Today, we will:

- Compare all the 1-D flows we have discussed qualitatively.
- Begin to discuss Fanno flow *quantitatively*: manipulate the equations to get them in a form applicable to the solution of Fanno flow problems

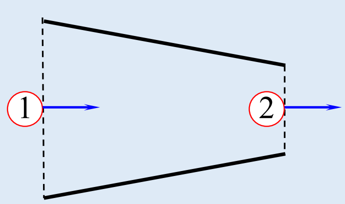
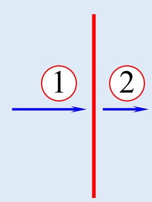
Qualitative Comparison of One-Dimensional Flows:

See Handout – all on one page (here are the first two flows)

Qualitative Comparison: Property Changes in Various One-Dimensional Compressible Flows

KNOW THESE!

Prepared by Professor John M. Cimbala, Penn State University. Latest revision: 13 November 2019

	Converging duct	Normal shock
Flow → Change →	 Area change	 Shock
Cons. Eqns.		
Mass	$\rho_1 V_1 A_1 = \rho_2 V_2 A_2$	$\rho_1 V_1 = \rho_2 V_2$
Momentum	Used only if need to know force on duct	$P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2$
Energy	$T_{01} = T_{02}$	$T_{01} = T_{02}$

Property ↓	Subsonic	Supersonic	Subsonic	Supersonic
V	↑	↓	Not Applicable	↓
M	↑	↓		↓
s	No change	No change		↑
T	↓	↑		↑
ρ	↓	↑		↑
P	↓	↑		↑
P_0	No change	No change		↓
T_0	No change	No change		No change

P, T, ρ go together (all 3 ↑ or all 3 ↓)

See Handout – all on one page (here are the second two flows)

Qualitative Comparison: Property Changes in Various One-Dimensional Compressible Flows

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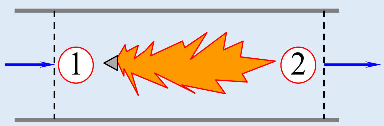
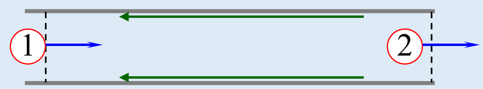
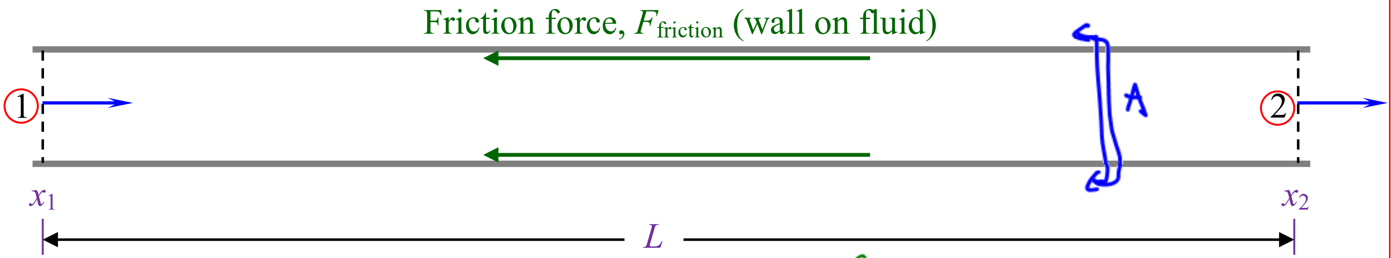
	Rayleigh flow (with heat *addition*)		**Fanno flow**	
Flow →				
Change →	Heat added, $\dot{Q}$		Friction on walls	
Cons. Eqns.				
Mass	$\rho_1 V_1 = \rho_2 V_2$		$\rho_1 V_1 = \rho_2 V_2$	
Momentum	$P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2$		$P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2 + \frac{F_{\text{friction}}}{A}$	
Energy	$q = \frac{\dot{Q}}{\dot{m}} = c_P\left(T_{02} - T_{01}\right)$		$T_{01} = T_{02}$	
Property ↓	**Subsonic**	**Supersonic**	**Subsonic**	**Supersonic**
V	↑	↓	↑	↓
M	↑	↓	↑	↓
s	↑	↑	↑	↑
T	↑ then ↓	↑	↓	↑
ρ	↓	↑	↓	↑
P	↓	↑	↓	↑
P_0	↓	↓	↓	↓
T_0	↑	↑	No change	No change

Quantitative Analysis of Fanno Flow

Review from previous lecture: Conservation equations for Fanno flow:



mass

$$\rho_1 V_1 = \rho_2 V_2 \qquad \text{or} \qquad \rho V = \text{constant} \quad (1)$$

energy

$$T_{01} = T_{02} \qquad \text{or} \qquad c_P T_1 + \frac{V_1^2}{2} = c_P T_2 + \frac{V_2^2}{2} \quad (2)$$

$$P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2 + \frac{F_{\text{friction}}}{A} \quad (3)$$

mom

A = cross-sectional area

state eq

+ A's needed: T-ds eq. Ideal gas $\frac{T_0}{T}, \frac{P_0}{P}$ eqs @ 1 point

$$\boxed{\frac{F_{fric}}{A} = \frac{\int_{x_1}^{x_2} \tau_w \cdot \text{perimeter} \cdot dx}{A} = \frac{\text{perimeter}}{A}\int_{x_1}^{x_2} \tau_w \, dx} \quad (4)$$

Recall, $\boxed{f = \text{Darcy friction factor} = \frac{8\tau_w}{\rho V^2}}$ V = avg speed

$$\boxed{\frac{F_{fric}}{A} = \frac{1}{2D_h}\int_{x_1}^{x_2} f \rho V^2 \, dx} \quad (5)$$

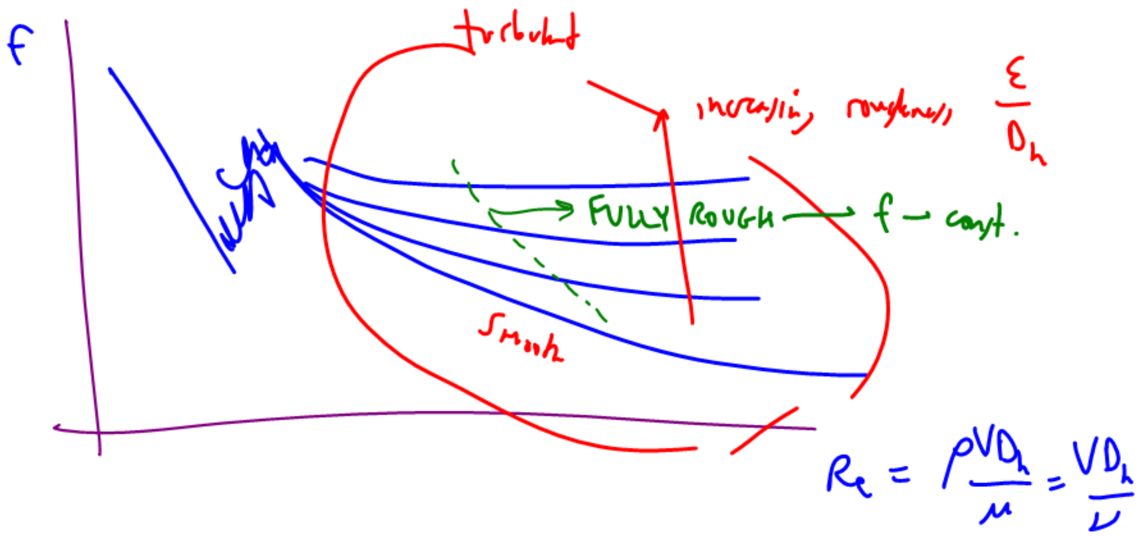
need f

D_h = Hydraulic diameter $\boxed{D_h = \frac{4A}{\text{perimeter}}}$ = D for a round duct

Round duct $A = \frac{\pi D_h^2}{4}$ Perimeter = πD_h

Consider only
- Long pipes ⇒ Fully Developed
- Fast flows (since compressible) ⇒ Turbulent

Recall <u>Moody Chart</u>



- Assume $f \approx$ constant

Major Losses Friction Factor (for losses in long straight sections of pipe or duct):

Graphical estimate – the Moody Chart: [Figure from Cengel and Cimbala, *Fluid Mechanics: Fundamentals and Applications*, Ed. 4.]

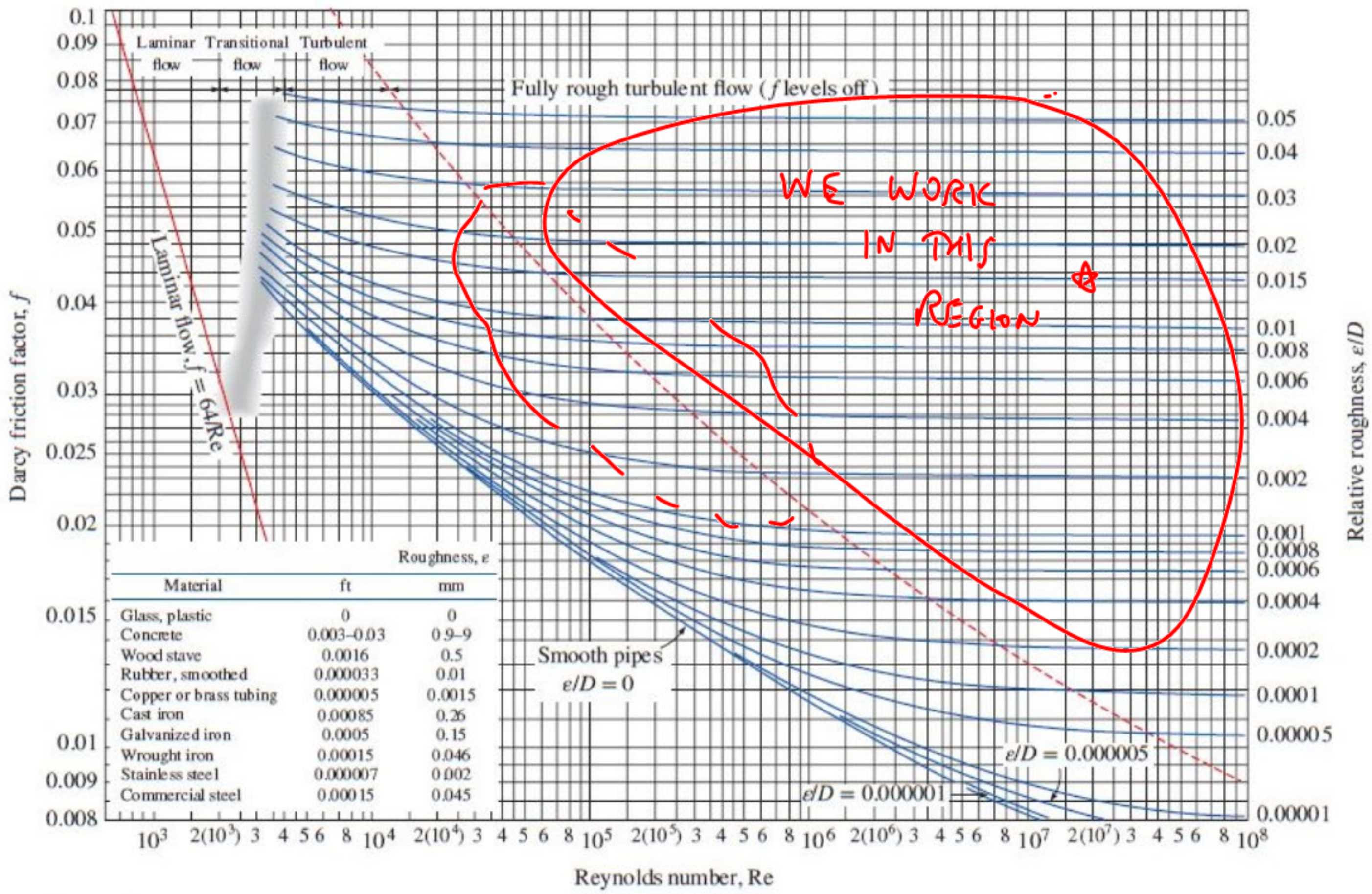


FIGURE A–12

implicit to find f

The Moody chart for the friction factor for fully developed flow in circular pipes for use in the head loss relation $h_L = f\frac{L}{D}\frac{V^2}{2g}$. Friction factors in the turbulent flow are evaluated from the Colebrook equation $\frac{1}{\sqrt{f}} = -2\log_{10}\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$.

WE WILL USE $\boxed{f = \text{fn}(\text{Re}, \varepsilon/D)}$ ✓ EXPLICIT IN f

Empirical equation – the Churchill Equation: $f = 8\left[\left(\frac{8}{\text{Re}}\right)^{12} + \left(A+B\right)^{-1.5}\right]^{\frac{1}{12}}$,

where $A = \left\{-2.457\cdot\ln\left[\left(\frac{7}{\text{Re}}\right)^{0.9} + 0.27\frac{\varepsilon}{D}\right]\right\}^{16}$, $B = \left(\frac{37530}{\text{Re}}\right)^{16}$, $\text{Re} = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$ — use D_h if not round

and f is the **Darcy friction factor**, $h_{L,\text{major}} = f\frac{L}{D}\frac{V^2}{2g}$.

Note: Use **hydraulic diameter** D_h everywhere in place of D when the duct is not round.

Back to
$$\boxed{\frac{F_{fric}}{A} = \frac{f}{2D_h}\int_{x_1}^{x_2} \rho V^2\,dx}$$

I moved f outside the integral since f = const

Plug into (3) (mom. eq.)

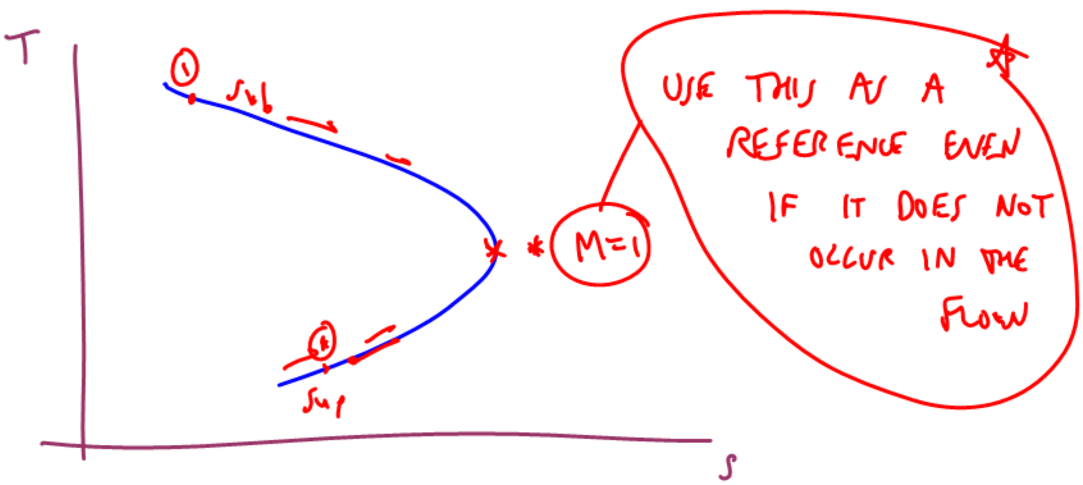
$$\boxed{P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2 + \frac{f}{2D_h}\int_{x_1}^{x_2} \rho V^2\,dx}$$

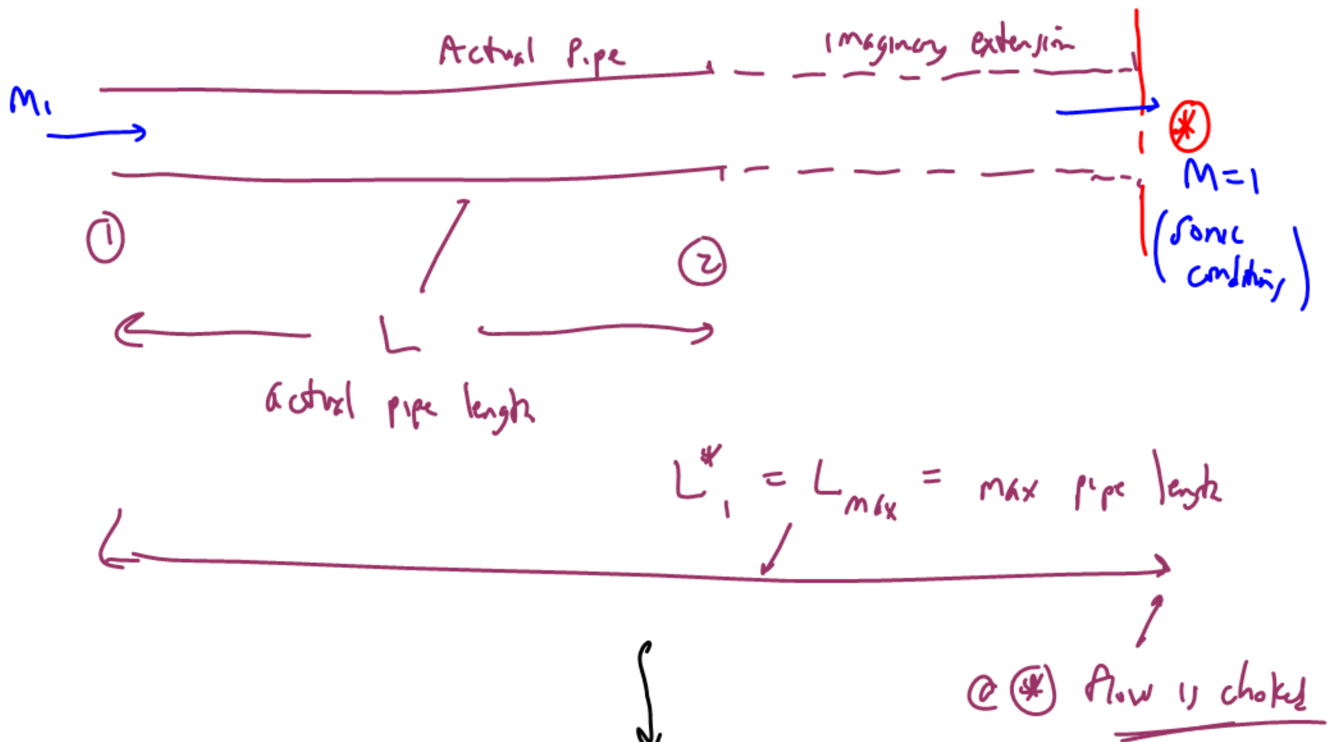
ρ changes V changes (Fanno flow)

Solve for

$$\boxed{f\frac{(x_2 - x_1)}{D_h} = \left[-\frac{1}{\gamma M^2} - \frac{\gamma+1}{2\gamma}\ln\left(\frac{M^2}{1+\frac{\gamma-1}{2}M^2}\right)\right]_{M_1}^{M_2}} \quad (6)$$

- Eq (6) holds between any 2 locations x_1 & x_2 in Fanno flow





let $x_2 = L^*_1$; $M_2 = 1$ in Eq (6)

$$\boxed{\frac{fL^*_1}{D_h} = \frac{1-M_1^2}{\gamma M_1^2} + \frac{\gamma+1}{2\gamma}\ln\left[\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right]} \qquad (7)$$

(nondimen Fanno flow parameter)

Workhorse Eq for Fanno flow

Procedure for a simple Fanno flow problem: (eg. HW 9)

Given: M_1, L, D_h, f

To do: Calc. L_{max} to just choke the flow

Soln: set $L = L_{max} = L^*_1$ = choked → USE EQ (7)

$$\boxed{L_{max} = \frac{fL_{max}}{D_h}\,\frac{D_h}{f}}$$

(EQ 7)