Today, we will:

- Finish Rayleigh flow equations
- Generate a procedure to solve Rayleigh flow problems
- Provide a brief biography – Who was Lord Rayleigh?
- Discuss applications of Rayleigh flow

Summary of equations for Rayleigh flow (for ideal gas only; see also Equation Sheet):

Conservation laws:

mass

$$\boxed{\rho_1 V_1 = \rho_2 V_2}$$

mom

$$\boxed{P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2}$$

$$\boxed{q = \frac{\dot{Q}}{\dot{m}} = c_P\left(T_{02} - T_{01}\right) = c_P\left(T_2 - T_1\right) + \frac{V_2^2 - V_1^2}{2}}$$

Maximum possible heat transfer: $\boxed{q_{\max} = \frac{\dot{Q}_{\max}}{\dot{m}} = c_P\left(T_0^* - T_{01}\right)}$

"MANIPULATE" THIS EQ:

$$\rho V^2 = \rho\left(a^2 M^2\right) = \underbrace{\rho \gamma R T}_{=P} M^2$$

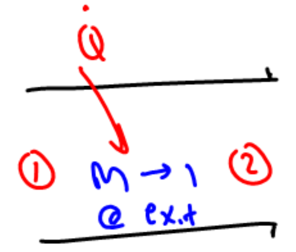
$$\boxed{\rho V^2 = \gamma P M^2}$$

$$P_1 + \gamma P_1 M_1^2 = P_2 + \gamma P_2 M_2^2 \rightsquigarrow \boxed{\frac{P_2}{P_1} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2}}$$

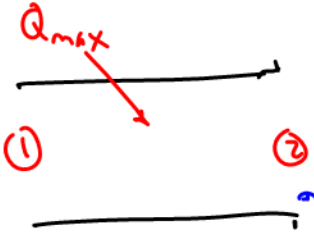
ALT. DERIVATION

Rayleigh eq !!

Holds between any 2 pts on the Rayleigh curve



$M_2 \to 1$ at the exit as $\dot{Q}\uparrow$



@ ②, $M_2 = 1$ (sonic, critical)

② = ⓔ = ⊛

It is convenient to use * conditions in solving problems even if * conditions do not actually occur in the problem

Static properties:

$$\boxed{\frac{P_2}{P_1} = \frac{1+\gamma M_1^2}{1+\gamma M_2^2}} \to \boxed{\frac{P}{P^*} = \frac{1+\gamma}{1+\gamma M^2}}$$

We can do more algebra to derive equations for temperature ratios, density ratios, velocity ratios, etc., along with stagnation property ratios.

Let ② = * (set $M_2 = 1$)
① = anywhere else in the flow ($M_1 = M$) & take reciprocal

We can derive similar eqs for $\frac{T}{T^*}$, $\frac{V}{V^*}$, etc.

& mass → $$\frac{\rho_1}{\rho_2} = \frac{V_2}{V_1} = \frac{M_2\sqrt{\gamma R T_2}}{M_1\sqrt{\gamma R T_1}} = \frac{M_2}{M_1}\sqrt{\frac{T_2}{T_1}}$$

$$P = \rho R T \rightarrow T = \frac{P}{\rho R} \Rightarrow \frac{T_2}{T_1} = \frac{P_2}{P_1}\frac{\rho_1}{\rho_2}$$

$$\boxed{\frac{T_2}{T_1} = \left[\frac{M_2\left(1+\gamma M_1^2\right)}{M_1\left(1+\gamma M_2^2\right)}\right]^2}$$ general eq.

Let ② = * ① = any other state

$$\boxed{\frac{T}{T^*} = \left[\frac{M(1+\gamma)}{1+\gamma M^2}\right]^2}$$

$$\boxed{\frac{\rho^*}{\rho} = \frac{V}{V^*} = \frac{M^2(1+\gamma)}{1+\gamma M^2}}$$

$$\frac{T_0^*}{T^*} = 1+\frac{\gamma-1}{2}(1)^2 = \frac{\gamma+1}{2}$$

$$\therefore \frac{T^*}{T_0^*} = \frac{2}{\gamma+1}$$

Stagnation Properties : $$\frac{T_0}{T_0^*} = \frac{T_0}{T}\,\frac{T}{T^*}\,\frac{T^*}{T_0^*}$$

$$\left(1+\frac{\gamma-1}{2}M^2\right)\left(\frac{M(1+\gamma)}{1+\gamma M^2}\right)^2\left(\frac{2}{\gamma+1}\right)$$

$$\boxed{\frac{T_0}{T_0^*} = \frac{\left(2+(\gamma-1)M^2\right)(\gamma+1)M^2}{\left(1+\gamma M^2\right)^2}}$$ ☆

Similarly

$$\boxed{\frac{p_0}{p_0^*} = \frac{\gamma+1}{1+\gamma M^2}\left[\frac{2+(\gamma-1)M^2}{\gamma+1}\right]^{\frac{\gamma}{\gamma-1}}}$$

Summary of equations for Rayleigh flow (for ideal gas only; see also Equation Sheet):

Conservation laws:

$$\boxed{\rho_1 V_1 = \rho_2 V_2} \quad \boxed{P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2} \quad \boxed{q = \frac{\dot{Q}}{\dot{m}} = c_P\left(T_{02} - T_{01}\right) = c_P\left(T_2 - T_1\right) + \frac{V_2^{\,2} - V_1^{\,2}}{2}}$$

Maximum possible heat transfer: $\boxed{q_{\max} = \frac{\dot{Q}_{\max}}{\dot{m}} = c_P\left(T_0^{\,*} - T_{01}\right)}$

Static properties:

$$\boxed{\frac{P_2}{P_1} = \frac{1+\gamma M_1^{\,2}}{1+\gamma M_2^{\,2}}} \rightarrow \boxed{\frac{P}{P^*} = \frac{1+\gamma}{1+\gamma M^2}} \qquad \boxed{\frac{T_2}{T_1} = \left[\frac{M_2}{M_1}\frac{1+\gamma M_1^{\,2}}{1+\gamma M_2^{\,2}}\right]^2} \rightarrow \boxed{\frac{T}{T^*} = \left[\frac{(1+\gamma)M}{1+\gamma M^2}\right]^2}$$

Combined with ideal gas law, speed of sound, and cons. of mass: $\boxed{\frac{\rho^*}{\rho} = \frac{V}{V^*} = \frac{(1+\gamma)M^2}{1+\gamma M^2}}$

Stagnation properties:

$$\boxed{\frac{T_0}{T_0^{\,*}} = \frac{\left[2+(\gamma-1)M^2\right](1+\gamma)M^2}{\left[1+\gamma M^2\right]^2}} \qquad \boxed{\frac{P_0}{P_0^{\,*}} = \left[\frac{2+(\gamma-1)M^2}{1+\gamma}\right]^{\frac{\gamma}{\gamma-1}} \frac{(1+\gamma)}{1+\gamma M^2}}$$

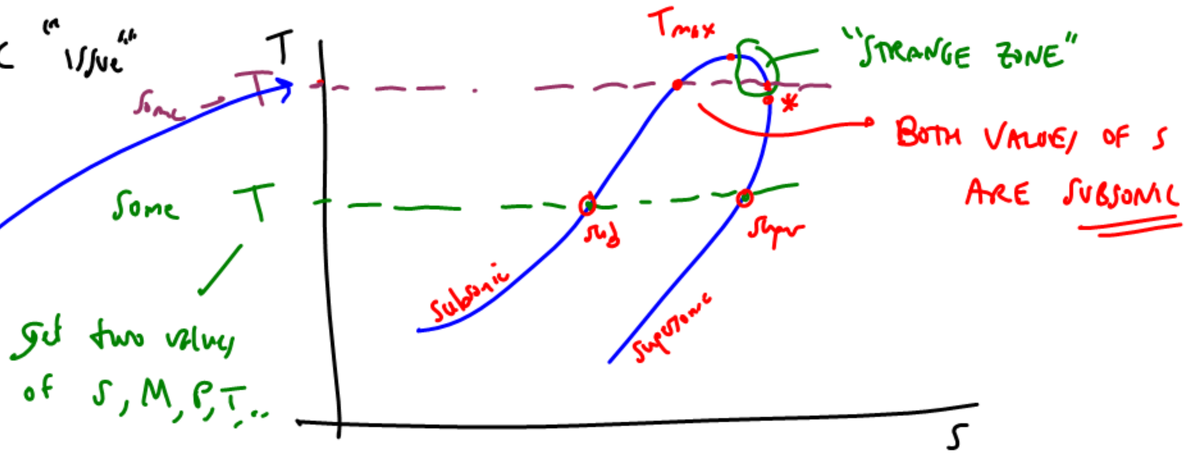
See Rayleigh flow also the on-line ***Compressible Aerodynamics Calculator***! ☆

Example : compare with CAC

@ $M = 0.3$; air $\gamma = 1.40$

My calcs	Agree with CAC ?
$\frac{T_0}{T_0^*} = 0.34686$	✓
$\frac{P_0}{P_0^*} = 1.1985$	✓
$\frac{T}{T^*} = 0.40887$	✓ ☺

CAC "issue"



IN CAC, "below T_{max}" means to left of T_{max} on T-s diagram
"above T_{max}" " — — right " — — —

Example Let $\boxed{\frac{T}{T^*} = 1.01}$ above T^* on our plot

CAC. $\frac{T}{T^*}$ (below T_{max}) ⇒ $\underline{M = 0.7382}$

$\frac{T}{T^*}$ (above T_{max}) ⇒ $\boxed{M = 0.9675}$

Both SUBSONIC

* This one is in "strange zone"

Procedure to solve Rayleigh flow problems:

1. For known conditions at 1 and known rate of heat transfer, use $\boxed{q = \frac{\dot{Q}}{\dot{m}} = c_P\left(T_{02} - T_{01}\right)}$ to calculate T_{02}.
2. Calculate T_{01}/T_0^* from the ratio equation: $\boxed{\frac{T_{01}}{T_0^*} = \frac{\left[2+(\gamma-1)M_1^2\right](1+\gamma)M_1^2}{\left[1+\gamma M_1^2\right]^2}}$.
3. Calculate T_{02}/T_0^* from clever use of ratios: $\boxed{\frac{T_{02}}{T_0^*} = \frac{T_{02}}{T_{01}}\frac{T_{01}}{T_0^*}}$.
4. Use the ratio equation for stagnation temperature (inversely) to calculate $\boxed{M_2}$:

$$\boxed{\frac{T_{02}}{T_0^*} = \frac{\left[2+(\gamma-1)M_2^2\right](1+\gamma)M_2^2}{\left[1+\gamma M_2^2\right]^2}}$$

*IMPLICIT EQ –
1) False Position Method
2) Newton's "
3) Goal seek, etc...

5. Use the remaining Rayleigh flow equations, ideal gas law, speed of sound equation, etc. to calculate other desired properties at state 2, such as T_2, P_2, V_2, a_2, h_2, ρ_2, etc.

Who was Lord Rayleigh?

- Real name: John William Strutt, the 3rd Baron Rayleigh
- Lived 1842-1919

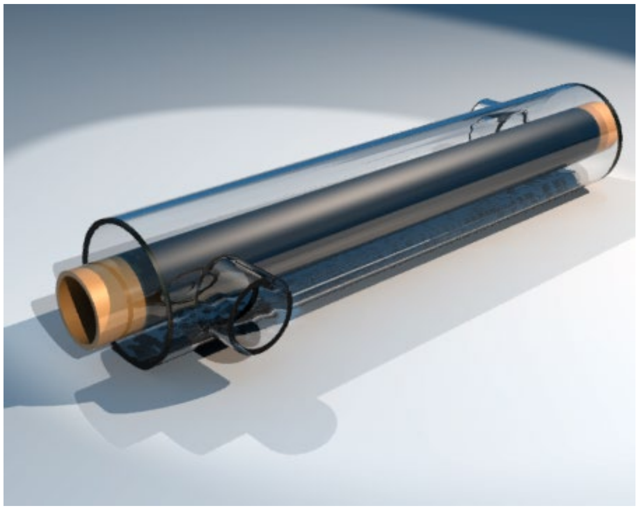
Accomplishments:

- Nobel Prize for Physics (1904) for the discovery of argon gas
- Rayleigh scattering: why the sky is blue
- Rayleigh waves
- Rayleigh number
- Rayleigh-Taylor instability
- Rayleigh-Benard convection
- Rayleigh-Pitot formula (we discussed previously)
- Rayleigh flow
- Wrote *The Theory of Sound*
- Developed Dimensional Analysis
- Christian

Applications of Rayleigh flow

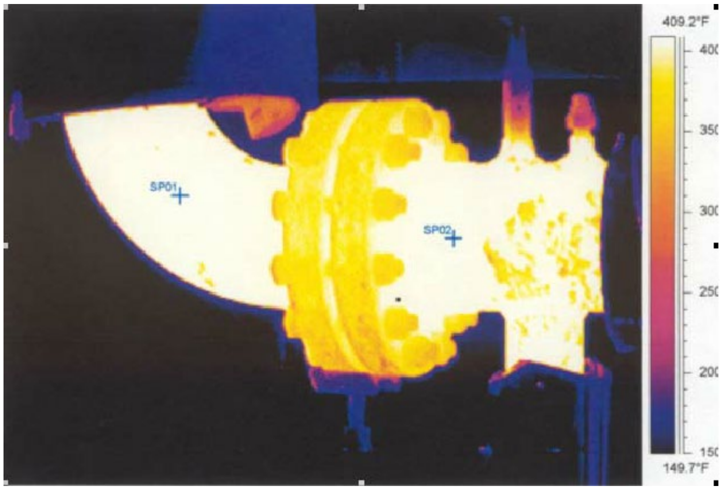
Gas heat exchangers: (Heating or cooling)

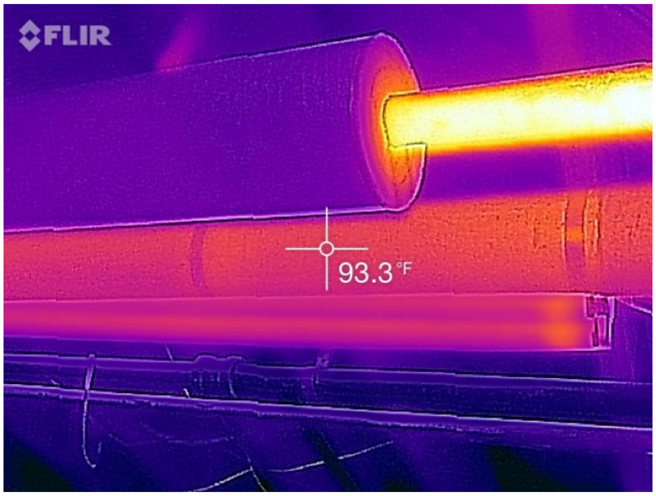
* Tubes cannot be too long or friction on walls becomes significant



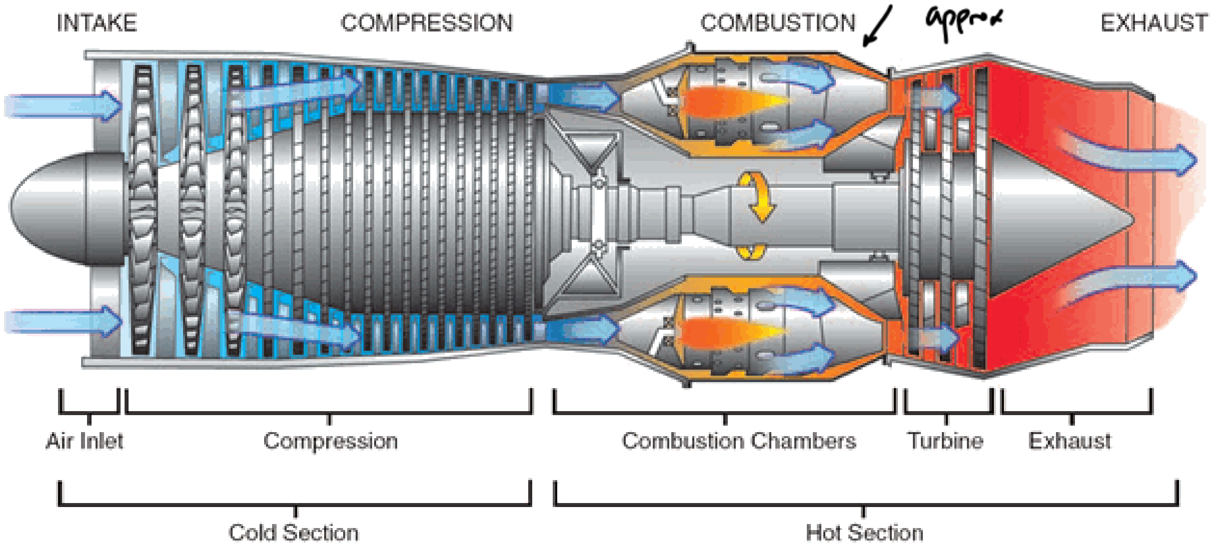
crossflow heat exchanger

Gas flowing in uninsulated pipes (e.g., steam pipes):

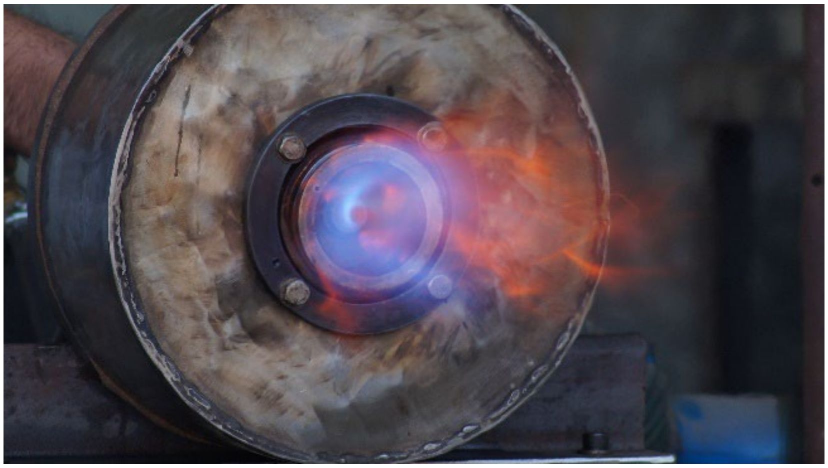




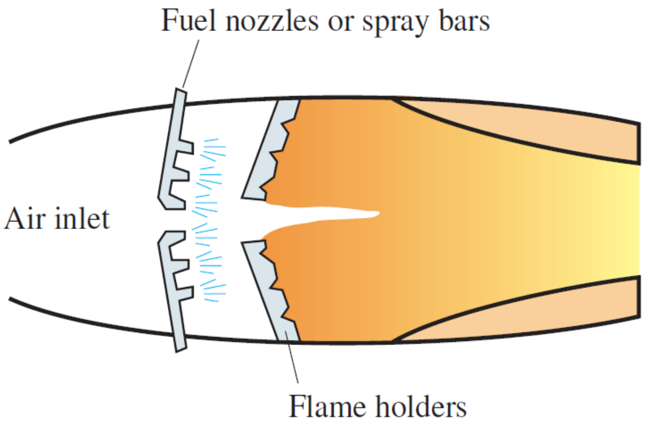
Combustors in a jet engine or gas turbine engine:

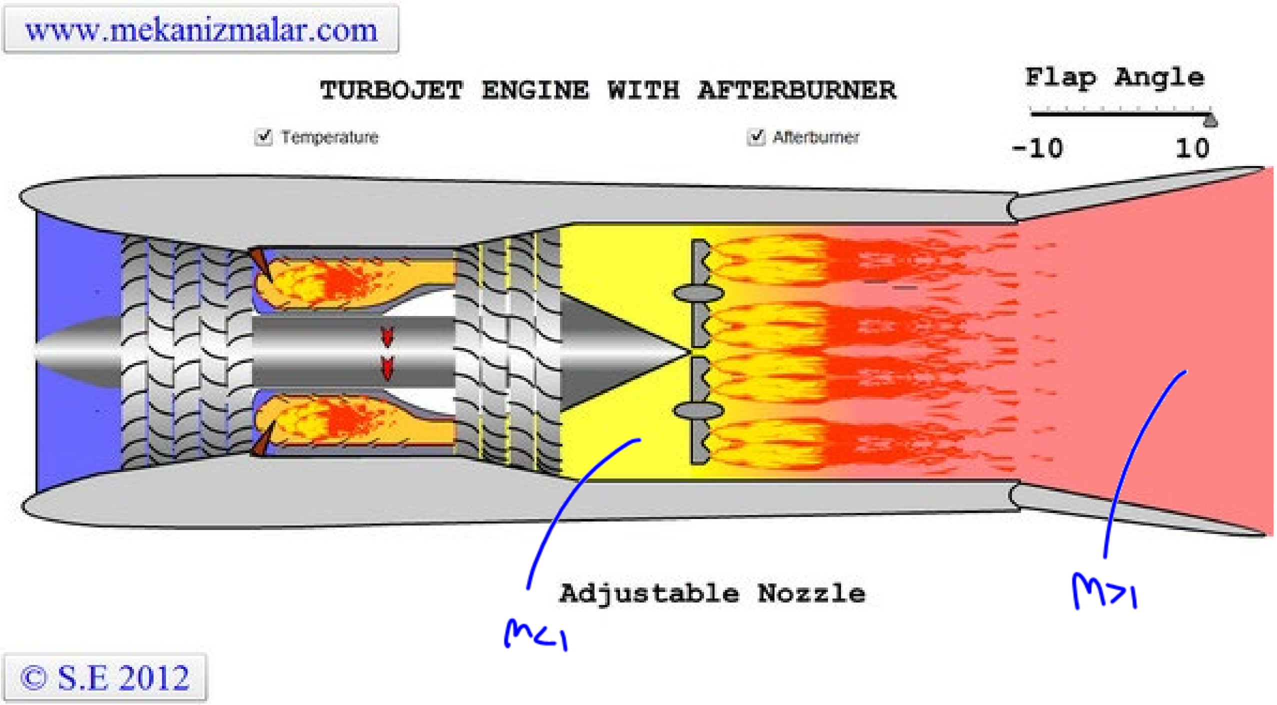


Rayleigh is good approx



Afterburners in a jet engine:




www.mekanizmalar.com

TURBOJET ENGINE WITH AFTERBURNER

☑ Temperature

☑ Afterburner

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