Today, we will:

- Do an example problem, Rayleigh flow
- Begin to discuss **Fanno Flow**: flow in a pipe with *friction* but no heat transfer

Example: Rayleigh flow

Given:

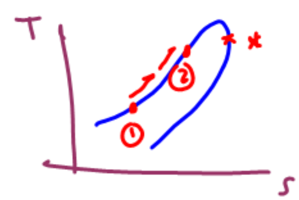
- Air and fuel enter a 15-cm diameter tube at 550 K, 480 kPa, and 80.0 m/s.
- The fuel is burned between locations 1 and 2 in the tube, as sketched, and in the process, 4514 kW of heat is added.

To do: Estimate the temperature, pressure, velocity, and Mach number at location 2.

Solution:

Assumptions and Approximations (consistent with our simplified Rayleigh flow analysis):

1. The air/fuel mixture is an ideal gas with the same properties as air alone, and the properties do not change due to combustion products.
2. The flow is steady and one-D.
3. Friction along the tube walls is negligible.



Inlet conditions:

Given: $V_1 = 80.0$ m/s, $T_1 = 550$ K, $P_1 = 480$ kPa, $A_1 = A_2 = \pi D^2/4 = 0.01767$ m^2.

Calculate: $\rho_1 = 3.0409$ kg/m^3, $a_1 = 470.10$ m/s, $M_1 = 0.17018$. ← SUBSONIC BRANCH

($\rho_1 = P_1/(RT_1)$), ($a_1 = \sqrt{\gamma R T_1}$), ($M_1 = V_1/a_1$)

Calculate: $T_{01} = 553.19$ K. I will show two ways to do this.

A] $$T_{01} = \text{stag. } T \text{ @ } ① = T_1 + \frac{V_1^2}{2c_p} = 550\text{ K} + \frac{(80.0\ \text{m/s})^2}{2\left(1004.5\ \frac{\text{J}}{\text{kg}\cdot\text{K}}\right)}\left(\frac{\text{J}}{\text{N}\cdot\text{m}}\right)\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)$$
$$= 553.19\text{ K}$$

B] $$T_{01} = \frac{T_{01}}{T_1}T_1 = \left(1 + \frac{\gamma-1}{2}M_1^2\right)T_1 = 553.19\text{ K}$$

So $T_{01} = 553.19$ K.

Can calculate other properties at the inlet if needed.

Heat transfer analysis (Rayleigh flow):

Given: $\dot{Q} = 4514$ kW

$\dot{m} = \rho_1 V_1 A_1 = 4.2989 \ \frac{kg}{s} = \dot{m}$

Calculate: $q = \frac{\dot{Q}}{\dot{m}} = \frac{\dot{Q}}{\rho_1 V_1 A_1} = 1050.0 \ \frac{\text{kJ}}{\text{kg}} = \frac{4514 \text{ kW}}{4.2989 \frac{kg}{s}} \left(\frac{kJ}{s \cdot kW}\right) = 1050.0 \ \frac{kJ}{kg} = q$

Calculate: $T_{02} = T_{01} + \frac{q}{c_P} = 1598.5$ K. $= 553.19 \text{ K} + \frac{1050.0 \text{ kJ/kg}}{1.0045 \text{ kJ/kg·K}} = 1598.5 \text{ K}$

Sonic (critical or *) reference values analysis (Rayleigh flow):

Calculate T_{01}/T_0^* from the ratio equation: $\frac{T_{01}}{T_0^*} = \frac{\left[2+(\gamma-1))M_1^2\right](1+\gamma)M_1^2}{\left[1+\gamma M_1^2\right]^2} = 0.12914$.

Calculate $\frac{T_{02}}{T_0^*} = \frac{T_{02}}{T_{01}}\frac{T_{01}}{T_0^*} = 0.37315$. $= \frac{1598.5 \text{ K}}{553.19 \text{ K}}(0.12914) = 0.37315$

Calculate M_2 (inversely any way you can!):

$$\frac{T_{02}}{T_0^*} = \frac{\left[2+(\gamma-1))M_2^2\right](1+\gamma)M_2^2}{\left[1+\gamma M_2^2\right]^2} = 0.37315 \quad \rightarrow \quad M_2 = 0.31429$$

THIS IS AN IMPLICIT EQ

Solve any way you can.

To be completed in class.

$$\dot{Q}_{max} = \dot{m} c_p (T_{02} - T_{01}) = \rho_1 V_1 A_1 c_p (T_0^* - T_1)$$

($T_{02} = T_0^*$)

$$\dot{Q}_{max} = 16110 \text{ kW}$$

our $\dot{Q} = 4514 \text{ kW} < \dot{Q}_{max}$ ✓ (otherwise the flow would have choked)

$$T_0^* = \frac{T_{01}}{(T_{01}/T_0^*)} = 4283.8 \text{ K} = T_0^*$$

Rest of props @ ② can now be calculated since we know M_2

e.g., $\frac{T_2}{T^*} = \left[\frac{M_2(1+\gamma)}{1+\gamma M_2^2}\right]^2$ $\qquad \frac{T_1}{T^*} = \left[\frac{M_1(1+\gamma)}{1+\gamma M_1^2}\right]^2$

$$T_2 = \frac{T_2}{T^*}\left(\frac{T_1}{T^*}\right)^{-1} T_1 = \boxed{1567.6\ K = T_2}$$

OR

$$\frac{T_{02}}{T_2} = 1 + \frac{\gamma - 1}{2} M_2^2 \rightarrow T_2 = \frac{T_{02}}{(T_{02}/T_2)} = \underline{1567.6\ K}$$

ü

OR

$$\frac{T_2}{T_1} = \left[\frac{M_2}{M_1}\ \frac{1+\gamma M_1^2}{1+\gamma M_2^2}\right]^2$$

(on eq sheet for Rayleigh flow)

$$\downarrow$$

$$\underline{1567.5\ K}$$

ü

Summary: (3 digits)

$T_2 = 1570\ K$	$T \uparrow$	$T_1 = 550\ K$
$P_2 = 439\ K$	$P \downarrow$	$P_1 = 480\ kPa$
☆ $V_2 = \underline{249}\ m/s$	$V \uparrow$	$V_1 = 80.0\ m/s$
$M_2 = \underline{0.314}$	$M \uparrow$	$M_1 = 0.170$

If we added **MORE** $\dot{Q}$, $M_2 \rightarrow 1$

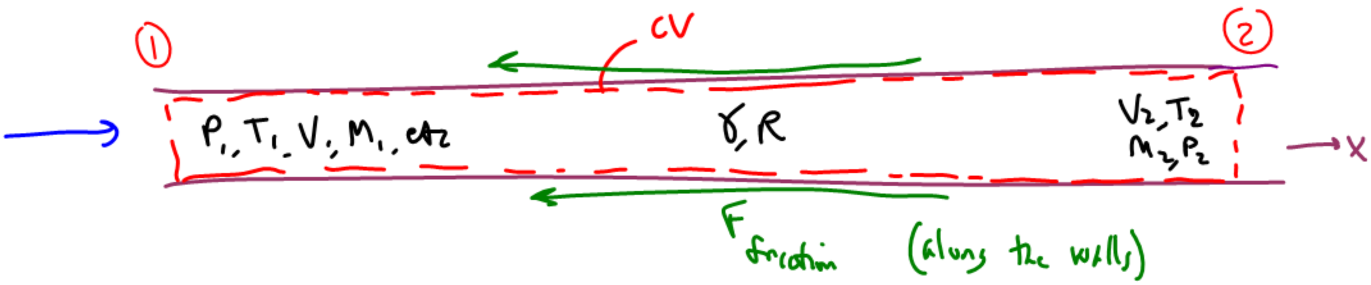
until $\boxed{\dot{Q}_{max} = 16{,}110\ kW}$

☆ FANNO FLOW

Compressible one-D flow w/ friction but no heat transfer (adiabatic)

if heat transfer ≠ 0 → diabatic

- ideal gas, constant properties
- No area change $A = A_1 = A_2$



CONS. LAWS

- Mass

$$\rho_1 V_1 \cancel{A_1} = \rho_2 V_2 \cancel{A_2}$$

$$\boxed{\rho_1 V_1 = \rho_2 V_2} \quad \text{or} \quad \boxed{\rho V = \text{constant}} \qquad (1)$$

- Energy

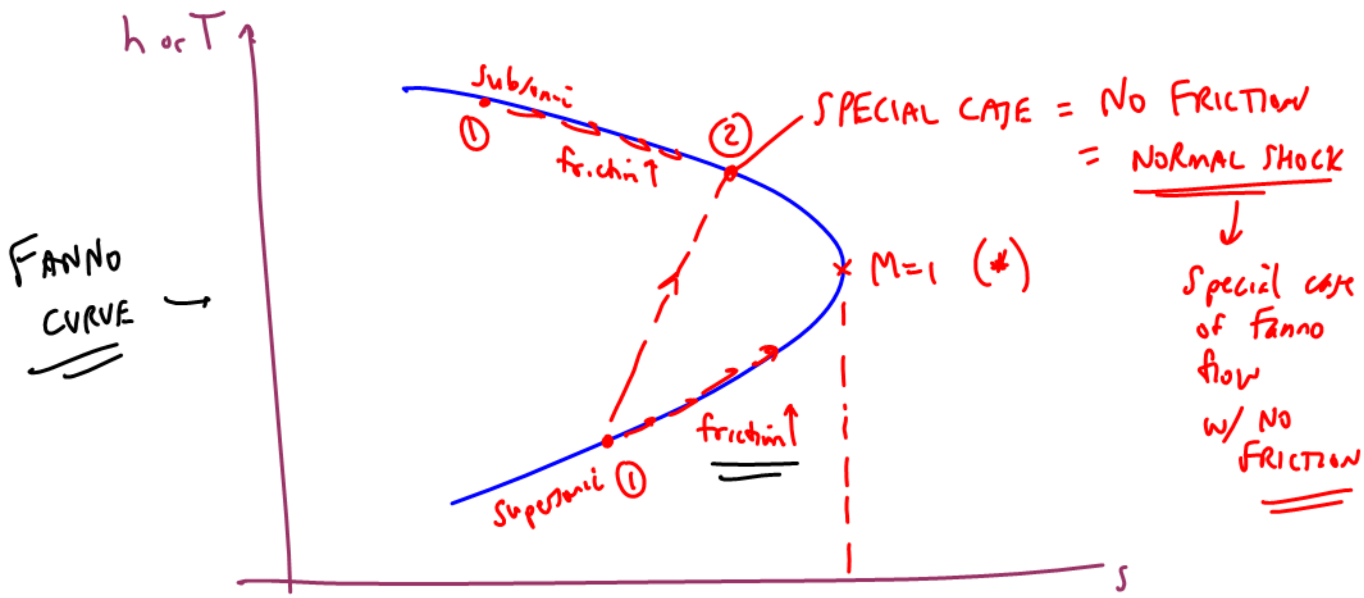
$$\boxed{h_{o_1} = h_{o_2}} \quad \text{since adiabatic}$$

$$\boxed{h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}}$$

$$\boxed{T_{o_1} = T_{o_2}}$$

$$\boxed{c_p T_1 + \frac{V_1^2}{2} = c_p T_2 + \frac{V_2^2}{2}} \qquad (2)$$

☆ Recall, FANNO CURVE = Locus of states that satisfy (1) & (2)



- For a given state ①, we move to another pt along the Fanno curve depending on friction force in the duct

- Compare to Rayleigh → Rayleigh s ↑ or ↓

  Fanno s always ↑

  (friction is irreversible)

- Cannot go past M=1 (*) because s would go ↓

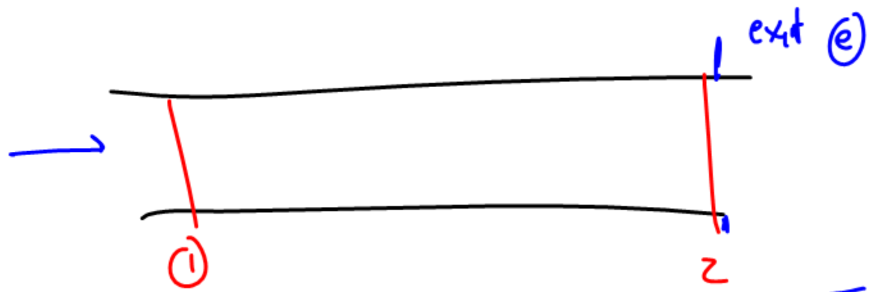
FANNO FLOW CAN CHOKE LIKE RAYLEIGH FLOW

IF ADD TOO MUCH FRICTION

- We typically plot T-s not h-s diagrams for Fanno

$[h = C_p T]$

EXAMPLE



$T_2 = T_e$
$P_2 = P_e$

- Air
- $M_1 = 0.2$ $\boxed{T_1 = 300\,K}$ $P_1 = 240\ kPa$
- Pipe is long enough to just choke the flow @ (2)

Calc T_2

$$\frac{T_{0_1}}{T_1} = 1 + \frac{\gamma-1}{2} M_1^2 \Rightarrow T_{0_1} = 302.4\,K$$

$T_{0_1} = T_{0_2}$

$$\frac{T_{0_2}}{T_2} = 1 + \frac{\gamma-1}{2} M_2^2$$

$M_2 = 1$ if choked

$$\boxed{T_2 = 252\,K}$$

$T \downarrow$ for subsonic flow