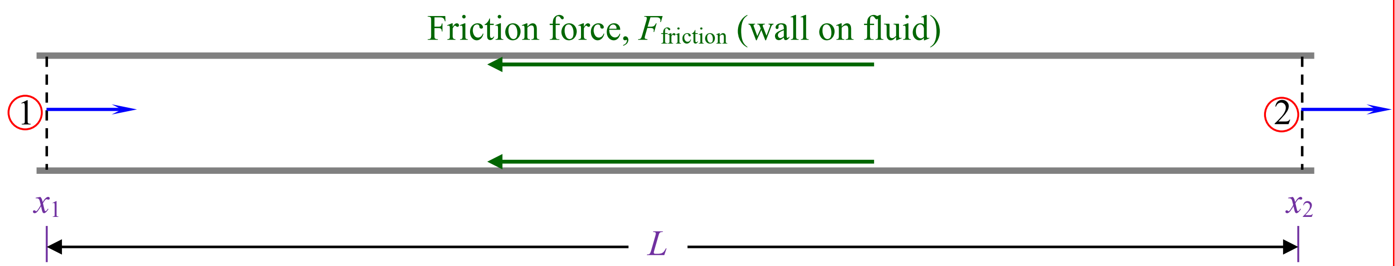
Today, we will:

- Continue to discuss Fanno flow, look at the momentum equation
- Discuss Fanno flow *qualitatively*, and compare with other 1-D flows we have discussed

Fanno Flow (continued):



Review from last time… Conservation equations:

ENERGY

MASS

$$\boxed{\rho_1 V_1 = \rho_2 V_2 \qquad \text{or} \qquad \rho V = \text{constant}} \quad (1)$$

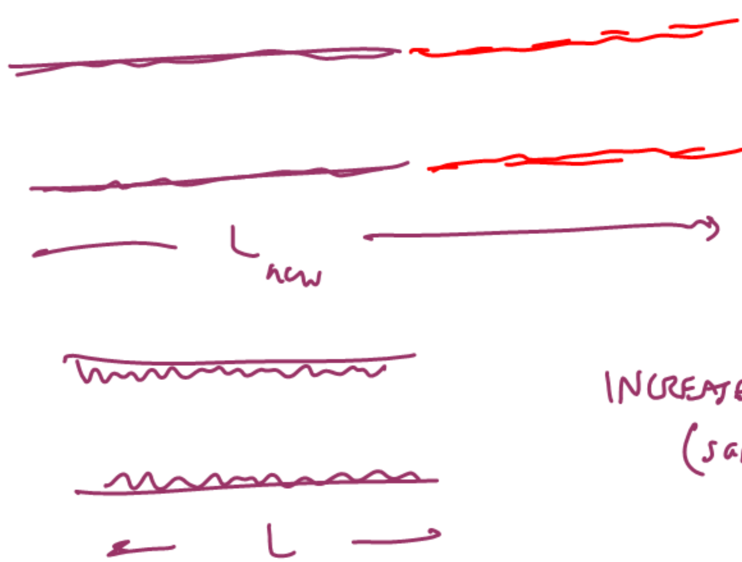
ONE-D APPROX

$$\boxed{T_{01} = T_{02} \qquad \text{or} \qquad c_P T_1 + \frac{V_1^2}{2} = c_P T_2 + \frac{V_2^2}{2}} \quad (2)$$

adiabatic

Now we need to look at the **momentum equation**.

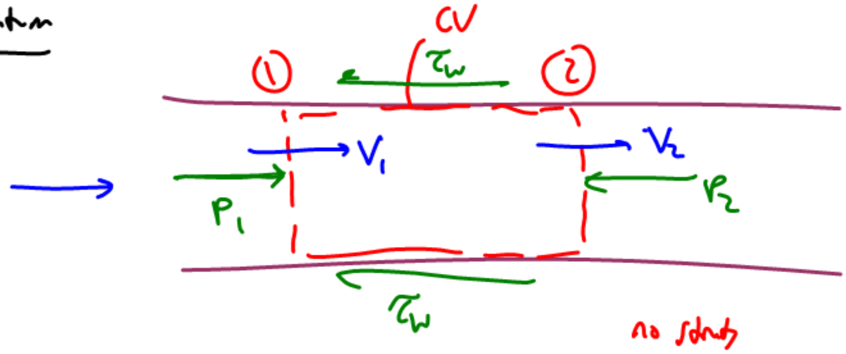
TWO WAYS TO INCREASE FRICTION



INCREASE LENGTH (same roughness)

INCREASE ROUGHNESS (same length)

X-momentum



β = momen. flux correction factor

no shear

$$\sum F_x = \underbrace{\sum F_{x_{press}}}_{} + \cancel{\sum F_{x_{grav}}} + \underbrace{\sum F_{x_{visc}}}_{} + \cancel{\sum F_{x_{other}}} = \sum_{out} \beta \dot{m} V - \sum_{in} \beta \dot{m} V$$

$$P_1 A_1 - P_2 A_2 - F_{friction} = \beta_2 \underbrace{\dot{m}}_{\rho_2 V_2 A_2} V_2 - \beta_1 \underbrace{\dot{m}}_{\rho_1 V_1 A_1} V_1$$

$(A_1 = A_2 = A$ everywhere$)$

$\beta \neq 1$ for fully developed duct flow

turb.

$\beta \approx 1.03$ for fully dev. turb flow

☆ BUT → AUTHORS SET $\beta = 1$ FOR FANNO FLOW

Let $\beta = 1$

$$\boxed{P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2 + \frac{F_{friction}}{A}} \qquad (3)$$

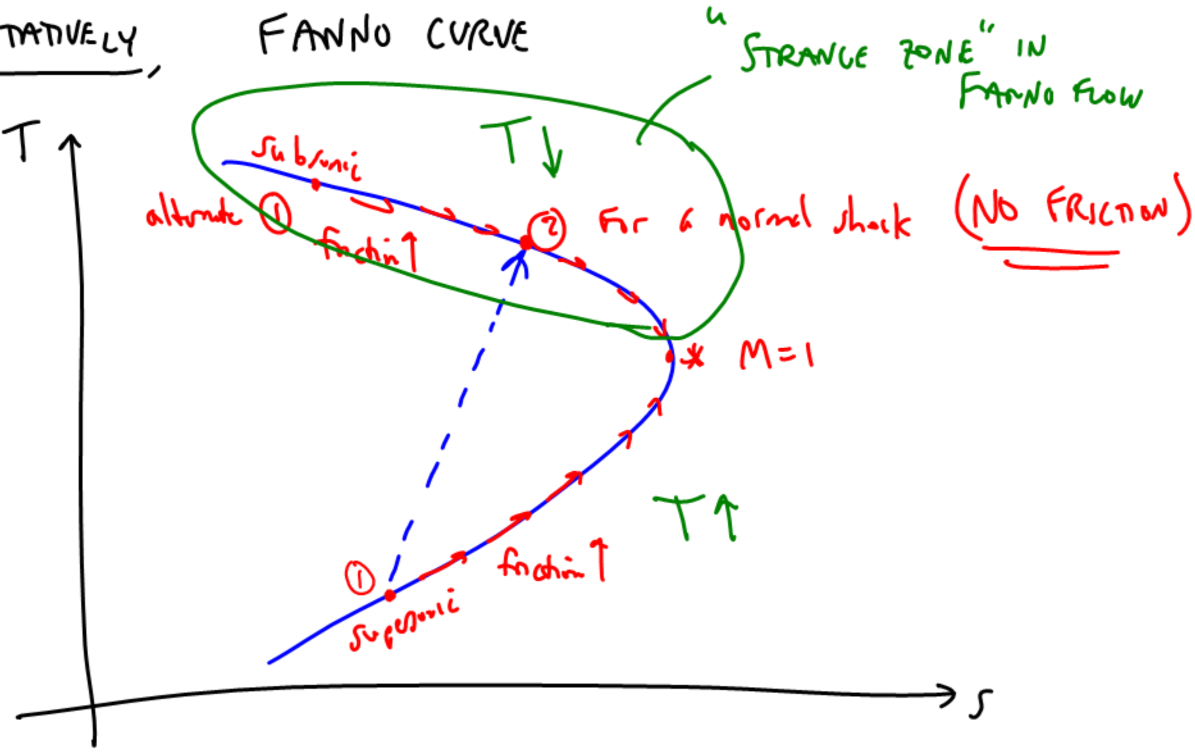
- All terms here dimensions $\left\{\frac{\text{Force}}{\text{Area}}\right\}$
- If $F_{friction} = 0$ → get same mom. eq. as we had for a normal shock [Eqs (1), (2), (3)]

Other Eqs:

T-ds $\boxed{T\,ds \rightarrow s_2 - s_1 = c_p \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1}}$ (4)

Ideal gas $\boxed{\frac{P_1}{\rho_1 T_1} = \frac{P_2}{\rho_2 T_2}}$ or $\boxed{\frac{P}{\rho T} = \text{const}}$ (5)

QUALITATIVELY, FANNO CURVE



SUBSONIC FANNO FLOW $M\uparrow$ $T\downarrow$ $M \to 1$

SUPERSONIC " " $M\downarrow$ $T\uparrow$ $M \to 1$

$V\uparrow$ as $M\uparrow$ $\therefore T\downarrow$

Look @ Energy eq.

$$C_p T + \frac{V^2}{2} = \text{constant}$$

Subsonic branch $V\uparrow$ as $M\uparrow$ $\rho\downarrow$

T must go down to keep this eq constant

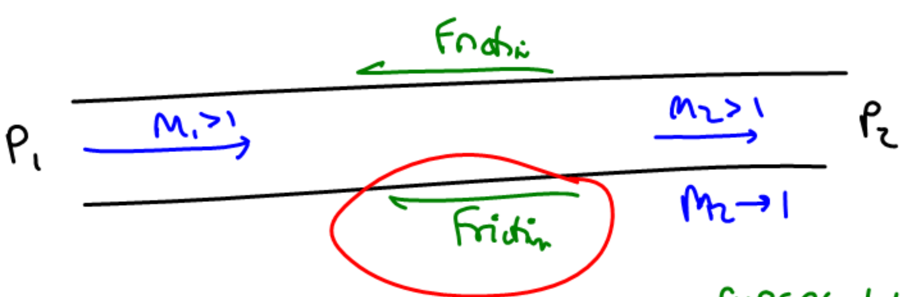
mass $\rho V = \text{const}$ so, $\rho\downarrow$ as $V\uparrow$

ideal gas $\frac{P}{\rho T} = \text{const}$ Subsonic branch $T\downarrow$ $\rho\downarrow$

$\therefore \boxed{P\downarrow}$

Supersonic $T\uparrow$, $\rho\uparrow$

$\therefore \boxed{P\uparrow}$



$P_2 > P_1$!

SUPERSONIC STRANGENESS !

Mom eq. (7)

$$\boxed{P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2 + \frac{F_{fric}}{A}}$$

$$(P_1 - P_2) + \underbrace{\rho_1 V_1}_{\text{const}} V_1 - \underbrace{\rho_2 V_2}_{\text{const}} V_2 = \frac{F_{fric}}{A}$$

mass $\rho V = \text{constant}$

$\therefore \rho_1 V_1 = \rho_2 V_2 = \text{const}$

rewrite as

$$\text{const}(V_1 - V_2) + (P_1 - P_2) = \frac{F_{fric}}{A}$$

Signs: $+$ | can be either $+$ or $-$ | ALWAYS $+$

Supersonic branch of Fanno: $V\downarrow$, $M\downarrow$

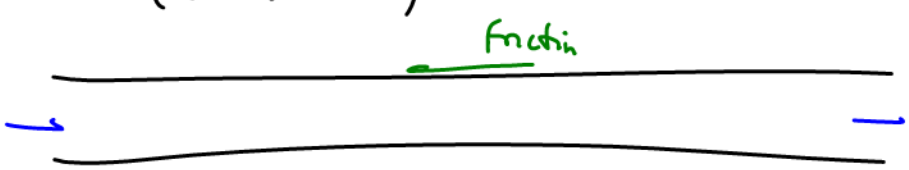
IF V_1 is much greater than V_2, first term dominates the second term

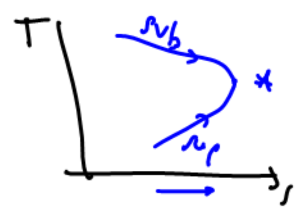
$\therefore P_1 - P_2$ can be $-$

THIS IS THE CASE

$\boxed{P\downarrow \quad V\uparrow}$

SUMMARY (QUALITATIVE)





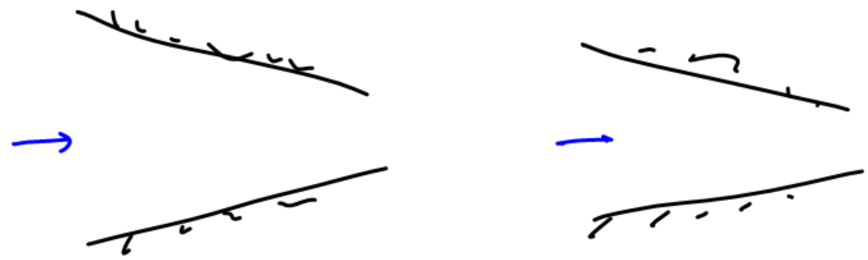
PROPERTIES	SUBSONIC	SUPERSONIC
V	$\uparrow$	$\downarrow$
M	$\uparrow$ $(\rightarrow 1)$	$\downarrow$ $(\rightarrow 1)$
s	$\uparrow$	$\uparrow$
T (i.h)	$\downarrow$	$\uparrow$
ρ	$\downarrow$	$\uparrow$
P	$\downarrow$	$\uparrow$
P_0	$\downarrow$	$\downarrow$
T_0	const	const

SAME FOR SUB & SUPER (others are opposite)

ADIABATIC $\rightarrow$ T_0

T, P, ρ always go together

COMMENT: VERY SIMILAR BEHAVIOR FOR ISENTROPIC CONVERGING DUCT



SUBSONIC	SUPERSONIC
$V \uparrow$	$V \downarrow$
$M \uparrow$	$M \downarrow$
$T \downarrow$	$T \uparrow$
$\rho \downarrow$	$\rho \uparrow$
$P \downarrow$	$P \uparrow$
s const	s const
T_0 const	T_0 const
P_0 const	P_0 const

NEXT TIME — FINISH EQUATIONS FOR FANNO

— SIMILAR COMPARISON OF PROPERTIES

- Variable area duct
- Normal Shock
- Rayleigh flow
- Fanno flow