STATIONARY SHOCK F.O.R. (OLD STUFF)

$$\frac{P_{o_2}}{P_{o_1}} = \text{fnc}(M_1, \gamma) \quad \text{Here } M_1 = 2.5425$$

$$\frac{P_{o_2}}{P_{o_1}} = 0.48220$$

where
$$P_{o_1} = \frac{P_{o_1}}{P_1} P_1 = \left(1 + \frac{\gamma-1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}} P_1 = 1849.42 \text{ kPa}$$

Similarly
$$T_{o_1} = \frac{T_{o_1}}{T_1} T_1 = \left(1 + \frac{\gamma-1}{2} M_1^2\right) T_1 = 683.62 \text{ K}$$

$$P_{o_2} = 891.79 \text{ kPa}$$ ☆ $P_{o_2} < P_{o_1}$ as before

$$T_{o_2} = 683.62 \text{ K}$$ $T_{o_2} = T_{o_1}$ ''

☆ MOVING SHOCK F.O.R. $(P_{GS} = P_2)$

$$P_{o_{GS}} = \frac{P_{o_{GS}}}{P_{GS}} P_{GS} = \left(1 + \frac{\gamma-1}{2} M_{GS}^2\right)^{\frac{\gamma}{\gamma-1}} (747.273 \text{ kPa}) = $$

$$P_{o_{GS}} = 1843.2 \text{ kPa}$$

Can use isentropic eq. after the shock

$$T_{o_{GS}} = \frac{T_{o_{GS}}}{T_{GS}} T_{GS} \rightsquigarrow T_{o_{GS}} = 841.20 \text{ K}$$

NOTICE: $P_{o_{GS}} \neq P_{o_2}$. In fact $P_{o_{GS}} > P_{o_2}$

$T_{o_{GS}} \neq T_{o_2}$. '' $T_{o_{GS}} > T_{o_2}$

$h_{o_{GS}} \neq h_{o_2}$ '' $h_{o_{GS}} > h_{o_2}$

$$h_{o_{gs}} = c_p T_{o_{gs}} = \left(1004.5 \frac{J}{kg\,K}\right)\left(841.20\,K\right) = \boxed{844{,}990 \frac{J}{kg} = h_{o_{gs}}}$$

OR,

$$h_{o_{gs}} = h_{gs} + \frac{V_{gs}^2}{2}$$

$$= c_p T_{gs} + \frac{V_{gs}^2}{2} = \left(1004.5 \frac{J}{kg\,K}\right)\left(649.94\,K\right) + \frac{\left(619.89\ m/s\right)^2}{2}$$

$$\left(\frac{J}{N\,m}\right)\left(\frac{N\,s^2}{kg\,m}\right)$$

$$= \boxed{844{,}990 \frac{J}{kg}}$$ ☺

Compare to stationary shock F.O.R.

$$\boxed{h_{o_2} = h_{o_1}} \rightarrow \boxed{h_{o_2} = h_{o_1} = 686{,}700 \frac{J}{kg}}$$

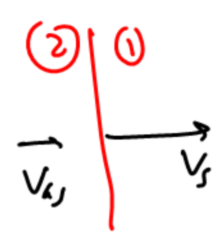
$$\underline{h_{o_{gs}} > h_{o_2}}$$

Recall 3 cons. eqs for a moving shock (in moving shock F.O.R.)

mass
$$\rho_1 V_s = \rho_2 \left(V_s - V_{gs}\right) \quad (1)$$

mom
$$P_1 + \rho_1 V_s^2 = P_2 + \rho_2 \left(V_s - V_{gs}\right)^2 \quad (2)$$

en
$$h_1 + \frac{1}{2} V_s^2 = h_2 + \frac{1}{2}\left(V_s - V_{gs}\right)^2 \quad (3)$$



Combine these 3 eqs & manipulate as necessary

Similar to our previous development for Hugoniot eq.

Get:

$$u_2 - u_1 = \frac{p_1 + p_2}{2}\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)$$

$\rho_2 = \rho_{sy}$

$p_2 = p_{sy}$ etc.

SAME AS BEFORE!

look back → Hugoniot Eq ☆

☆ HUGONIOT EQ IS INDEPENDENT OF F.O.R. !!

For ideal gas,

$$\frac{p_2}{p_1} = \frac{\frac{\gamma+1}{\gamma-1}\frac{\rho_2}{\rho_1} - 1}{\frac{\gamma+1}{\gamma-1} - \frac{\rho_2}{\rho_1}}$$

☆ SAME HUGONIOT AS BEFORE!

BECAUSE ALL THESE ARE STATIC PROPERTIES

YOU CAN DESCRIBE SHOCK STRENGTH BY $\frac{\rho_2}{\rho_1}$ or $\frac{p_2}{p_1}$ (we use this) instead of M_1

or

$$\frac{\rho_2}{\rho_1} = \frac{1 + \frac{\gamma+1}{\gamma-1}\frac{p_2}{p_1}}{\frac{\gamma+1}{\gamma-1} + \frac{p_2}{p_1}}$$

⋮

$$\frac{T_2}{T_1} = \frac{\frac{\gamma+1}{\gamma-1} + \frac{p_2}{p_1}}{1 + \frac{\gamma+1}{\gamma-1}\frac{p_2}{p_1}} \cdot \frac{p_2}{p_1}$$

∴ Calc M_s as a fnc of $\frac{P_2}{P_1}$ & γ

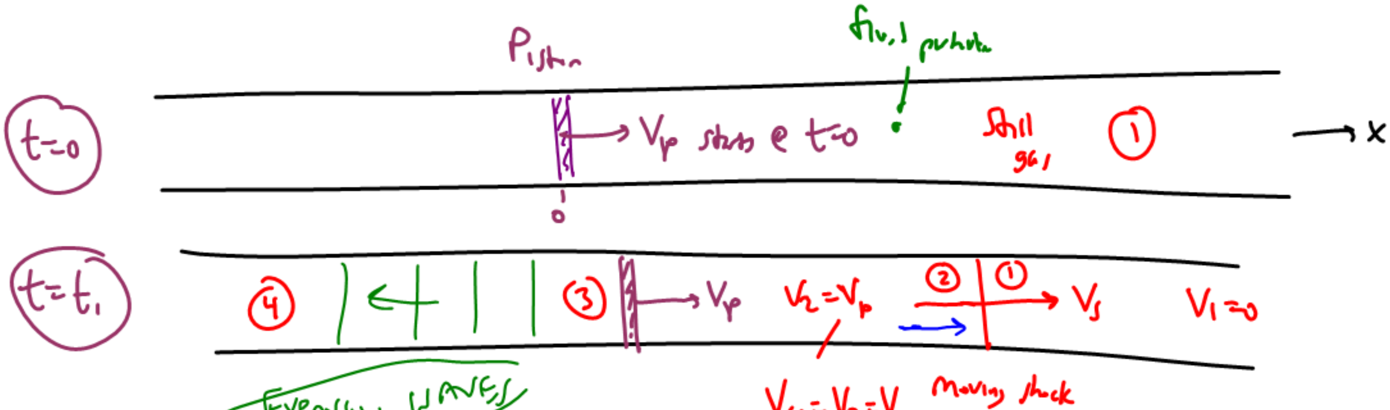
$$M_s = \sqrt{1 + \frac{\gamma+1}{2\gamma}\left(\frac{P_2}{P_1} - 1\right)}$$

Then V_s from $M_s = \frac{V_s}{a_1}$

$$V_s = a_1\sqrt{1 + \frac{\gamma+1}{2\gamma}\left(\frac{P_2}{P_1} - 1\right)}$$

☆ Moving Shocks & Expansion Waves & x-t diagrams

Consider an insulated (adiabatic) tube filled w/ gas

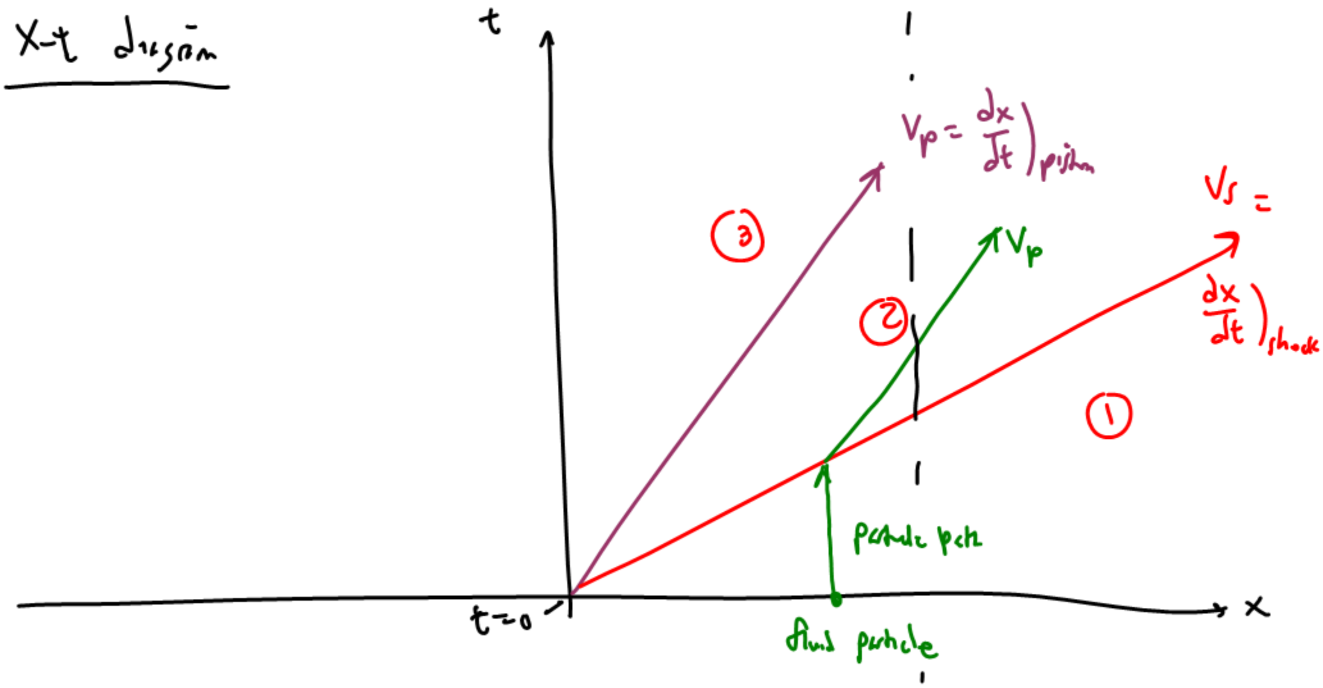


Expansion waves move @ speed of sound

$P_2 > P_1$

$\rho_2 > \rho_1$ $T_2 > T_1$ as previously

x-t diagram



x-P plot @ some time (instantaneous plot) @ t_1