Today, we will:

- Discuss Rayleigh flow [heat added to (or removed from) a duct] qualitatively
- Discuss Rayleigh flow quantitatively

Rayleigh flow

Introduction: Approximations and Assumptions:

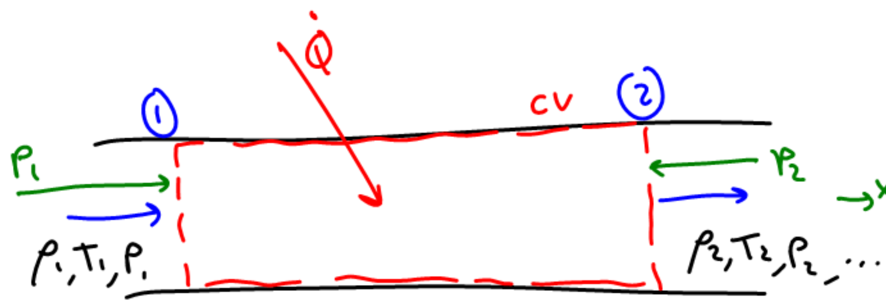
- One-D flow (ignore boundary layers – V approx. constant at any cross-section of the duct, i.e, at any x location; so, $V = V(x)$ only
- Ideal gas
- Constant area duct (straight section of pipe)
- Constant gas properties (γ, C_P, R, etc.) *even if chemical reactions or combustion provides the heat input* (different gas properties of the combustion products and/or different mixture of gases after a reaction)
- Negligible friction along duct walls

otherwise need numerical solution

– can average the constants after combustion & before combustion

& all other irreversibilities (separation, turbulent dissipation, etc)

Control volume analysis:



$\dot{Q}$ = rate of heat transfer in

$\dot{Q} > 0$ if heat added

$\dot{Q} < 0$ if heat removed

even if we inject fuel & burn to generate $\dot{Q}$ we ignore the additional $\dot{m}$

CONS. LAWS FOR THIS CV

• mass $\quad \dot{m}_1 = \dot{m}_2 = \dot{m} = \text{constant}$ (no leaks)

Since A = const, $\boxed{\rho_1 V_1 = \rho_2 V_2}$ (1)

• Linear mom $\quad \sum F_x = \sum_{out} \beta \dot{m} V - \sum_{in} \beta \dot{m} V$

ONE-D approx makes $\beta = 1$

$$\sum F_x = \sum F_{x_{pressure}} + \cancel{\sum F_{x_{grav}}} + \cancel{\sum F_{x_{friction}}} + \cancel{\sum F_{x_{other}}}$$

ignore neglect no struts, etc.

$P_1A_1 - P_2A_2$ in x-direction

$\rightarrow A_1 = A_2 = A$ here

x-mom reduces to $P_1A - P_2A = \dot{m}V_2 - \dot{m}V_1$

$\dot{m} = \rho_2 V_2 A$, $\dot{m} = \rho_1 V_1 A$

A's cancel,

$$\boxed{P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2} \qquad (2)$$

☆ **Eq. (1) + (2) = SAME AS WE HAD FOR A NORMAL SHOCK**

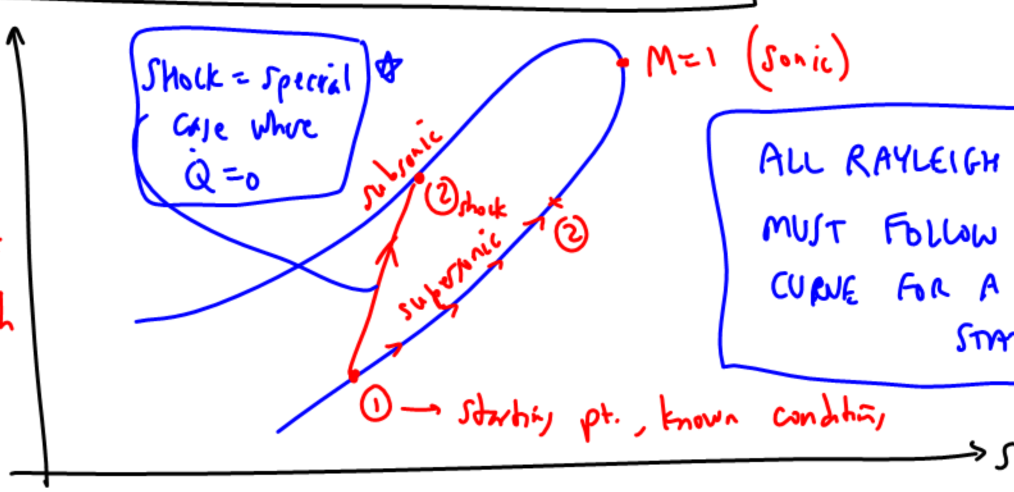
Also – same as $\underline{\text{Rayleigh curve}}$ previously discussed

we discussed for a normal shock

SAME RAYLEIGH CURVE AS PREVIOUSLY !

For Rayleigh flow we typically plot T instead of h

$\boxed{h = C_pT}$



☆ ALL RAYLEIGH FLOWS MUST FOLLOW THIS CURVE FOR A GIVEN STATE ①

SUMMARY: • NORMAL SHOCK = ADIABATIC $(\dot{Q}=0)$

$\therefore h_{o_1} = h_{o_2} \Rightarrow T_{o_1} = T_{o_2}$

(special case of Rayleigh flow)

THIS IS AT ONE SPECIAL POINT ② shock ON OUR PLOT

• RAYLEIGH FLOW (in general) $\dot{Q} \neq 0$

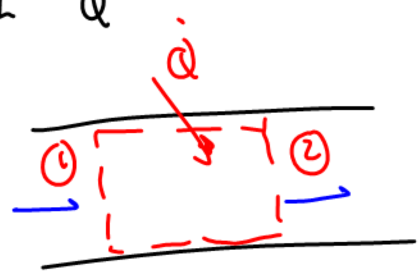
$\therefore$ $\boxed{T_{o_2} \neq T_{o_1}}$ $\boxed{h_{o_2} \neq h_{o_1}}$

$\therefore$ we follow the curve as we add or remove heat

Ⓠ Where do we end up on the Rayleigh curve (state ②)?

Ⓐ Depends on how much $\dot{Q}$

ENERGY EQ



$$\dot{Q} + \dot{m}\underbrace{\left(h_1 + \frac{V_1^2}{2}\right)}_{h_{o_1}} = \dot{m}\underbrace{\left(h_2 + \frac{V_2^2}{2}\right)}_{h_{o_2}}$$

$\div \dot{m}$: Let * $\boxed{q = \frac{\dot{Q}}{\dot{m}}}$ [q = heat transfer per unit mass of fluid]

"specific heat transfer"

$$\boxed{q = c_p\left(T_{o_2} - T_{o_1}\right)} \quad (3a)$$

OR (since) $h_1 + \frac{V_1^2}{2} = c_p T_1 + \frac{V_1^2}{2}$

ALTERNATE (3) is $\boxed{q = c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2}}$ (3b)

COMMENTS:

- (3a) → $T_0 \uparrow$ for $\oplus q$ (heating) $T_{0_2} > T_{0_1}$

  $T_0 \downarrow$ for $\ominus q$ (cooling) $T_{0_2} < T_{0_1}$

- Here $T_{0_2} \neq T_{0_1}$ as we've been using all semester

- (3b) T may go ↑ or ↓ or stay the same

  when $q \oplus$

  [strange case → can add heat ($\oplus q$) but $T \downarrow$]

  (if $V_2 \uparrow$ very rapidly compared to T change)

Other eqs:

- one of our Tds eq:

$$\boxed{s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}} \quad (4)$$

- Ideal gas law

$$\boxed{\frac{P_1}{\rho_1 T_1} = \frac{P_2}{\rho_2 T_2}} \quad (5)$$

- ENTROPY recall from thermo, $\boxed{ds = \frac{\delta q_{reversible}}{T}}$ ✱

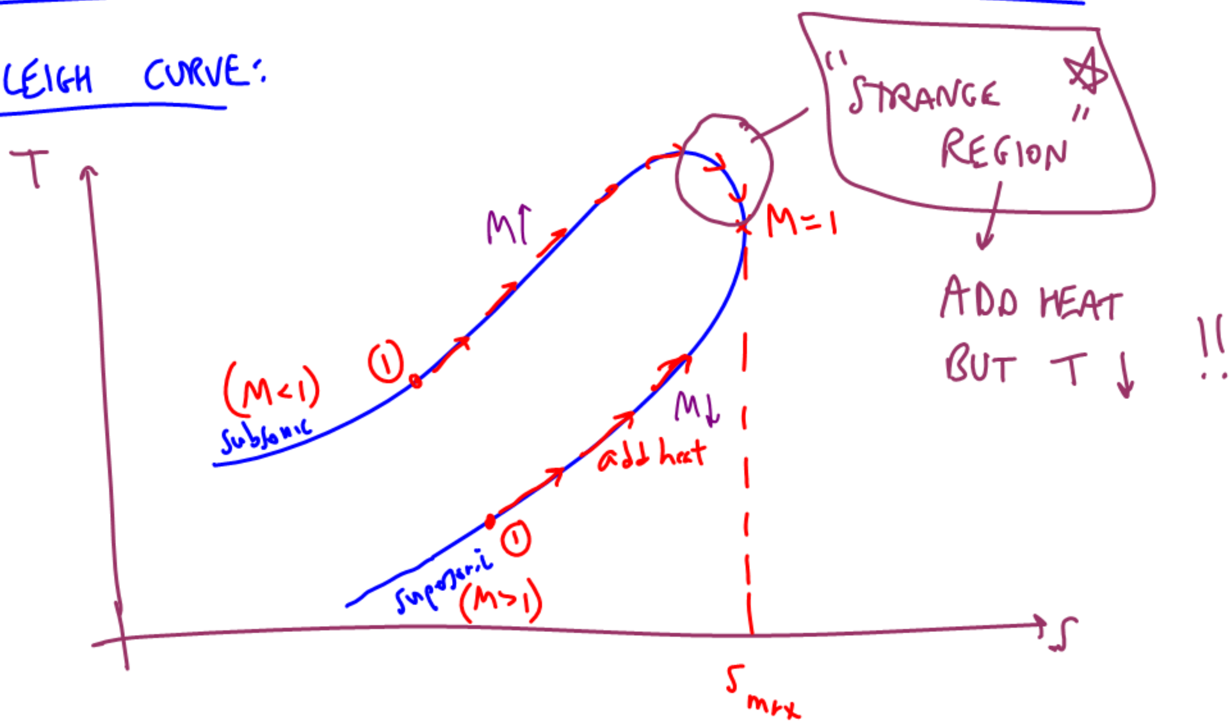
Practically we write $\boxed{ds = \frac{\overset{\text{actual } \delta q}{\delta q}}{T} + ds_{irreversible}}$ ✱

$ds_{irreversible}$ → always > 0 (2^{nd} law)

HERE, THERE ARE NO IRREVERSIBILITIES BY OUR A & A

Bottom line → $\boxed{\begin{matrix} s\uparrow & \text{for} & q\oplus \\ s\downarrow & \text{for} & q\ominus \end{matrix}}$

RAYLEIGH CURVE:



- If flow is supersonic, @ ①: M ↓ with x (or with additional $\dot{Q}$); M heads toward M=1; it cannot become subsonic (or s ↓)
- If flow is subsonic, @ 1: M ↑ with x (or added $\dot{Q}$); M heads toward M=1; it cannot become supersonic! (or s ↓)