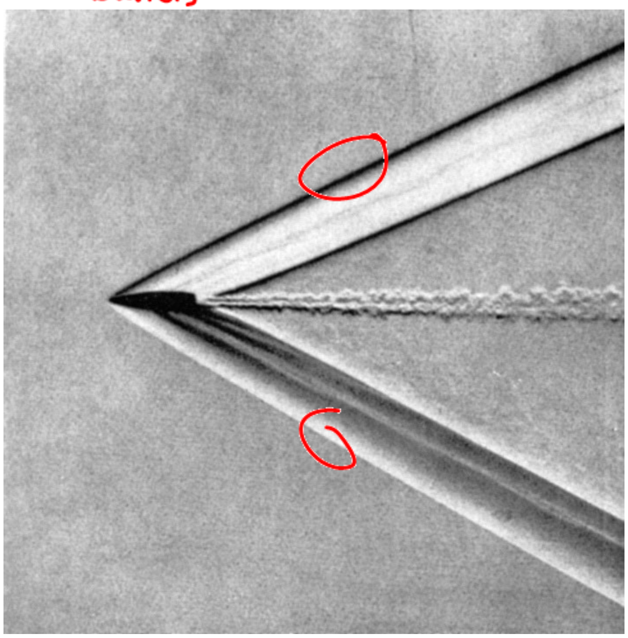
Today, we will: SO FAR – ONE-DIMENSIONAL FLOWS

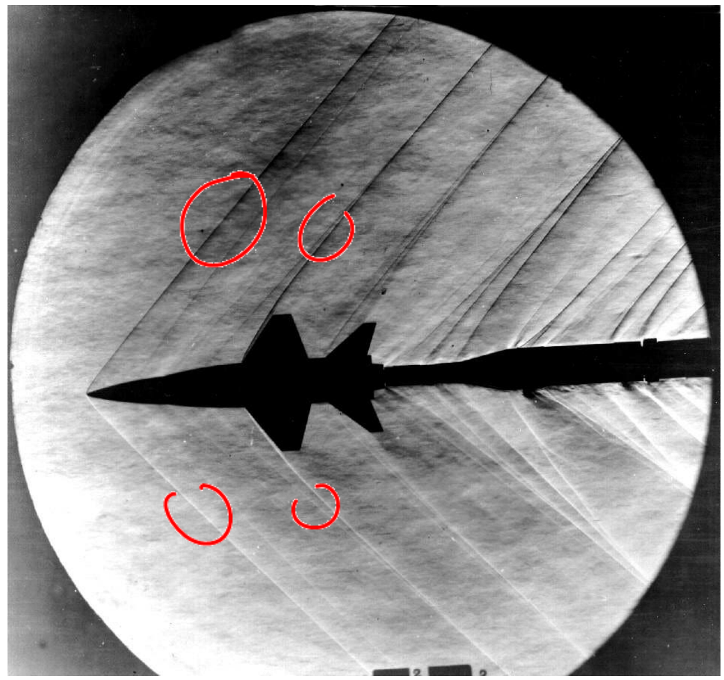
- Discuss ***oblique shocks*** in two-dimensional compressible flow – *qualitatively* first
- Begin to derive the equations for 2-D oblique shocks

Introduction: Oblique shocks

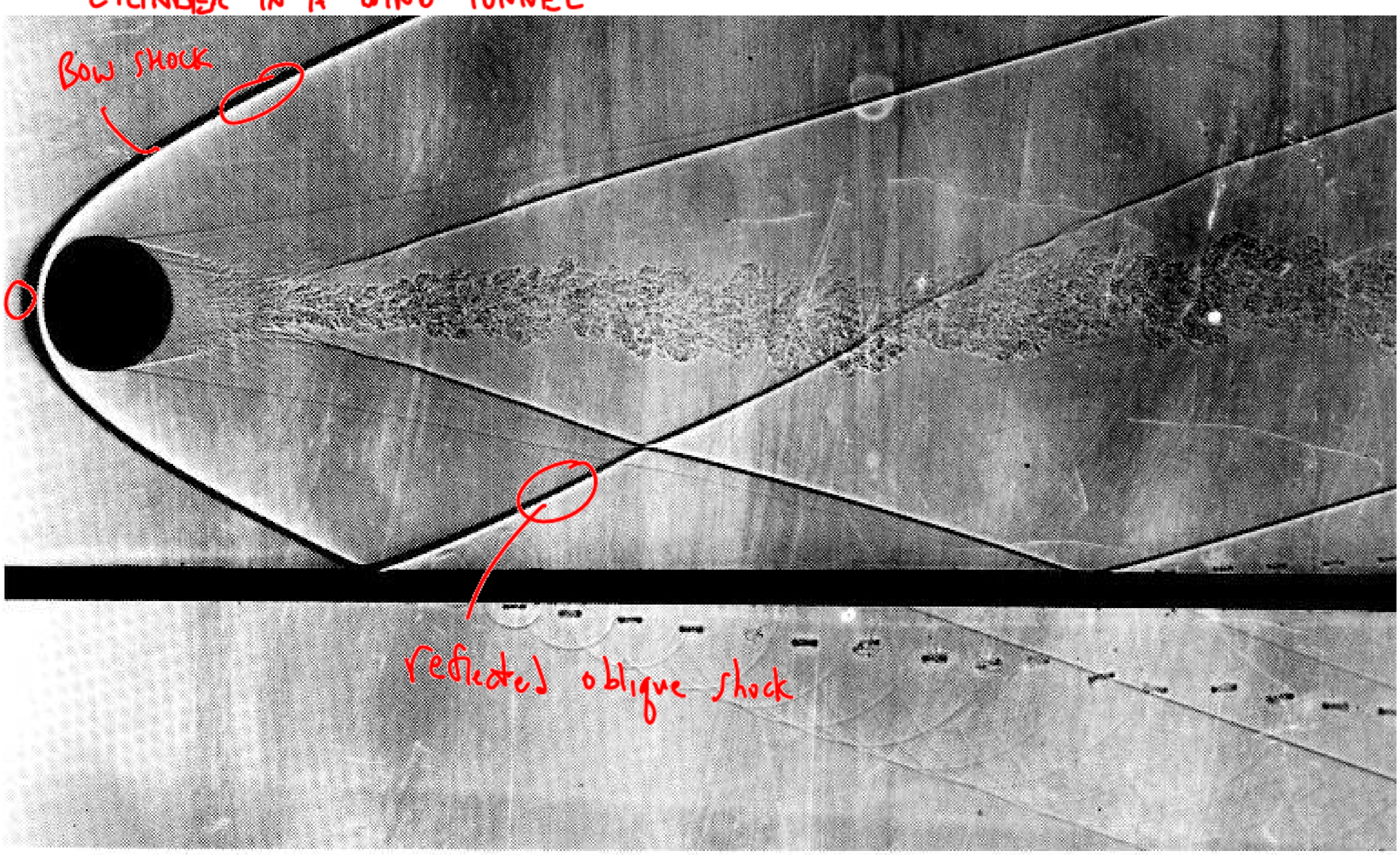
SUPERSONIC AIRCRAFT IN A WIND TUNNEL

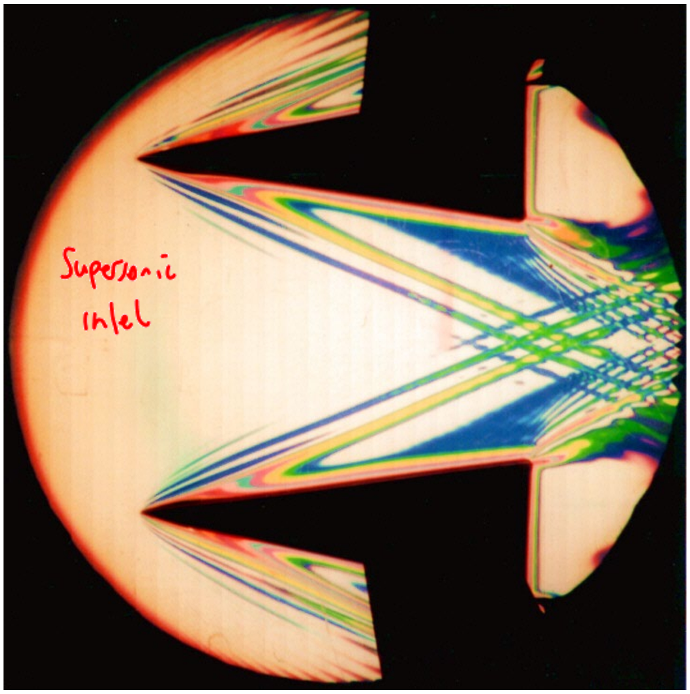
Bullets

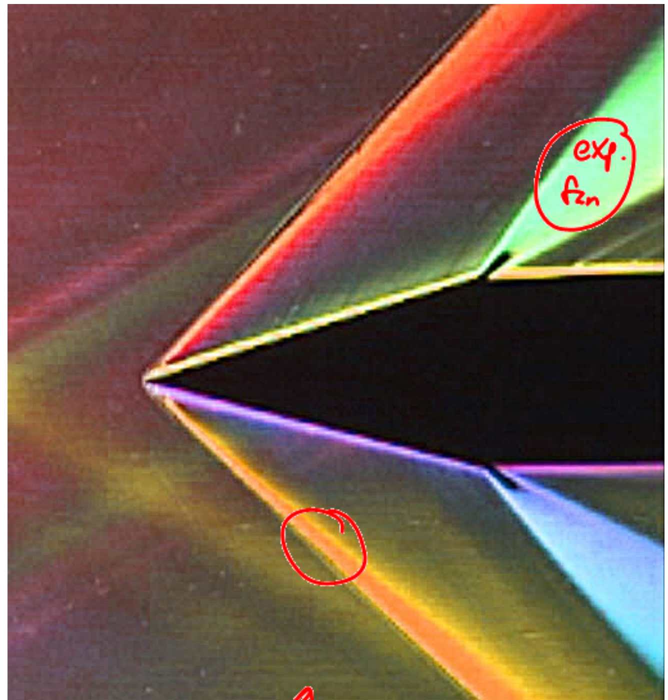




CYLINDER IN A WIND TUNNEL

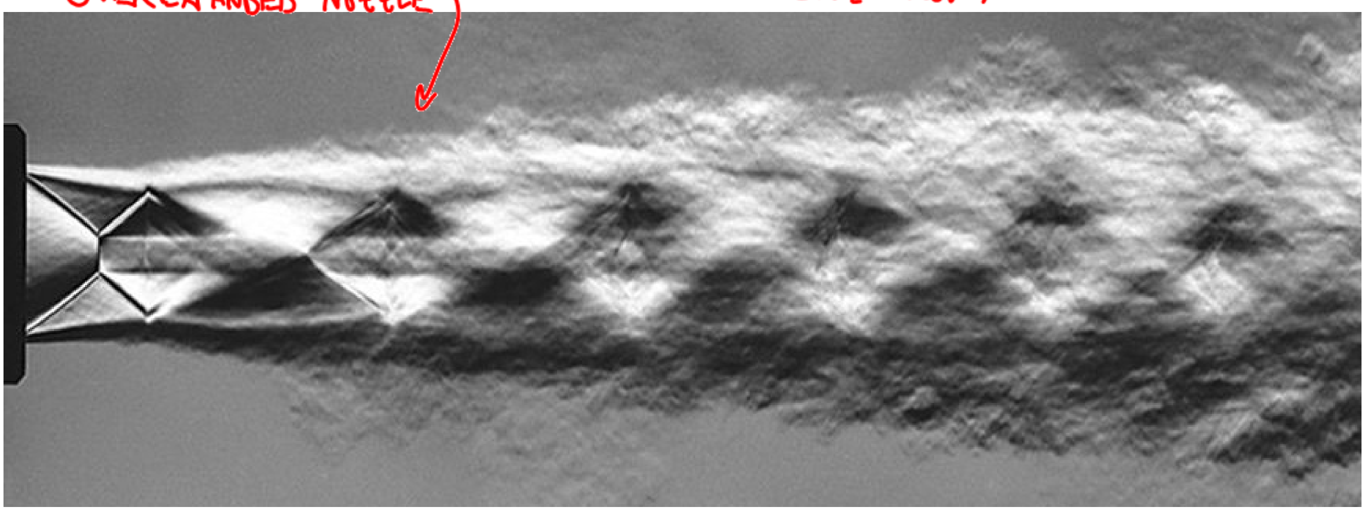


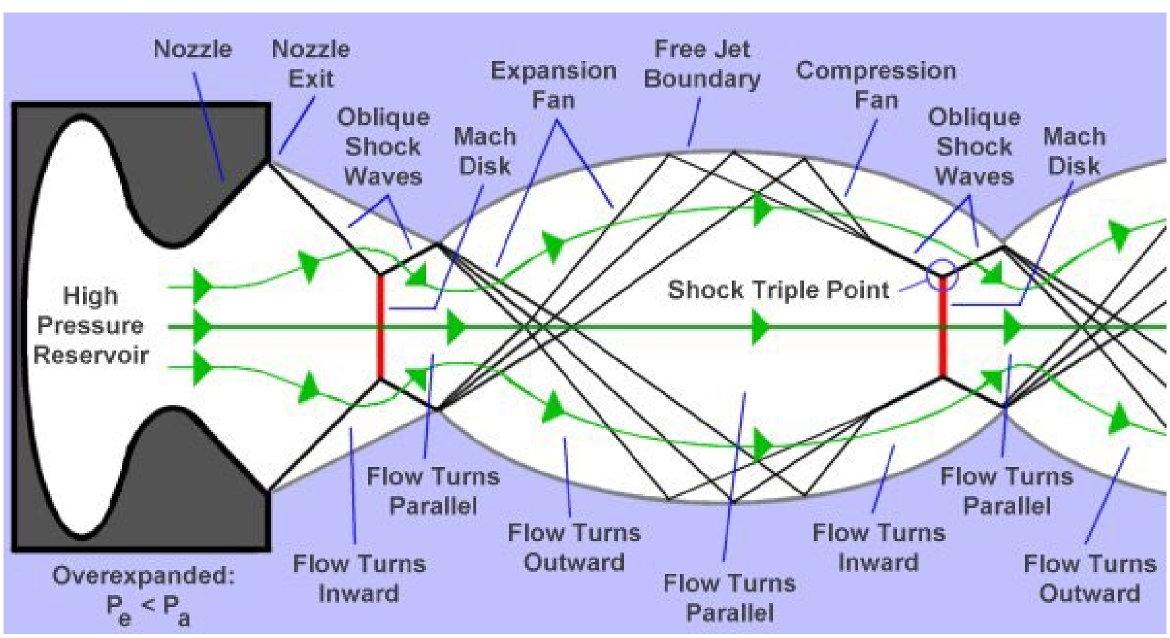




OVEREXPANDED NOZZLE

WEDGE FLOW

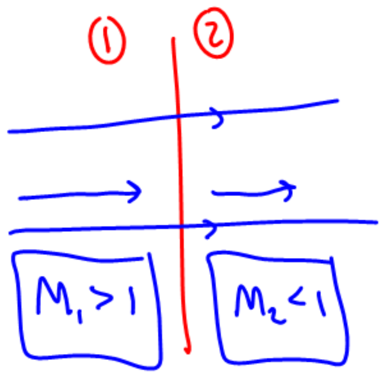




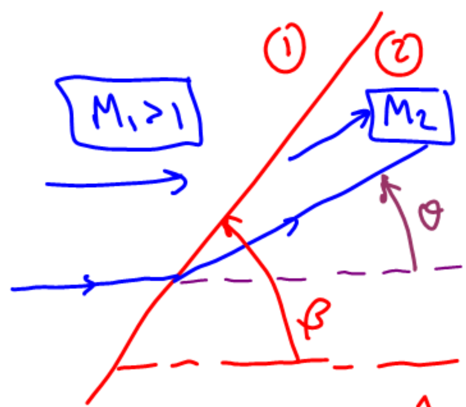
☆ Qualitative analysis of oblique shocks

A:A 2-D, ideal gas, ignore BLs on walls

Normal shock



Oblique shock

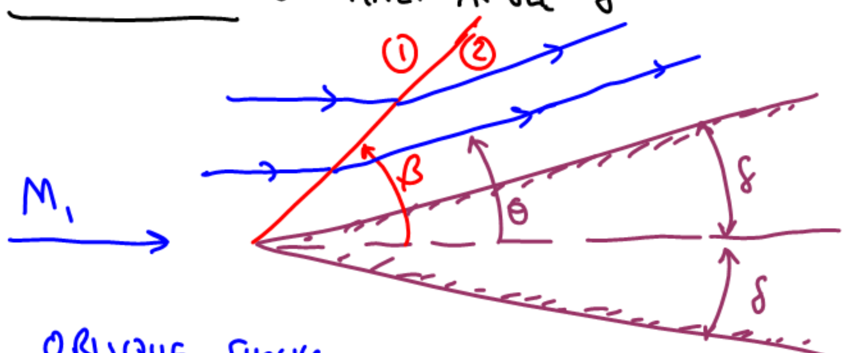


β = shock angle

θ = turning angle ☆

$M_2 < M_1$, M_2 can be < 1 or > 1 ☆

2-D wedge of half-angle δ



Here, $\theta = \delta$

θ = turning angle or deflection angle

β = shock angle

δ = wedge half-angle

Oblique shocks:

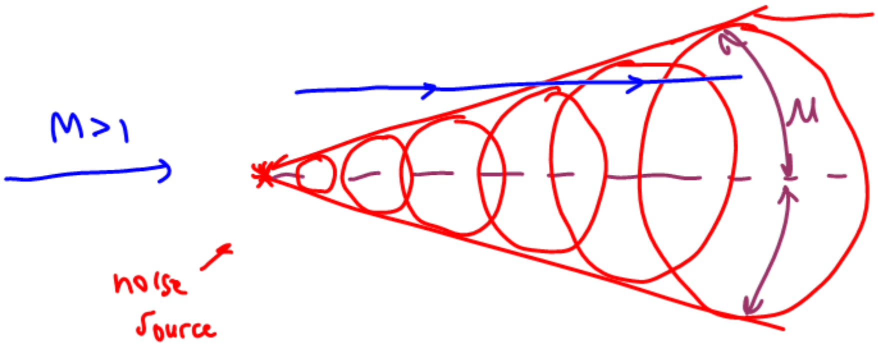
- Similar overall behavior as normal shocks

$$P_2 > P_1 \quad T_2 > T_1 \quad \rho_2 > \rho_1 \quad S_2 > S_1$$

$$T_{0_1} = T_{0_2} \quad P_{0_2} < P_{0_1} \quad M_2 < M_1$$

- All our eq's for normal shocks still apply but we must account for the angles

RECALL, SOUND WAVE IN SUPERSONIC FLOW



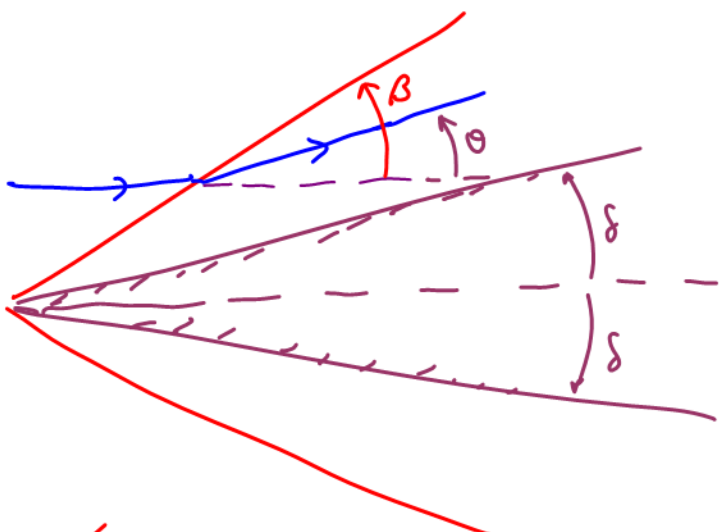
$$\boxed{\mu = \sin^{-1}\frac{1}{M}}$$

= MACH ANGLE

A MACH WAVE IS AN INFINITESIMALLY WEAK OBLIQUE SHOCK!

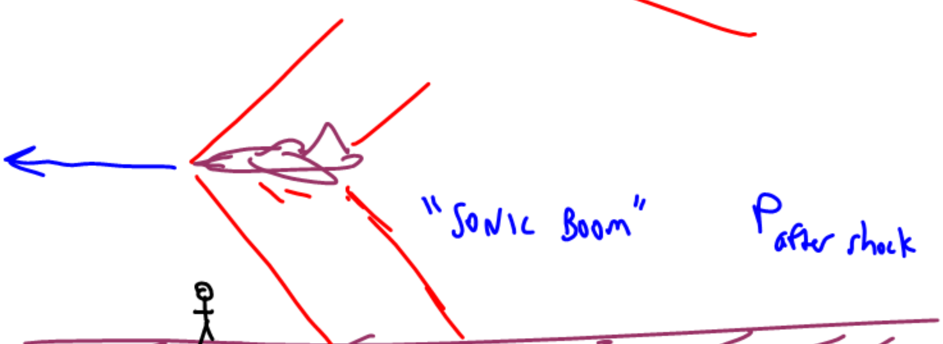
- Isentropic
- θ = turning angle = 0
- $\beta = \mu$

$\underline{M_1 = M_2}$



$$\boxed{\beta > \mu}$$

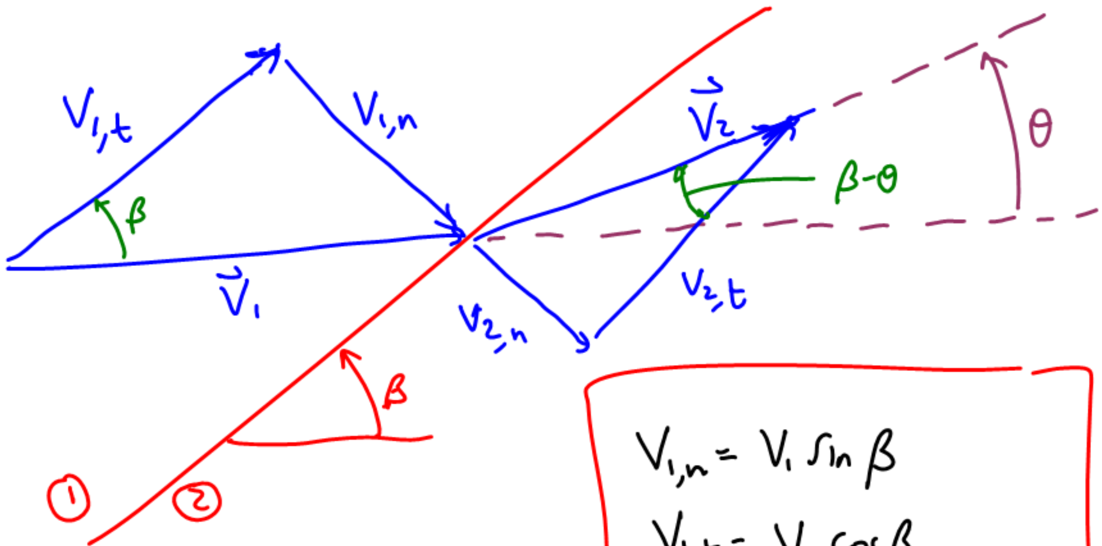
μ is the lower limit of β



"SONIC BOOM"

$P_{\text{after shock}} > P_{\text{before shock}}$

☆ QUANTITATIVE ANALYSIS OF OBLIQUE SHOCK



$$V_{1,n} = V_1 \sin\beta$$

$$V_{1,t} = V_1 \cos\beta$$

$$V_{2,n} = V_2 \sin(\beta-\theta)$$

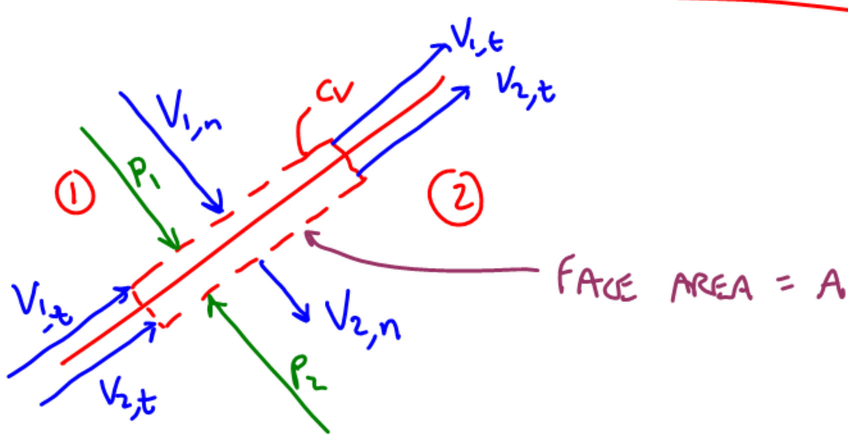
$$V_{2,t} = V_2 \cos(\beta-\theta)$$

KEY $\boxed{V_{1,t} = V_{2,t}}$ ☆

☆ THE TANGENTIAL COMPONENT OF THE OBLIQUE SHOCK IS NOT AFFECTED BY THE SHOCK!

☆ THUS, OUR OLD NORMAL SHOCK EQ'S STILL APPLY, BUT IN TERMS OF THE NORMAL COMPONENT OF VELOCITY

(Not V_1, but $V_{1,n}$)



CONS. OF MASS:

$$\rho_1 V_{1,n} A = \rho_2 V_{2,n} A$$

$$\boxed{\rho_1 V_{1,n} = \rho_2 V_{2,n}} \quad (1)$$

MOMENTUM EQ:

- In tangential direction, nothing changes
- In normal direction,

$$P_1 \cancel{A} - P_2 \cancel{A} = \sum_{out} \dot{m} V_n - \sum_{in} \dot{m} V_n$$

$$= \rho_2 V_{2,n} \cancel{A}\, V_{2,n} - \rho_1 V_{1,n} \cancel{A}\, V_{1,n}$$

$$\star \quad \boxed{P_1 - P_2 = \rho_2 V_{2,n}^2 - \rho_1 V_{1,n}^2} \quad (2)$$

CONS. OF ENERGY

$$\boxed{h_{o_1} = h_{o_2}} \quad \text{or} \quad \boxed{T_{o_1} = T_{o_2}}$$

(ideal gas)

$$\left(h_o = h + \tfrac{1}{2} V^2 \right)$$

$\star$ (TOTAL VELOCITY MAGNITUDE)^2

$V_{1,t} = V_{2,t}$

$$h_{o_1} = \underbrace{h_1 + \tfrac{1}{2} V_{1,n}^2 + \cancel{\tfrac{1}{2} V_{1,t}^2} = h_2 + \tfrac{1}{2} V_{2,n}^2 + \cancel{\tfrac{1}{2} V_{2,t}^2}} = h_{o_2}$$

$$\boxed{h_1 + \tfrac{1}{2} V_{1,n}^2 = h_2 + \tfrac{1}{2} V_{2,n}^2} \quad (3)$$

NORMAL SHOCK EQS

mass: $\rho_1 V_1 = \rho_2 V_2$

mom: $P_1 - P_2 = \rho_2 V_2^2 - \rho_1 V_1^2$

energy: $h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$

$\Longleftrightarrow$

OBLIQUE SHOCK EQS

$\rho_1 V_{1,n} = \rho_2 V_{2,n}$

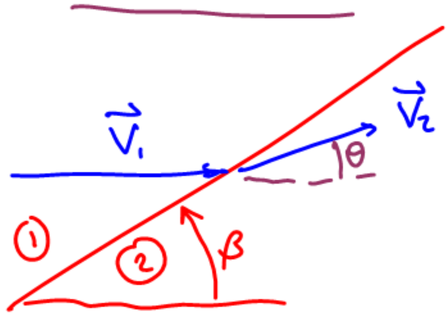
$P_1 - P_2 = \rho_2 V_{2,n}^2 - \rho_1 V_{1,n}^2$

$h_1 + \frac{V_{1,n}^2}{2} = h_2 + \frac{V_{2,n}^2}{2}$

EQS ARE IDENTICAL IF WE REPLACE V_1 BY $V_{1,n}$

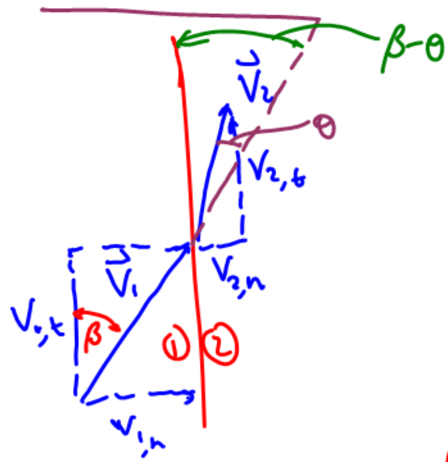
& V_2 BY $V_{2,n}$

STATIONARY F.O.R

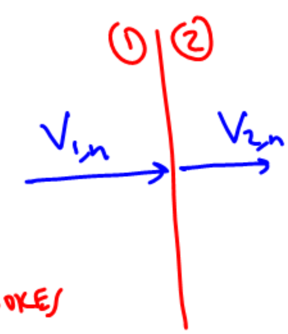


NOW ROTATE COUNTERCLOCKWISE BY ANGLE $90^\circ - \beta$

ROTATED F.O.R.



ROTATED F.O.R. & MOVING UP @ SPEED $V_{1,t}$ (or $V_{2,t}$)



NOW IT LOOKS LIKE A TYPICAL NORMAL SHOCK!

Let $M_{1,n} = \frac{V_{1,n}}{a_1} = \underbrace{\frac{V_1}{a_1}}_{M_1} \sin\beta \Rightarrow \boxed{M_{1,n} = M_1 \sin\beta}$ (4)

$M_{2,n} = \frac{V_{2,n}}{a_2} = \frac{V_2}{a_2}\sin(\beta-\theta) \Rightarrow \boxed{M_{2,n} = M_2 \sin(\beta-\theta)}$ (5)

SO – WE USE THE SAME NORMAL SHOCK EQS WE USED BEFORE EXCEPT USE $M_{1,n}$ instead of M_1

& USE $M_{2,n}$ instead of M_2

e.g., $\frac{P_2}{P_1} = \text{fnc}(M_1, \gamma)$ For a normal shock

$\parallel$

$\frac{P_2}{P_1} = \text{fnc}(M_{1,n}, \gamma)$ For an oblique shock

SAME EQ!

$M_{1,n}$ must be > 1 SUPERSONIC

$M_{2,n}$ must be < 1 SUBSONIC

BUT ACTUAL M_2 can be $<, =, > 1$