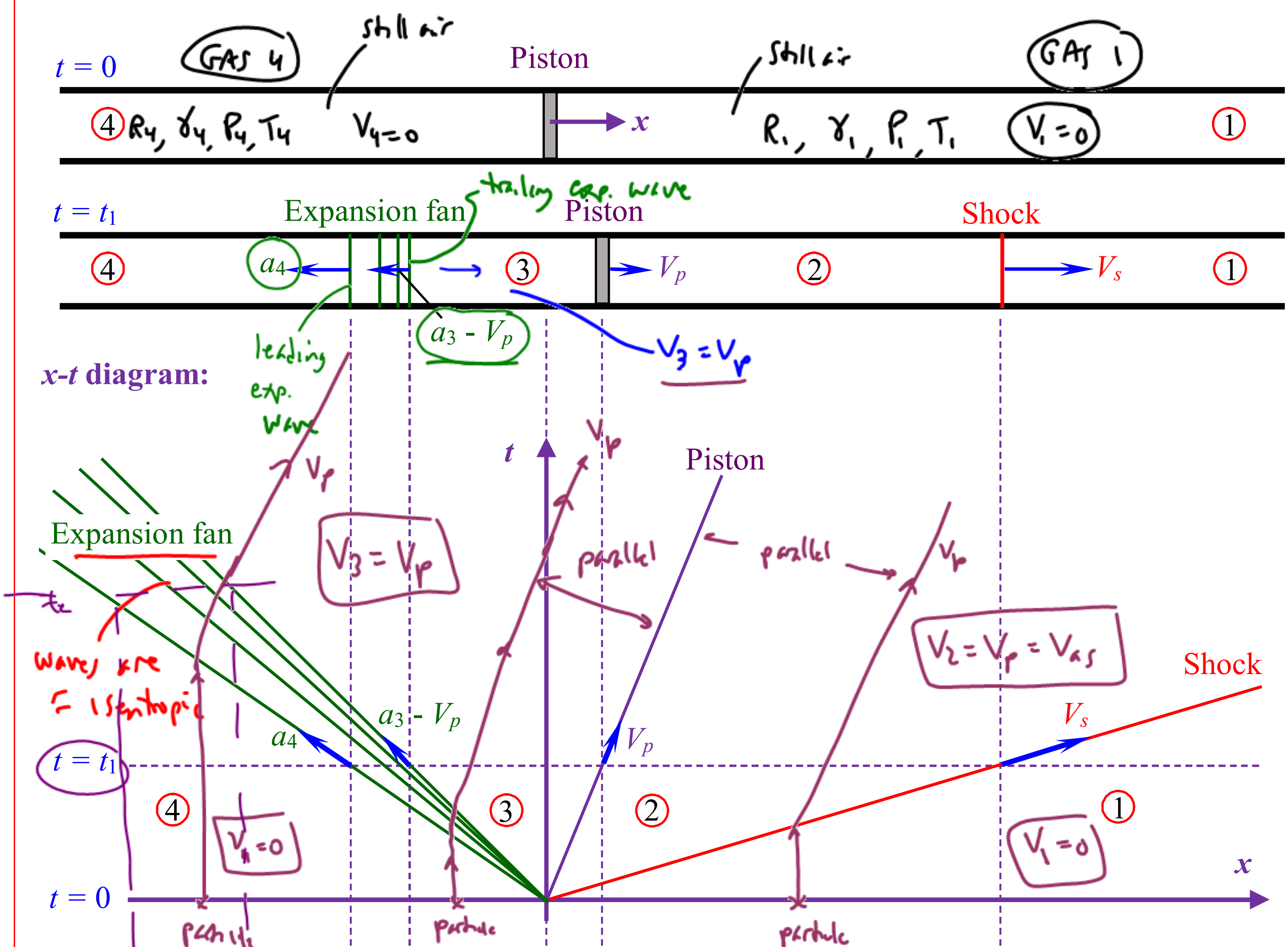
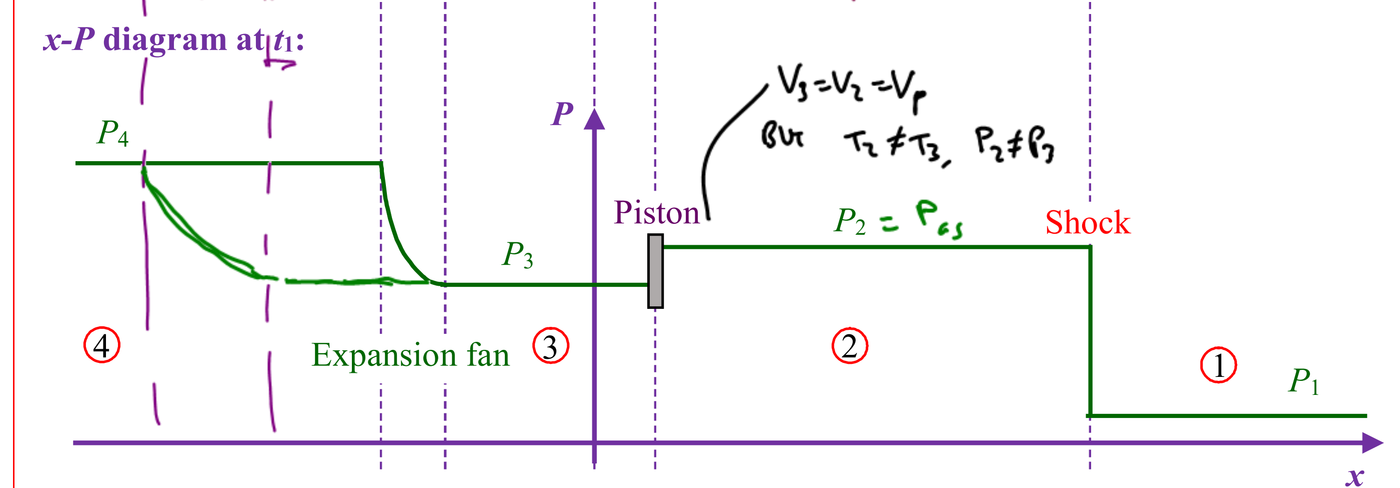
Today, we will:

- Finish discussing piston-driven expansion fans in long tubes; *x-t* and *x-P* diagrams
- Discuss shock tubes – qualitatively first, then quantitatively

Review: Carefully sketched *x-t* and *x-P* diagrams for piston-driven flow in long tubes





Comments:

- expansion waves move @ local speed of sound relative to the moving gas
- $\boxed{T_4 > T_3}$ → $a_4 > a_3$ $\quad a_4 = \sqrt{\gamma_4 R_4 T_4}$

Leading (Front) wave is fastest
Trailing (back) wave is slowest
∴ exp. fan expands with time (spreads out)

- Expansion fan is ≈ <u>Isentropic</u> (sound waves)

NO DISCONTINUITY
NO IRREVERSIBILITIES

- V_2 must $= V_3$ must $= V_p$
- BUT P_2 not necessarily $= P_3$

CAN HAVE:
$P_2 < P_3$
$P_2 = P_3$
$P_2 > P_3$

- HOW TO CALC. P_3? (across exp. fan)

Let c = wave speed = a − local speed of the gas

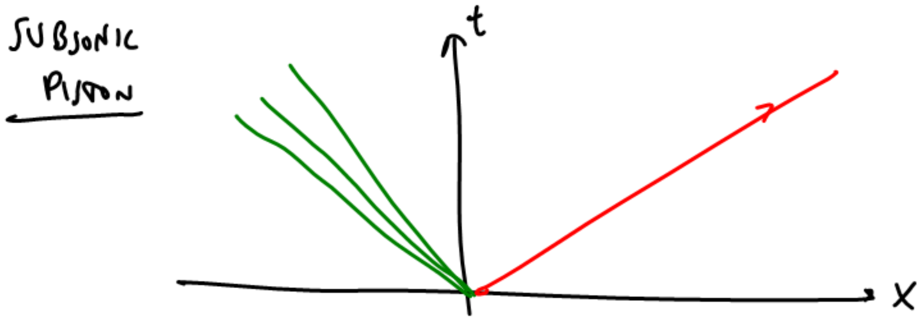
Here, $C_4 = a_4$ since $V_4 = 0$

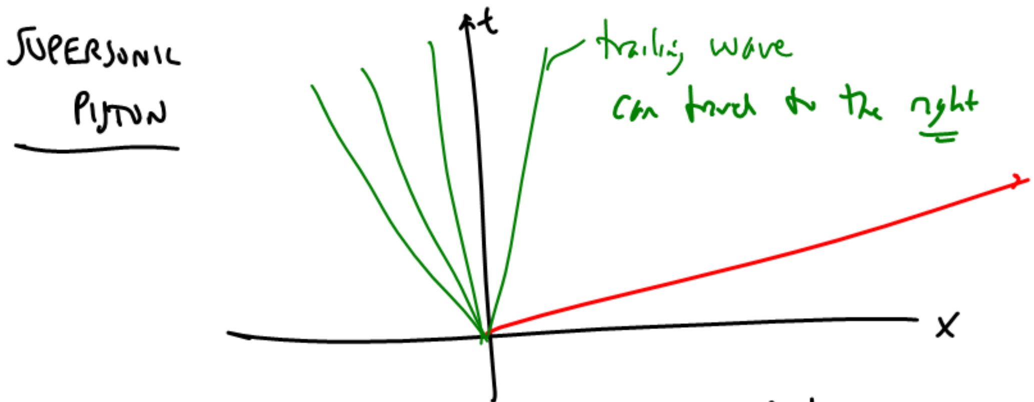
$C_3 = a_3 - V_3 = \underline{\underline{a_3 - V_p}}$

V_p can be subsonic or supersonic

On <u>x-t diagram</u> @ some $t = t_1$,

$X_4 = -C_4 t$
$X_3 = -C_3 t$





★ **ISENTROPIC WAVE THEORY** $\rightarrow$ $\underset{\text{wave speed}}{C} = a_{ref} \pm \frac{\gamma+1}{2} V$ (ideal gas)

For exp. fan, wave moves to left (leading wave)

$$C_4 = a_4 \pm \frac{\gamma+1}{2} \cancel{V_4} \quad \rightarrow \quad \boxed{C_4 = a_4} \text{ to left}$$

$V_4 = 0$

$$\boxed{C_3 = a_4 - \frac{\gamma+1}{2} V_3}$$

$$\boxed{C_3 = a_3 - V_3}$$

It turns out

$$\boxed{\frac{a_3}{a_4} = 1 - \frac{\gamma-1}{2} \frac{V_3}{a_4}} \; ★$$

Then — use isentropic relations $\frac{P}{P_0} = \left(\frac{T}{T_0}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{a}{a_0}\right)^{\frac{2\gamma}{\gamma-1}}$

At 4 $V_4 = 0 \rightarrow a_4 = a_0$; $p_4 = p_0$ @ region 4

$$\boxed{\frac{p_3}{p_4} = \left(\frac{a_3}{a_4}\right)^{\frac{2\gamma}{\gamma-1}}}$$

$$\boxed{\frac{p_3}{p_4} = \left(1 - \frac{\gamma-1}{2}\frac{V_3}{a_4}\right)^{\frac{2\gamma}{\gamma-1}}}$$

$\gamma = \gamma_4$ here

Pressure ratio across an expansion fan

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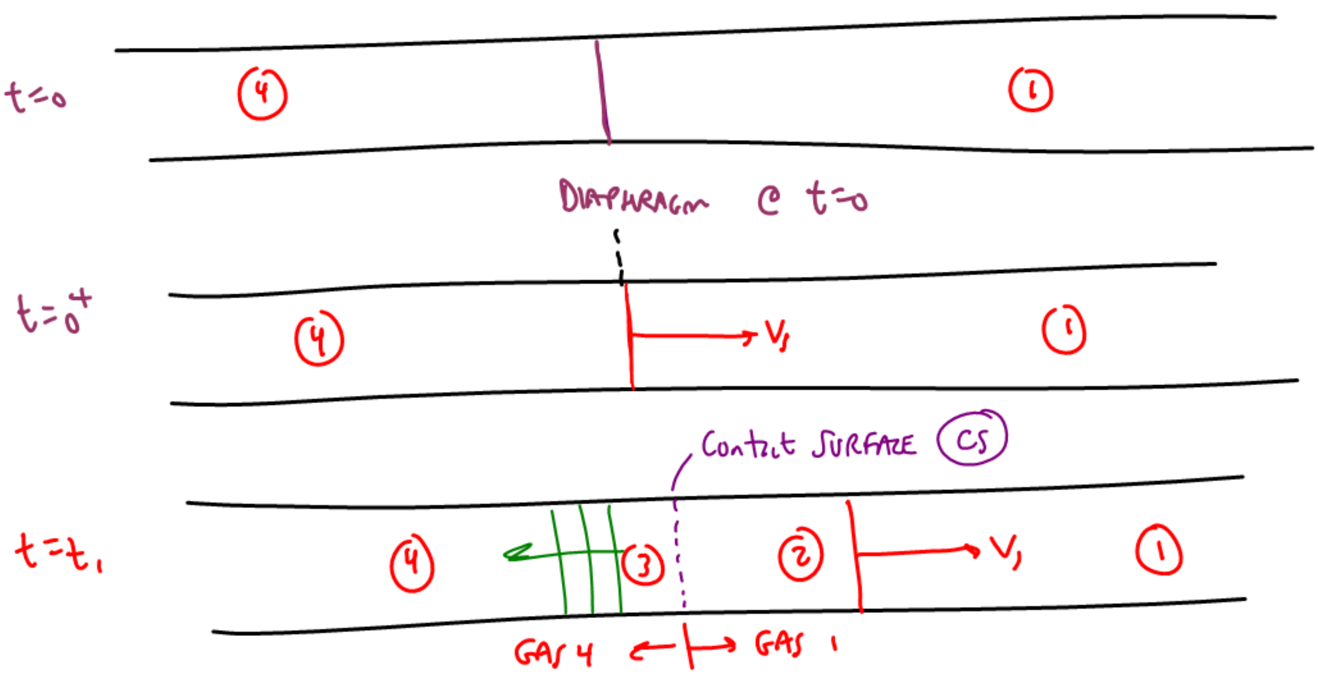
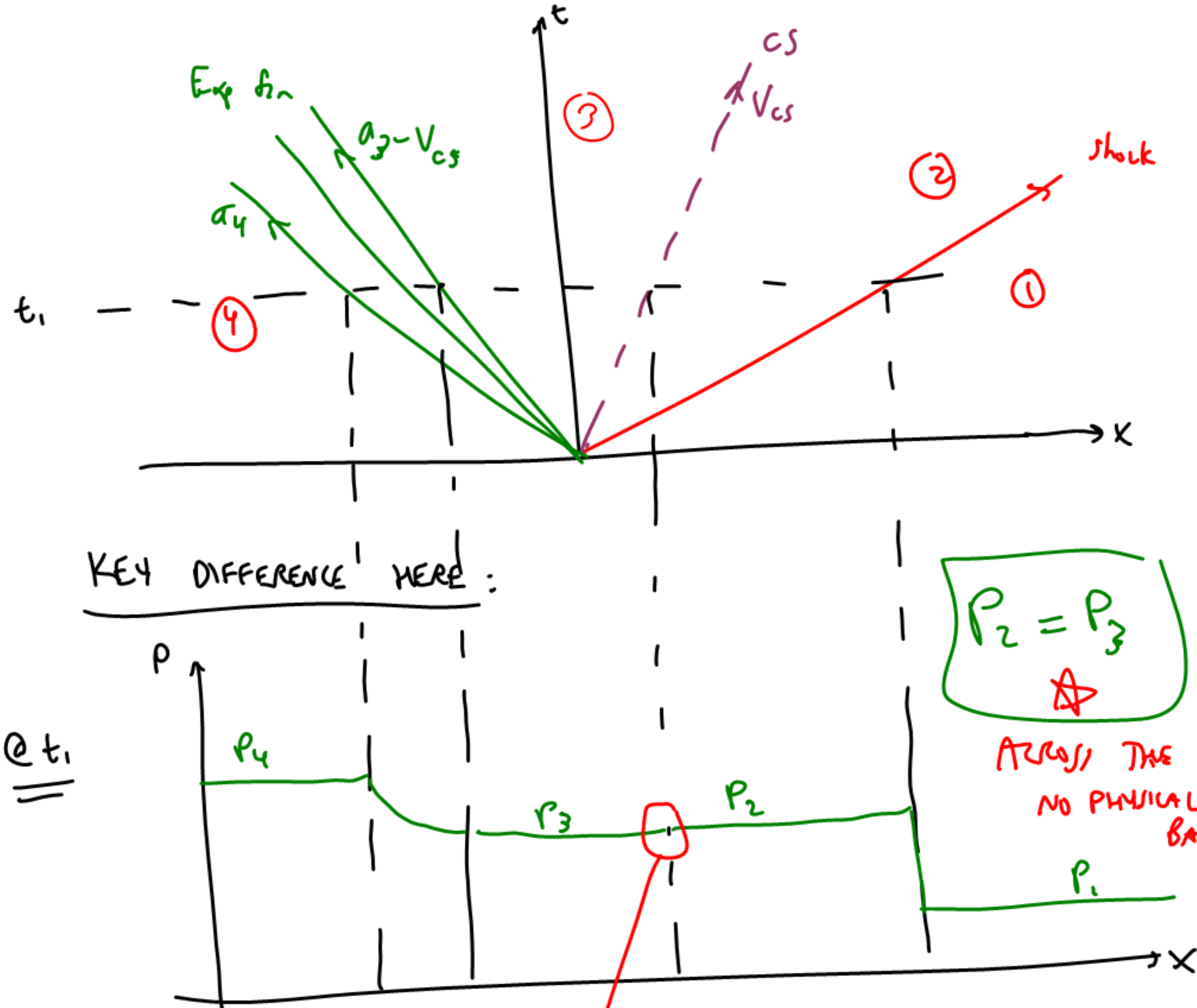
* **SHOCK TUBES** → REPLACE PISTON BY A DIAPHRAGM THAT INSTANTANEOUSLY BURSTS (DISAPPEARS) AT $t = 0^+$



CONTACT SURFACE IS LIKE OUR PISTON → SEPARATES GAS 1 FROM GAS 4

OUR ANALYSIS IS IDENTICAL TO THE PISTON CASE

V_{cs} replaces V_p



KEY DIFFERENCE HERE:

$$\boxed{P_2 = P_3}$$

ACROSS THE CS NO PHYSICAL BARRIER

For Piston case, P_2 not necessarily $= P_3$
For shock tube, P_2 must $= P_3$

ON NEXT PAGE IS A COMPUTER-DRAWN PLOT OF THE ABOVE

Shock tube *x-t* and *x-P* diagrams: