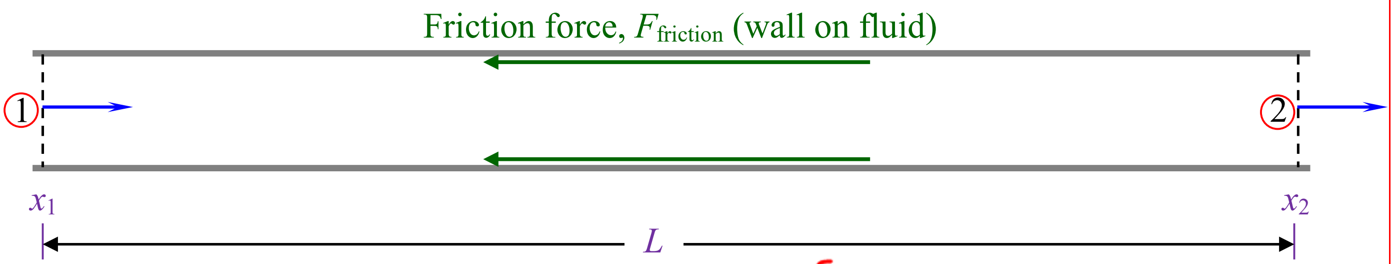
Today, we will:

- Continue to discuss Fanno flow *quantitatively*: manipulate the equations to get them in a form applicable to the solution of Fanno flow problems
- Generate a procedure for solving general Fanno flow problems
- Do some example problems – Fanno flow
- Do Candy Questions for Candy Friday

Review from previous lecture, Fanno flow (ideal gas):



mass

$$\boxed{\rho_1 V_1 = \rho_2 V_2 \quad \text{or} \quad \rho V = \text{constant}} \quad (1)$$

Energy

$$\boxed{T_{01} = T_{02} \quad \text{or} \quad c_P T_1 + \frac{V_1^2}{2} = c_P T_2 + \frac{V_2^2}{2}} \quad (2)$$

$$\boxed{P_1 + \rho_1 V_1^2 = P_2 + \rho_2 V_2^2 + \frac{F_{\text{friction}}}{A}} \quad (3) \qquad \boxed{\frac{F_{\text{friction}}}{A} = \frac{\text{perimeter}}{A}\int_{x_1}^{x_2} \tau_w dx} \quad (4) \quad \text{where} \quad \boxed{f = \frac{8\tau_w}{\rho V^2}}$$

Momentum

We defined hydraulic diameter $\boxed{D_h = \dfrac{4A}{\text{perimeter}}}$, thus $\boxed{\dfrac{F_{\text{friction}}}{A} = \dfrac{1}{2D_h}\displaystyle\int_{x_1}^{x_2} f \rho V^2 dx}$ (5)

USE CHURCHILL EQ

$f = f(Re, \frac{\varepsilon}{D_h})$

After plugging (5) into (3) and doing a *lot* of algebra,

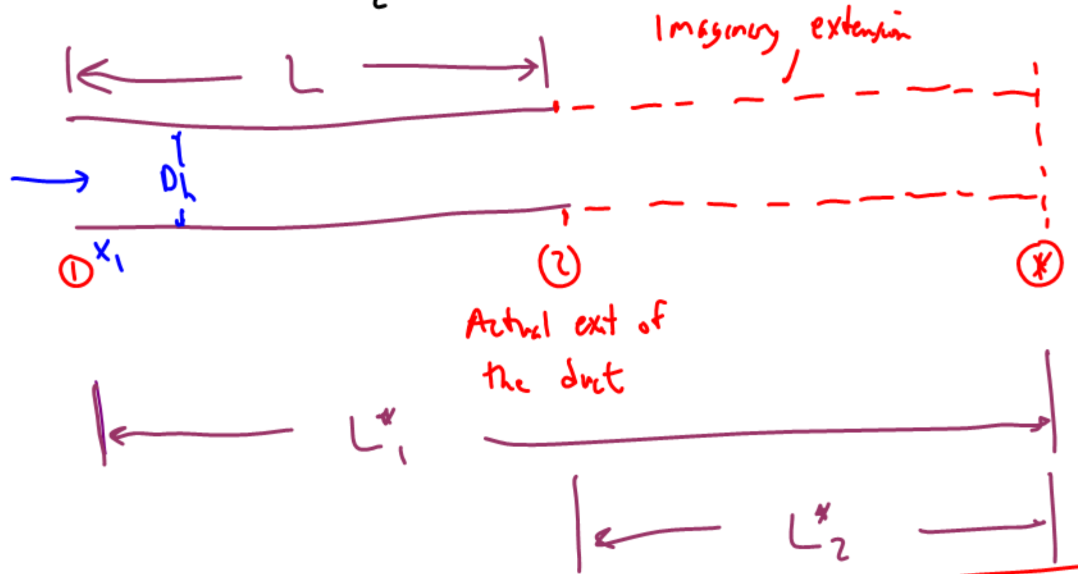
$$\boxed{\frac{f}{D_h}(x_2 - x_1) = \left[-\frac{1}{\gamma M^2} - \frac{\gamma+1}{2\gamma}\ln\left(\frac{M^2}{1-\frac{\gamma-1}{2}M^2}\right)\right]_{M_1}^{M_2}} \quad (6)$$

Finally, consider the **choked case**, where $\boldsymbol{x_2 = L^*_1}$ and set $\boldsymbol{M_2 = 1}$ since the flow is choked at the exit. After a *little* more algebra, Eq. (6) becomes

$$\boxed{\frac{fL^*_1}{D_h} = \frac{1-M_1^2}{\gamma M_1^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right)} \quad (7)$$

Use as a reference even if * does not occur in the flow.

Define L_1^* & L_2^*



f & D_h are constants

$$\boxed{L = L_1^* - L_2^*} \Rightarrow \boxed{\frac{fL}{D_h} = \left(\frac{fL^*}{D_h}\right)_1 - \left(\frac{fL^*}{D_h}\right)_2}$$

☆ KEY

Eq (7) @ any location is

$$\boxed{\frac{fL^*}{D_h} = \frac{1-M^2}{\gamma M^2} + \frac{\gamma+1}{2\gamma}\ln\left[\frac{(\gamma+1)M^2}{2+(\gamma-1)M^2}\right]} \quad (8)$$

☆ workhorse eq.

can apply @ 1, 2, anywhere

☆ Tabulated in the C.A.C. online

E.g. @ $M = 0.3$ & $\gamma = 1.40 \rightarrow (8) \rightarrow \boxed{5.2993} = \frac{fL^*}{D_h}$ @ $M = 0.3$

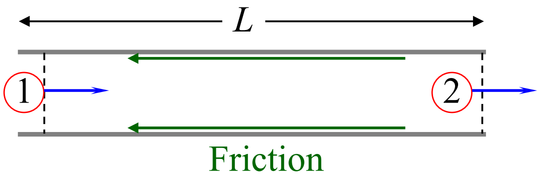
$$\boxed{\text{CAC's } \frac{4fL^*}{D} = \text{same as our } \frac{fL^*}{D_h}}$$ ☆

IGNORE THIS 4

E.g., $M = 2.0 \rightarrow \boxed{\frac{fL^*}{D_h} = 0.305 \quad @ \quad M = 2.0}$

Example: Fanno flow (simple case)

Given: Air at $M_1 = 0.300$ enters a well-insulated (adiabatic) duct of length L and hydraulic diameter 0.250 m. The average Darcy friction factor through the length of the pipe is 0.0230. $= f$



To do: Estimate the maximum possible duct length that will not affect the inlet conditions.

Solution:

Assumptions and Approximations (consistent with our simplified Fanno flow analysis):

1. The air is an ideal gas, and the properties do not change with temperature or pressure.
2. The flow is steady and one-D.
3. The flow is adiabatic but friction along the tube walls is *not* negligible.
4. The Darcy friction factor f is approximated as constant based on conditions at the inlet.

First we calculate

$$\frac{fL_1^*}{D_h} = \frac{1-M_1^2}{\gamma M_1^2} + \frac{\gamma+1}{2\gamma}\ln\left(\frac{(\gamma+1)M_1^2}{2+(\gamma-1)M_1^2}\right) = 5.2993$$

To be completed in class.

$$L_{max} = L_1^* = \frac{5.2993\, D_h}{f} = \frac{(5.2993)(0.250\ \text{m})}{0.0230}$$

$$\boxed{L_{max} = 57.6\ \text{m}}$$

★ PROCEDURE FOR GENERAL FANNO FLOW

Given

@ ① Know $P_1, T_1, (M_1 \text{ or } V_1)$ → can calc $\rho_1, a_1, \ldots$, (ε known) (D_h known)

To do: Calc flow properties @ ②

Sol'n: Assume $L < L_1^*$ (not choked)

PROCEDURE:

(1) Calc μ_1 @ T_1 USE SUTHERLAND'S LAW (on eq sheet)

Calc $\boxed{Re_1 = \frac{\rho_1 V_1 D_h}{\mu_1}}$ & $\frac{\varepsilon}{D_h}$

Use Churchill Eq. to get f_1

(2) Use Eq (8) @ ① to get $\left(\frac{fL^*}{D_h}\right)_1$

(3) Use our "key" Eq. to get

$$\boxed{\left(\frac{fL^*}{D_h}\right)_2 = \left(\frac{fL^*}{D_h}\right)_1 - \left(\frac{fL}{D_h}\right)}$$

(4) Use Eq (8) again to get M_2 INVERSELY

Implicit eq

choose subsonic answer

(5) Calc T_2 recall $T_0 = \text{const}$ (adiabatic)

$$\boxed{T_2 = \frac{T_2}{T_0}\frac{T_0}{T_1}T_1} = \left(1 + \frac{\gamma-1}{2}M_2^2\right)^{-1}\left(1 + \frac{\gamma-1}{2}M_1^2\right)T_1$$

(6) Calc. other properties @ (2)

$$\boxed{a_2 = \sqrt{\gamma R T_2}}$$

$$\boxed{V_2 = a_2 M_2}$$

$$\boxed{\rho_2 = \frac{\rho_1 V_1}{V_2}}$$

$$\boxed{P_2 = \rho_2 R T_2}$$

$$\boxed{\frac{P_{0_2}}{P_2} = \left(1 + \frac{\gamma-1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}}}$$

(M=1 @ *)

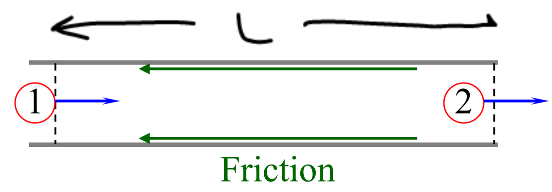
OR

$$\frac{T}{T^*} = \frac{T}{T_0}\frac{T_0}{T^*} = \left(1+\frac{\gamma-1}{2}M^2\right)^{-1}\left(1+\frac{\gamma-1}{2}(1)^2\right)$$

$$\boxed{\frac{T}{T^*} = \frac{\gamma+1}{2+(\gamma-1)M^2}}$$

Similarly for $\frac{\rho^*}{\rho}$ $\frac{p}{p^*}$ etc.

Example: Fanno flow



Given:

L

- Air enters a 5.0-cm diameter, 27.0-m long tube at 450 K, 220 kPa, and 85.0 m/s. The average roughness height of the inside wall of the pipe is 0.08 mm. $= \varepsilon$
- The pipe is well-insulated, but we need to be concerned about friction in the pipe since it is so long.

To do: Estimate the temperature, pressure, velocity, and Mach number at location 2.

Solution:

Assumptions and Approximations (consistent with our simplified Fanno flow analysis):

1 ~~5~~. The air is an ideal gas, and the properties do not change with temperature or pressure.
2 ~~6~~. The flow is steady and one-D.
3 ~~7~~. The flow is adiabatic but friction along the tube walls is *not* negligible.
4 ~~8~~. The Darcy friction factor f is approximated as constant based on conditions at the inlet, and the Churchill equation is used to calculate f.

To be completed in class.

PROCEDURE:

(1) @ ① $M_1 = \frac{V_1}{a_1} = \boxed{0.200 = M_1}$

Sutherland $\mu_1 = 2.499 \times 10^{-5} \ \frac{kg}{m \cdot s}$ @ T_1

$$Re_1 = \frac{\rho_1 V_1 D_h}{\mu_1} \approx 289{,}660$$

With Re_1 & $\frac{\varepsilon}{D_h}$ → Can get $\boxed{f_1 = 0.02296}$ from Churchill Eq.

(2) $\left(\frac{fL^*}{D_h}\right)_1$ from Eq(8) → $\boxed{\left(\frac{fL^*}{D_h}\right)_1 = 14.55}$

(3) "Key" eq.

Actual L = 27.0 m

$$\left(\frac{fL^*}{D_h}\right)_2 = \left(\frac{fL^*}{D_h}\right)_1 - \left(\frac{fL}{D_h}\right)$$

$$= 14.550 - 12.398 = 2.151$$

(4) @ $\left(\frac{fL^*}{D_h}\right)_2 = 2.151 \longrightarrow$ Solve (8) inversely to get $\boxed{M_2 = 0.4090}$

(5) $T_2 = \frac{T_2}{T_0}\frac{T_0}{T_1}T_1 \longrightarrow \boxed{T_2 = 438.9 \text{ K}}$

(6) $a_2 = \sqrt{\gamma R T_2} \longrightarrow \boxed{171.8 \text{ m/s} = a_2}$

This should be V2 not a2 (typo error)

$\rho_2 = \frac{\rho_1 V_1}{V_2} \longrightarrow \boxed{P_2 = \rho_2 R T_2 = 106.2 \text{ kPa}}$

Alternate way: $\frac{T_1}{T^*} = \frac{\gamma+1}{2+(\gamma-1)M_1^2} = 1.1905$

$\frac{T_2}{T^*} = \sim\sim_{M_2} = 1.1611$

$$\boxed{T_2 = \frac{T_2}{T^*}\frac{T^*}{T_1}T_1 = 438.9 \text{ K}}$$

OPTIONAL STEP (7) → since $T_2 \neq T_1$ $\mu_2 \neq \mu_1$ $Re_2 \neq Re_1$

$\therefore f_2 \neq f_1$

we assumed f = constant from ① to ②

Calc f_2 using Churchill @ Re_2 & $\frac{\varepsilon}{D_h}$

I get $\underline{f_2 = 0.02293}$ recall $\underline{f_1 = 0.02296}$

Calc avg. f $\quad f = \dfrac{f_1 + f_2}{2}$

Repeat the analysis