Today, we will:

- Continue our discussion of Rayleigh flow: discuss ***choked*** Rayleigh flow
- Derive the equations needed to solve Rayleigh flow problems
- Do Candy Questions for Candy Friday

RAYLEIGH FLOW (continued)

THERE ARE TWO WAYS TO IMAGINE A RAYLEIGH FLOW EXPERIMENT:

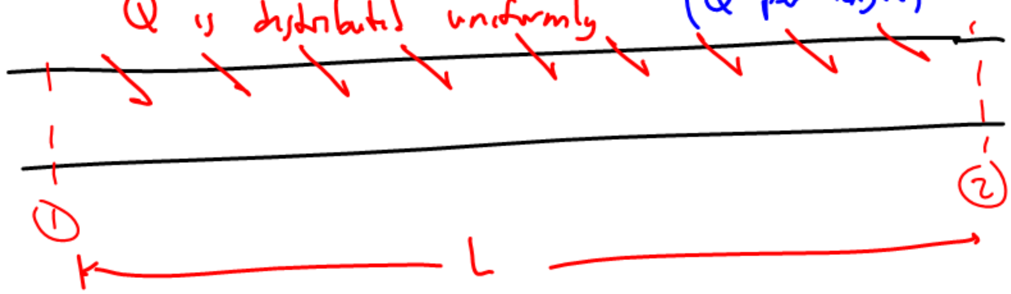
(A) Short tube ; $\dot{Q}$ added @ a "point" FLAME

$\dot{Q}$ determines where we move on Rayleigh line

(L does not matter)

(B) long tube ; add $\dot{Q}$ slowly

$\dot{Q}$ is distributed uniformly ($\dot{Q}$ per length)



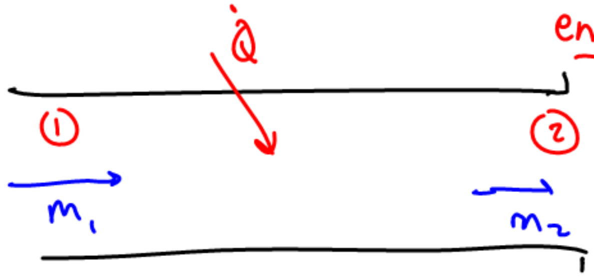
L does matter

L determines where you move along the curve.

Since we are neglecting friction, tube length is irrelevant in our equations

$\boxed{\dot{Q} \text{ is all that matters}}$ in the equations

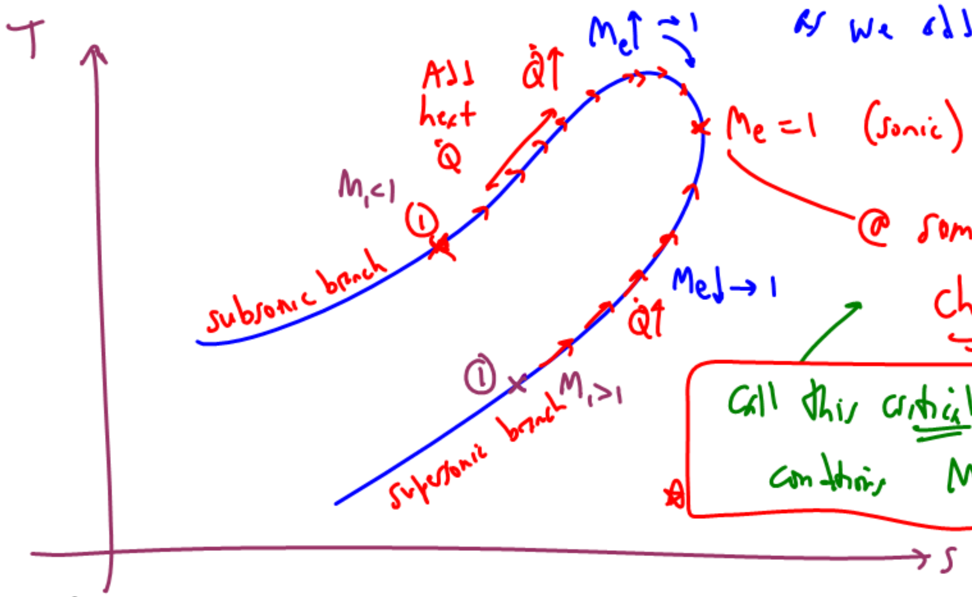
★ CHOKED RAYLEIGH FLOW



end of duct, or exit plane (e)

$M_2 = M_e$

But M_2 or $M_e \to 1$ (sonic) as we add more heat



@ some $\dot{Q}_{max}$, we <u>choke</u> the flow!

★ Call this <u>critical</u> or <u>sonic</u> conditions $M_2 = 1$, $T_2 = T^*$, $T_{o_2} = T_o^*$

· <u>SUBSONIC BRANCH</u>:

What happens if we add more heat after the flow gets choked? ($M_e = 1$)

· It cannot go supersonic

$\boxed{\text{So – the upstream conditions must change}}$ ★ Conditions @ 1 will change

★ Get a different Rayleigh curve that then chokes @ exit plane

Eqs: <u>When choked</u> $\dot{Q} = \dot{Q}_{max}$ $T_2 = T^*$ $T_{o_2} = T_o^*$ $P_2 = P^*$ etc.

recall, energy eq. → $q = \dfrac{\dot{Q}}{\dot{m}} = c_p\left(T_{o_2} - T_{o_1}\right)$

When choked,

$$\boxed{q = q_{max} = c_p\left(T_0^* - T_{0_1}\right)}$$

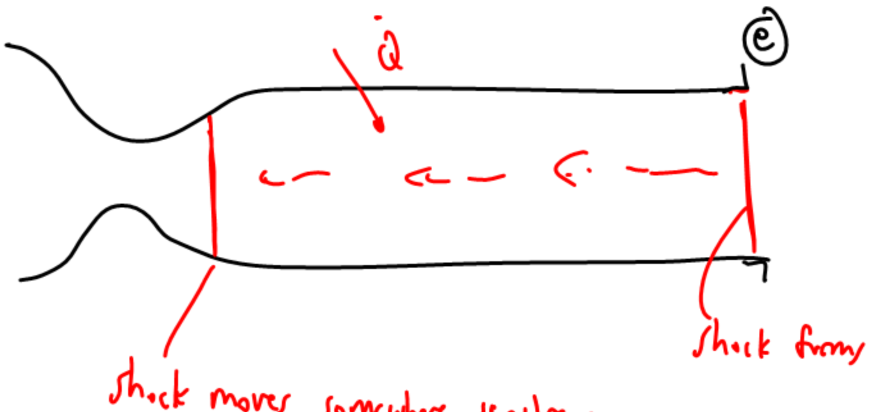
choked, max possible q

- SUPERSONIC BRANCH $M_1 > 1$ → as $\dot{Q}\uparrow$, $M_e\downarrow$ and approaches $M_e = 1$ (sonic)

When choked, $M_e = 1$ we have * conditions

(Q) What happens if it is choked & we add more heat?

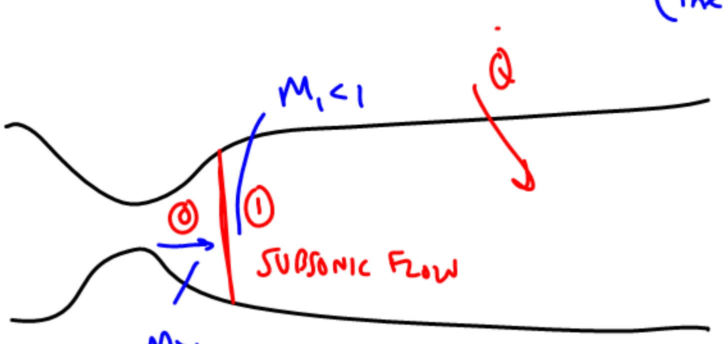
(A) A shock forms → shock forms @ exit plane & quickly moves upstream



Flow becomes subsonic after the shock

(the shock is "swallowed")

The Rayleigh curve changes to a different one



Now we are back to the subsonic branch

What happens to T_0? & T?

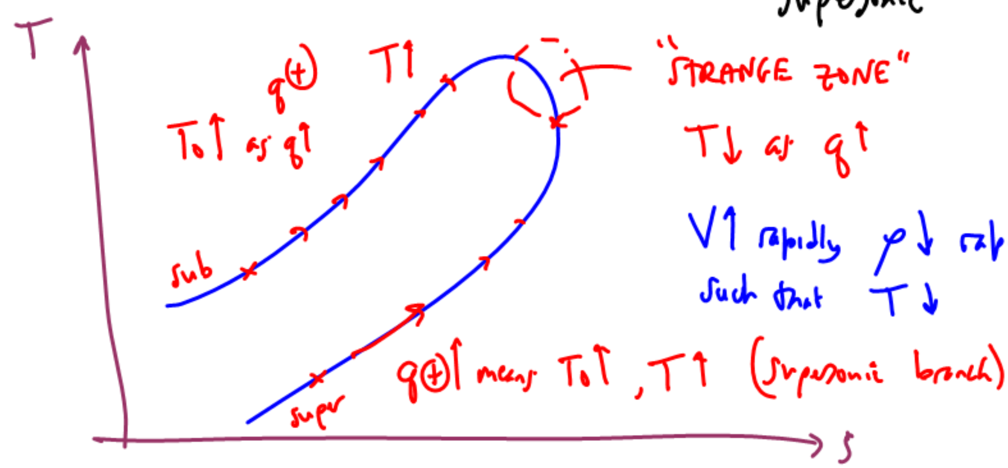
eq $\quad h_0 = h + \frac{V^2}{2}$

$$C_pT_0 = C_pT + \frac{V^2}{2}$$

$$\boxed{T_0 = T + \frac{V^2}{2C_p}}$$

& we have $\boxed{q = C_p(T_{0_2} - T_{0_1})}$ Rayleigh

For $q \oplus$, T_0 must go up $\uparrow$ as $\dot{Q}\uparrow$ whether subsonic or supersonic



Other Variables:

Eqs

Cons of mass $\quad$ OLD Eq (1) $\rightarrow$ $\boxed{\rho_1 V_1 = \rho_2 V_2} = \text{const}_1$

linear mom $\quad$ OLD Eq (2) $\rightarrow$ $\boxed{P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2 = \text{constant}_2}$

$$P_1 + \underbrace{\rho_1 V_1}_{\text{const}_1} V_1 = P_2 + \underbrace{\rho_2 V_2}_{\text{const}_1} V_2$$

$$P + \text{const}_1 V = \text{const}_2$$

$$\boxed{P = \text{const}_2 - \text{const}_1 V}$$

If $V\uparrow$, $P\downarrow$

If $V\downarrow$, $P\uparrow$

(just like Beloved Bernoulli)

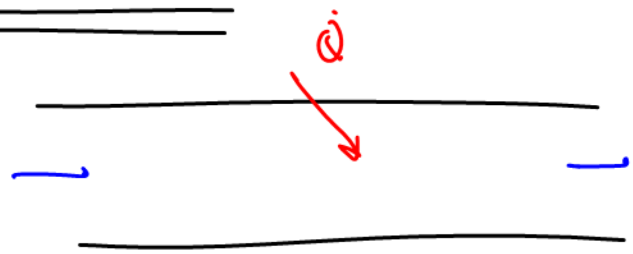
Here, for Rayleigh flow,

- Subsonic branch $V \uparrow$ as $q \uparrow \therefore P \downarrow$
- Supersonic branch $V \downarrow$ as $q \uparrow \therefore P \uparrow$ ☆

We know from Eq (1)

- Subsonic → $\rho \downarrow$ as $V \uparrow$
- Supersonic $\rho \uparrow$ as $V \downarrow$ ☆

☆ QUALITATIVE SUMMARY



ADDING HEAT

q ⊕

SUBSONIC

go up	go down
$V \uparrow$	$\rho \downarrow$
$M \uparrow$	$P \downarrow$
$s \uparrow$	$P_0 \downarrow$
$(h_0 \uparrow) - T_0 \uparrow$	
$T \uparrow$ or $\downarrow$ (strange zone)	

SUPERSONIC

go up	go down
$P \uparrow$	$V \downarrow$
$\rho \uparrow$	$M \downarrow$
$s \uparrow$	$P_0 \downarrow$
$(h_0 \uparrow) - T_0 \uparrow$	
$T \uparrow$	

If we <u>remove</u> heat, $s \downarrow$ $h_{o_2} < h_{o_1}$ $h_o \downarrow$

behavior of variables is <u>opposite</u>

{Most practical problems involve $\oplus\ q$}

$\boxed{\text{END OF EXAM 2 MATERIAL}}$ ☆

Qualitative Rayleigh is fair game

Quantitative " " not on E2